HW #1 9/6/11 | Phys 425 | Claire Johnson

Problem 1.7

50/50

a) **Given**: The thermal expansion coefficient, β, of mercury

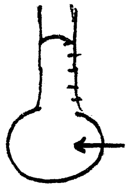
$$\beta \equiv \frac{\Delta V/V}{\Delta T} = \frac{1/550{,}000}{K} = 1.81\times10^{-4}\ K^{-1}$$

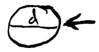
This is a measure of the fractional increase in volume per unit change in temperature

10/10

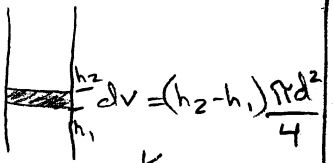
Find: The inside diameter of a mercury thermometer necessary for the thermometer to work. Assume a bulb size.

Solution:





← Inner area = $\frac{\pi d^2}{4}$



$dV = (h_2-h_1)\frac{\pi d^2}{4}$ ✓

Nice detail

Engineer a thermometer with a 0.5cm increase in height/1 degree increase in temperature.

$$\Delta V = (h_2-h_1)\frac{\pi d^2}{4} = \frac{(0.5cm)\pi d^2}{4} \qquad V = 2cm^3 \qquad \Delta T = 1K$$

$$\frac{\Delta V/V}{\Delta T} = \frac{[(0.5cm)\pi d^2/4]/(2cm^3)}{1\ K} = \underline{1.81\times10^{-6}\ K^{-1}}$$

error in problem (given as $1.81\times10^{-4}\ K^{-1}$)

$$d = 0.0030\ cm = \boxed{0.030mm}$$

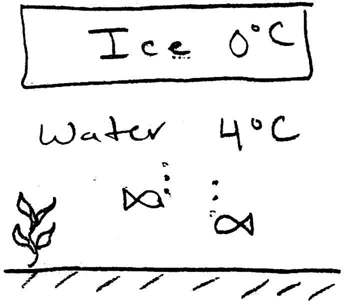
Interpretation: The inner diameter is very small due to the large thermal expansion coefficient of mercury. The response to fairly small changes in temperature is easily captured using a mercury thermometer, explaining why they were so commonly used before the toxic effects of mercury were known. ✓

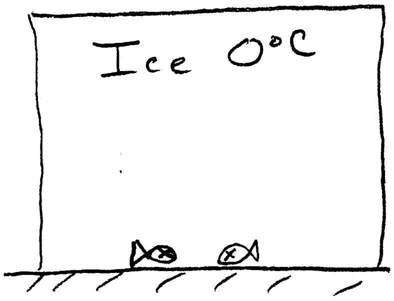
b) **Given**: β for water is $7.5\times10^{-4}\ K^{-1}$ @ 100°C, but decreases to zero at 4°C, only to become slightly negative at lower temperatures ($-0.68\times10^{-4}\ K^{-1}$ @ 0°C).

Find: How would the process of a lake freezing over differ if β were always positive?

Solution: ✓

Current Situation





If β were always positive

Because β for water becomes negative at temperatures lower than 4°C, water colder than 4°C begins to expand and become less dense. Thus during winter the 4°C water sinks while colder water and ice remain on the surface. This helps preserve aquatic life. If β were always positive, lakes would freeze from the bottom up, making them more likely to freeze through.

Problem 1.8

The linear thermal expansion coefficient, α, of a solid is defined as the fractional increase in length per degree change in temperature

$$\alpha \equiv \frac{\Delta L / L}{\Delta T}$$

a) **Given**: steel has an $\alpha = 1.1 \times 10^{-5}\,K^{-1}$

Find: The variation in length on a 1 km bridge between a cold winter night and hot summer day.

Solution: Assume bridge is located in Minneapolis, MN.

Record high = 42°C

Record low = −41°C

$\Delta T = 101\,°C = 101\,K$

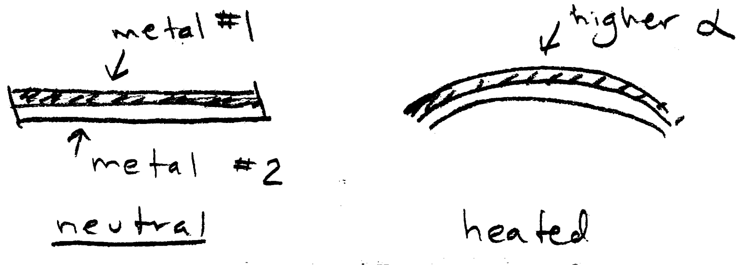
$$\left(\frac{\Delta L}{1000\,m}\right) / 101\,K = 1.1 \times 10^{-5}/K \qquad \Delta L = \boxed{1.1\,m}$$

Interpretation: An expansion that large would require special engineering techniques like expansion joints. ✓ There also could be problems with other materials on the bridge, like asphalt and concrete, expanding at different rates causing cracking and compromising the structure. ✓

b) **Find**: How does the bimetallic strip in a dial thermometer work?

✓

Solution: The two metals used in a dial thermometer have different α values, meaning they expand at different rates. This causes the bimetallic strip to bend, either closing a circuit (as in the temperature indicator ✓ in a rice cooker) or putting pressure on a spring connected to a dial (as in this case).

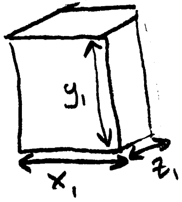


Choosing metals with a large difference in α would make a more sensitive instrument.

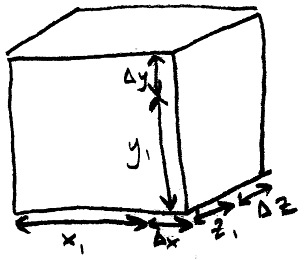
c) Given : A solid with volume expansion coefficient β and linear expansion coefficent α.

Find : Prove that β is equal to the sum of α in three directions

$$\beta = \alpha_x + \alpha_y + \alpha_z$$



$V_1 = (x_1)(y_1)(z_1)$



$y_2 = y_1 + \Delta y$
$x_2 = x_1 + \Delta x$
$z_2 = z_1 + \Delta z$

$$V_2 = (y_2)(x_2)(z_2)$$
$$= (y_1 + \Delta y)(x_1 + \Delta x)(z_1 + \Delta z)$$

$$\alpha_x = \frac{\Delta x / x_1}{\Delta T} \qquad \alpha_y = \frac{\Delta y / y_1}{\Delta T} \qquad \alpha_z = \frac{\Delta z / z_1}{\Delta T}$$

$$\Delta V = V_2 - V_1 = (y_1 x_1 + y_1 \Delta x + x_1 \Delta y + \Delta y \Delta x)(z_1 + \Delta z) - (x_1 y_1 z_1)$$
$$= y_1 z_1 \Delta x + x_1 z_1 \Delta y + z_1 \Delta y \Delta x + y_1 x_1 \Delta z + y_1 \Delta x \Delta z + x_1 \Delta y \Delta z + \Delta x \Delta y \Delta z$$

$$\beta = \frac{\Delta V / V_1}{\Delta T} = \frac{\left(\dfrac{y_1 z_1 \Delta x + x_1 z_1 \Delta y + z_1 \Delta y \Delta x + y_1 x_1 \Delta z + y_1 \Delta x \Delta z + x_1 \Delta y \Delta z + \Delta x \Delta y \Delta z}{x_1 y_1 z_1} \right)}{\Delta T}$$

$$\beta = \frac{\dfrac{\Delta x}{x_1} + \dfrac{\Delta y}{y_1} + \cancelto{0}{\dfrac{\Delta y \Delta x}{x_1 y_1}} + \dfrac{\Delta z}{z_1} + \cancelto{0}{\dfrac{\Delta x \Delta z}{x_1 z_1}} + \cancelto{0}{\dfrac{\Delta y \Delta z}{y_1 z_1}} + \cancelto{0}{\dfrac{\Delta x \Delta y \Delta z}{x_1 y_1 z_1}}}{\Delta T}$$

✱ Assume that the linear changes are very small. Therefore, $(\Delta y \Delta x)$, $(\Delta x \Delta z)$, $(\Delta y \Delta z)$ and $(\Delta x \Delta y \Delta z)$ are $\cong 0$.

$$\beta \cong \frac{\dfrac{\Delta x}{x_1} + \dfrac{\Delta y}{y_1} + \dfrac{\Delta z}{z_1}}{\Delta T} = \frac{\left(\dfrac{\Delta x}{x_1}\right)}{\Delta T} + \frac{\left(\dfrac{\Delta y}{y_1}\right)}{\Delta T} + \frac{\left(\dfrac{\Delta z}{z_1}\right)}{\Delta T} = \alpha_x + \alpha_y + \alpha_z$$

Interpretation - If you assume small linear changes per degree change in temperature, β can be approximated as the sum of the linear expansion coefficients, α, in all three directions ✓

a. What is temperature? Give three definitions.
According to the text, three definitions of temperature are:

1) What you measure with a thermometer.
2) The thing that's the same for two objects when they're in contact long enough.
3) A measure of the tendency of an object to spontaneously give up energy to its surroundings.

In a practical sense, temperature is a measure of the relative heat of an object or system, and is an expression of molecular activity.

b. What is meant by thermal equilibrium? Can we do thermodynamics on systems that are not in thermal equilibrium?
Thermal equilibrium is the tendency of two objects in contact to spontaneously reach the same temperature, given enough time. The object at a higher temperature spontaneously loses energy to the lower temperature object until equilibrium is achieved.

Since thermodynamics concerns the exchange of heat and energy to produce work, it's fair to say that we cannot do thermodynamics on systems that *are* in thermal equilibrium. Once a system is in thermal equilibrium, no energy exchange is taking place (or rather, the exchange is random and bidirectional).

c. What are two mechanisms for objects to use to come to thermal equilibrium?
Two mechanisms for objects to come to equilibrium are thermal convection and radiation. Thermal radiation is the movement of electromagnetic radiation in the form of particles (such as photons). Convection is the movement of molecules within a gas or liquid. In both cases, the bulk of the movement occurs from the object of higher temperature to the object of lower temperature.

d. In your experience, what is the relaxation time necessary for a hot cup of coffee, tea, or water to come to thermal equilibrium with at room at room temperature?
The relaxation time for a cup of coffee is about 30 minutes. This varies if the cup is open or closed like a thermos. The relaxation time for coffee in a thermos can be several hours because the thermos, which contains both foam and a vacuum chamber, is a poor ✓ conductor of heat. ✓

e. What is the most exotic type of thermometer you can think of?
The most exotic thermometer I can think of is the use of Mg/Ca ratios in formanifera shells to evaluate paleotemperatures in oceans. Forams are a class of protists that form tests, or shells, made of calcium carbonate. However, magnesium, which is more plentiful in the ocean than calcium, is readily substituted. This substitution is more likely to occur at higher temperatures. By examining the ratio of magnesium to calcium in formanifera tests, scientists can evaluate past ocean temperatures. The biggest problem with using this proxy measurement for temperature is establishing a calibration. Also, there is less magnesium available in the deep ocean, so different calibrations must be used for surface dwelling and benthic forams. ✓

Excellent!

Keep up the great work -
Very clear!

f. What would you rather touch, a piece of fluffy carpet or a large piece of aluminum, if both are at 353 K? Why?

I would rather touch the carpet because it is probably less conductive than the aluminum. Because the molecules in fluffy carpet are less densely packed, it would probably take longer to conduct heat from the carpet to your hand. The only exception would be if the melting point of fluffy carpet were less than 353 K (this is some seriously cheap carpet), in which case the liquid carpet might be a better conductor.

agree