3.7) Calculate the temperature of a black hole, in terms of its mass M. Evaluate the resulting expression for a one solar mass black hole. Also sketch the entropy as a function of energy, and discuss the shape + implications.

a) From 2.42 $\quad S = \frac{8\pi^2 G M^2}{hc} k \qquad U = Mc^2$

$$T = \left.\frac{\partial U}{\partial S}\right)_{NV} = \frac{\partial U}{\partial M} \cdot \left(\frac{\partial S}{\partial M}\right)^{-1} = c^2 \left(\frac{16\pi^2 G M}{hc} k\right)^{-1}$$

$$\boxed{T = \frac{hc^3}{16\pi^2 G M k}}$$

$h = 6.26 \times 10^{-34}$ J·s
$c = 3 \times 10^{8}$ m/s
$G = 6.67 \times 10^{-11}$ N m^2/kg^2
$M = 1.99 \times 10^{30}$ kg
$k = 1.38 \times 10^{-23}$ J/K

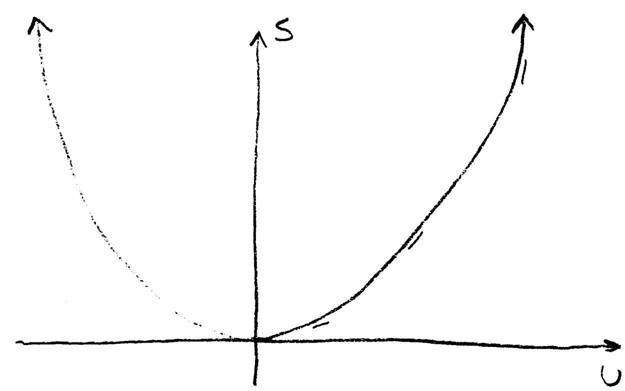
b)
$$T = \frac{(6.26\times10^{-34})(3\times10^{8})^3}{16\pi^2(6.67\times10^{-11})(1.99\times10^{30})(1.38\times10^{-23})} = \boxed{5.8\times10^{-8}\ K}$$

Check units

$$\frac{\cancel{kg}\,\cancel{m^2}\,\cancel{s}}{\cancel{s^2}} \cdot \frac{\cancel{m^3}}{\cancel{s^3}} \cdot \frac{\cancel{kg}\cdot \cancel{s^2}}{\cancel{m^3}} \cdot \frac{1}{\cancel{kg}} \cdot \frac{\cancel{s^2}\,K}{\cancel{kg}\,\cancel{m^2}} = K \quad \checkmark$$

c) @ $M=0 \quad U=0 \quad S=0$
$M\to\infty \quad U\to\infty \quad S\to\infty$
$M\to-\infty \quad U\to-\infty \quad S\to\infty$ (M2 always positive)

unlikely?? mcd?



✓

To gain entropy, a black hole will want to gain energy if U is positive and lose energy if U is negative. As entropy is gained, the slope increases, meaning the temperature decreases. The more massive a black hole, the greater its entropy and lower its temperature.

10/25

Homework 10 | Phys 425 | Claire Johnson 2/2

3.10 An ice cube (mass = 30g) at 0°C is left in a 25°C kitchen until it melts.

a) What is ΔS of the ice cube melting @ 0°C (ignore ΔV)?

$$\Delta S = \frac{\Delta U}{T} = \frac{Lm}{T} = \frac{(333\ J/g)\,30g}{273.15\ K} = 36.6\ J/K$$

b) Calculate the change in the entropy of the water from 0°C to 25°C

$$\Delta S = \int_{T_i}^{T_f} C_v/T\ dT = C_v \ln\left(\frac{T_f}{T_i}\right) = (4.2\ J/g\cdot K)(30g)\ln\left(\frac{298.15}{273.15}\right)$$

$$\Delta S = 11.03\ J/K$$

c) Calculate the change in the entropy of the kitchen

$$\Delta S = \frac{\Delta U}{T} \qquad \Delta U = -\Delta U \text{ to melt + warm ice} = -Q$$

no work done

$$Q = m(C_v\Delta T + L) = 30g\,(4.2\ J/g\cdot K\ (25K) + 333\ J/g)$$

$$Q_{WATER} = 13140\ J \qquad Q_{ROOM} = -13140\ J$$

$$\Delta S = -13140\ J / 298.15\ K = -44.1\ J/K$$

d) $S_{Tot} = 36.6 + 11.03 - 44.1 = +3.53\ J/K$

The net change in entropy is positive, meaning the process will tend to occur.
From what I know about ice melting in a warm room, a positive change in entropy is expected because the disorder of the ice increases as it goes to water.