A NEW MODEL FOR QUANTIFYING CLIMATE EPISODES

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ABSTRACT

When long records of climate (precipitation, temperature, stream runoff, etc.) are available, either from instrumental observations or from proxy records, the objective evaluation and comparison of climatic episodes becomes necessary. Such episodes can be quantified in terms of duration (the number of time intervals, e.g. years, the process remains continuously above or below a reference level) and magnitude (the sum of all series values for a given duration). The joint distribution of duration and magnitude is represented here by a stochastic model called BEG, for 'bivariate distribution with exponential and geometric marginals'. The model is based on the theory of random sums, and its mathematical derivation confirms and extends previous empirical findings. Probability statements that can be obtained from the model are illustrated by applying it to a 2300-year dendroclimatic reconstruction of water-year precipitation for the eastern Sierra Nevada–western Great Basin. Using the Dust Bowl drought period as an example, the chance of a longer or greater drought is 8%. Conditional probabilities are much higher, i.e. a drought of that magnitude has a 62% chance of lasting for 11 years or longer, and a drought that lasts 11 years has a 46% chance of having an equal or greater magnitude. In addition, because of the bivariate model, we can estimate a 6% chance of witnessing a drought that is both longer and greater. Additional examples of model application are also provided. This type of information provides a way to place any climatic episode in a temporal perspective, and such numerical statements help with reaching science-based management and policy decisions. Copyright © 2005 Royal Meteorological Society.

KEY WORDS: climatic change; stochastic hydrology; drought; tree rings; palaeoclimatology; western juniper; Sierra Nevada; Great Basin

1. INTRODUCTION

Demands for climate resources, such as fresh water, are expanding and changing world-wide, making it necessary to better represent uncertainty in climatic systems. As an example, drought is the most costly natural disaster (FEMA, 1995; Wilhite, 2000; Svoboda *et al.*, 2002). Improving the quality and reliability of probabilistic models used for drought monitoring and mitigation has, therefore, great value to human society, as was recognized in recent years with the establishment of the North American Drought Monitor (Redmond, 2002). Since 2002, the Climate Monitoring Branch of NOAA's National Climatic Data Center has begun collaborating with NOAA's Paleoclimatology section to incorporate pre-instrumental perspectives into the monthly and annual State of the Climate (SoC) Reports (Eakin *et al.*, 2003). These palaeoclimatic data provide a multi-century baseline from which users can better gauge recent hydroclimatic episodes relative to those of previous centuries. As longer and longer records become available, new methods need to be developed to quantify uncertainty in the climate system and to extract information on climatic variability that will be relevant to researchers, policy makers, and land managers.

Stochastic models (Salas, 1992) enable us to compute the likelihood of potentially disastrous phenomena, such as a drought more severe than all observed droughts, a flood with peak discharge higher than previously

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measured discharges, an El Niño episode with magnitude greater than all past ones, etc. Stochastic models also provide numerical tests useful for deciding whether two drought episodes are significantly different from one another or whether positive episodes behave differently from negative ones. Such information holds great value for, among others, water resource management, risk assessment, civil engineering projects, and the insurance industry (Pielke, 1999; Wilhite, 2000; Redmond *et al.*, 2002).

Climatic episodes can be quantified in terms of two random variables (Figure 1). First, duration is the number of time intervals (e.g. years) the process remains continuously above (or below) a reference level. Second, magnitude is the sum of all series values for a given duration, which is equivalent to the area under (or above) the reference level. 'Magnitude' has been referred to as 'intensity' (e.g. Kim *et al.*, 2003) or 'severity' (e.g. Dracup *et al.*, 1980; Shiau and Shen, 2001). In the latter case, the word 'magnitude' was actually used to denote the ratio Severity/Duration; see Dracup *et al.* (1980). Episodes above (or below) a reference level are usually called positive (or negative). Although it is useful to quantify the relative importance of prolonged spells in any climatic record, a stochastic framework is required to answer questions about the probability of occurrence and statistical significance of differences between episodes. For instance, a theoretical model provides answers to questions like: (1) What is the probability of a drought longer than all observed droughts? (2) Is the difference between two warm periods statistically significant? (3) Do low-flow episodes have different properties than high-flow ones? (4) What are the recurrence intervals for floods of a given size? Using a univariate model, geometric distributions were proposed for, and fit to, duration data by Yevjevich (1967), Sen (1976, 1980), Mathier *et al.* (1992), Fernández and Salas (1999) and Biondi *et al.* (2002). Again, in a univariate setting, the exponential distribution was shown to provide a good fit to magnitude data by Todorovic and Woolhiser (1976) for precipitation, by Zelenhasić and Salvai (1987) for drought, by Krstanovic and Singh (1987) and Correira (1987) for floods, and by Mathier *et al.* (1992) for water deficit. In all these studies the exponential model appeared because of its good empirical fit to the magnitude data, but without a theoretical justification. Furthermore, the models were univariate, i.e. fit separately to magnitudes and durations of episodes.

This paper presents a new stochastic model of the bivariate distribution of duration and magnitude of climatic episodes. Such joint modelling is of great interest because (a) it captures the co-variability of these two aspects of climatic excursions, thus allowing characterization of their nature for various management and decision-making applications, and (b) it better recognizes the underlying feature of slow climatic modes that often force persistent, multi-annual anomalies. Several workers have explicitly noted the need for multivariate, i.e. joint bivariate, models for duration and magnitude of droughts, floods, precipitation, etc. Yue *et al.* (1999)
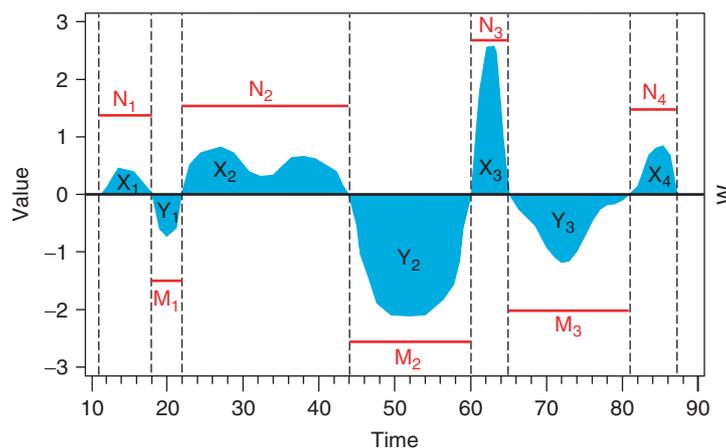

Figure 1. Time-series plot showing the durations (dashed vertical lines and horizontal segments) and magnitude (solid areas) of seven episodes (four positive and three negative) in a fictitious record with time-invariant reference level w (solid horizontal line). The third episode (N_2, X_2) has the longest duration, and the fourth episode (M_2, Y_2) has the largest magnitude. This figure is available in colour online at www.interscience.wiley.com/ijoc

and Correira (1987) mention that flood analysis has, so far, concentrated on studying flood peaks, whereas the solution of many hydrological problems also requires knowledge of the magnitude and duration. Yevjevich (1967) and Kim *et al.* (2003) emphasize that drought is a multivariate event characterized by its duration, magnitude, and intensity: separate analysis of such drought characteristics cannot uncover relationships among them. Various models have been proposed for the joint distribution of duration and magnitude (Krstanovic and Singh, 1987; Sackl and Bergmann, 1987; Goel *et al.*, 1998; Yue *et al.*, 1999; Yue, 2001; Kim *et al.*, 2003). These models use a continuous distribution for duration, but time is often considered a discrete variable (see Acreman, 1990); hence, one should use a discrete marginal for duration and a continuous marginal for magnitude. Such mixed distributions were proposed by Shiau and Shen (2001) and González and Valdés (2003). Shiau and Shen (2001) fit gamma models to the conditional distributions of drought magnitude given several values of the duration n, and estimated the shape parameters to be very close to n. The conditional gamma distribution for magnitude has also appeared in the univariate context (Acreman, 1990). None of these studies provide a comprehensive theoretical justification for their models, relying instead on an empirical fit.

The bivariate stochastic model proposed here is called BEG, for 'bivariate distribution with exponential and geometric marginals'. Its mathematical derivation explains the empirical fit of the gamma, exponential and geometric distributions described above. The model has geometric and exponential marginals, and the conditional distribution of magnitude for a given duration is gamma. The model is based on the stochastic theory of random sums, and is applied to a 2300-year dendroclimatic reconstruction of water-year (October through to September) total precipitation for the eastern Sierra Nevada–western Great Basin.

2. MODEL DEFINITION

Assume we have a stationary process $\{W_i, i \geq 1\}$, that at any time period i can be either above or below a threshold w (reference level). This process can be characterized by the duration and magnitude of its positive and negative episodes, which alternate above and below the reference level respectively. Therefore, there are four basic quantities associated with this process: positive and negative durations (N and M respectively), with values $N_1, M_1, N_2, M_2, N_3, M_3$, and so on, and the corresponding positive and negative magnitudes (X and Y respectively), in the sequence $X_1, Y_1, X_2, Y_2, X_3, Y_3$, and so on (Figure 1).

Duration, according to the theory of runs (Feller, 1957; Yevjevich, 1967), has a geometric distribution. This is consistent with defining duration as the waiting time until the process $\{W_i, i \geq 1\}$ crosses the reference level (see Feller (1957) and Sen (1980) for precipitation and Biondi *et al.* (2002) for any climate variable). The waiting time is the run length, i.e. the number of consecutive values of the process above (or below) the reference level. More generally, consider a two-state Markov chain $\{X_k, k \geq 1\}$ with states p (the value of the process is above the reference level, i.e. positive) and n (the value of the process is below the reference level, i.e. negative). If the transition probabilities $q = P(X_{k+1} = p | X_k = n)$ and $p = P(X_{k+1} = n | X_k = p)$ are independent of k, then the positive and negative durations are geometric variables with means $1/p$ and $1/q$ respectively (e.g. Sen, 1976, 1980).

As mentioned in Section 1, magnitude is defined as

$$X = \sum_{i=1}^{N} V_i \tag{1}$$

where each V_i is a conditional random variable W_i given that $W_i > w$. The number of terms N in the above sum is itself a random variable – the corresponding duration, which we assume has a geometric distribution. Sums of geometric numbers of random variables can be approximated by an exponential distribution (Biondi *et al.*, 2002). Using these ideas we now consider a bivariate model for magnitude X and duration N of the form

$$(X, N) \overset{\mathrm{d}}{=} \left(\sum_{i=1}^{N} V_i, N \right) \tag{2}$$

where the notation $\overset{\text{d}}{=}$ indicates equality in distribution, and N has a geometric distribution with the probability density function (PDF) $h(n) = P(N = n) = p(1 - p)^{n-1}$ for $n = 1, 2, \ldots$, denoted by $\mathcal{GEO}(p)$. Further, we assume that the V_i values are independent of N and, among themselves, independent and identically distributed (i.i.d.) exponential variables with $f(x) = \beta e^{-\beta x}$ for $x > 0$, denoted by $\mathcal{EXP}(\beta)$. Since the random sum in Equation (1) of i.i.d. exponential $\mathcal{EXP}(\beta)$ random variables V_i is itself exponential (e.g. Arnold, 1973), the variable X in Equation (2) has an exponential distribution as well, with parameter $p\beta$ (where p is the parameter of N). Thus, the random vector in Equation (2) has a mixed (neither continuous nor discrete) bivariate distribution with exponential and geometric marginals (BEG; Kozubowski and Panorska, in press), with parameters $\beta > 0$ and $p \in (0, 1)$, denoted by $\mathcal{BEG}(\beta, p)$. The distribution of the random sum X is approximately exponential even if the V_i values are not exactly exponential, but only satisfy the weak law of large numbers (see Biondi *et al.*, 2002).

Observe that, for a given $N = n$, the variable X in Equation (2) is a deterministic (i.e. non-random) sum of n i.i.d. exponential variables with parameter $\beta > 0$, and thus has gamma distribution $\mathcal{G}(n, \beta)$ with PDF

$$f_{X|N=n}(x) = \frac{\beta^n}{(n-1)!} x^{n-1} e^{-\beta x} \quad \text{for } x > 0$$

Consequently, as shown by Kozubowski and Panorska (in press), the joint PDF of $(X, N) \sim \mathcal{BEG}(\beta, p)$ is

$$g(x, n) = \frac{p\beta^n}{(n-1)!} [x(1 - p)]^{n-1} e^{-\beta x} \quad \text{for } x > 0 \text{ and } n = 1, 2, \ldots$$

Under this model, the pairs (X_i, N_i) and (Y_i, M_i) of positive and negative magnitudes and the corresponding durations are i.i.d. and follow the $\mathcal{BEG}(\beta_+, p_+)$ and $\mathcal{BEG}(\beta_-, p_-)$ distributions respectively. The two (bivariate) sequences are assumed to be independent. Although we have assumed the reference level to be a horizontal line (Figure 1), our model can be applied to time-varying levels as long as the geometric parameter p remains constant. This stochastic model has theoretical properties that are in agreement with the empirical findings mentioned in Section 1. To summarize, these properties include: (1) exponential/geometric marginals, (2) conditional gamma distribution with shape parameter n of the magnitude given a duration of size n, and (3) high correlation between durations and their corresponding magnitudes, and independence between positive magnitudes and negative durations, negative magnitudes and positive durations, positive and negative magnitudes, and positive and negative durations.

An important property of the BEG model is that it appears *naturally* from the mathematical definition of magnitude and duration. To illustrate what we mean by a 'natural' model, let us consider the normal (Gaussian) distribution. If a variable can be expressed as the sum of i.i.d. components, then the classical central limit theorem stipulates that the distribution of the sum is approximately normal (if the number of components is large enough). Further, under certain conditions, the sum is still approximately normal even when the components are no longer independent, or identically distributed. As one can see, the normal distribution is used to describe a large sum not only because of a good numerical fit, but, more importantly, because of the underlying stochastic mechanism of accumulating independent innovations. In this sense, the normal distribution is a 'natural' model to use when dealing with large sums, even though one might obtain a better empirical fit for a particular dataset when using another probability distribution with more parameters. For the BEG model, we first assume that episode duration is an integer-valued random variable with a geometric distribution. Then, episode magnitude is the sum of process values within a particular duration; consequently, a 'natural' probability distribution to describe magnitude is one that approximates a sum of a geometric number of random terms. The stochastic mechanism of accumulating a geometric number of random quantities produces the exponential distribution as the approximation to the sum (just as the normal distribution approximates a deterministic sum of random quantities).

There is another mathematical way to explain why the exponential model works well for magnitude. The variables V_i that appear in the BEG model are called *exceedances* over a threshold (Embrechts *et al.*, 1997; Reiss and Thomas, 2001). The extreme value theory states that, as the threshold increases, the distribution

of the exceedances is one of only three possible types: exponential, Pareto, and beta. The Pareto distribution will describe exceedances from a 'heavy tailed' or large variability process. The beta distribution works for bounded processes. Most climate-related processes are 'light tailed' and unbounded and thus will have exceedances well approximated by the exponential distribution (Balkema and de Haan, 1974; Pickands, 1975; Embrechts *et al.*, 1997).

3. MODEL APPLICATION

The BEG model was applied to a multi-century long dendroclimatic reconstruction of water-year total precipitation. Tree-ring chronologies from western juniper (*Juniperus occidentalis*) samples collected in the Walker River basin, at the boundary between California and Nevada (Figure 2), were built using standard techniques (e.g. Stokes and Smiley, 1996) applied to a total of 93 ring-width series, comprising 34 209 measurements. Mean segment length was 368 years, which allows for the accurate identification of interdecadal to intercentennial patterns (Cook *et al.*, 1995). Climatic data used for calibration/verification of the reconstruction (Table I; see Fritts (1976) for details) are October through to September precipitation totals in California Climate Division 3 (Northeast Interior Basins; NOAA, 2002), which includes the headwaters of a few eastern Sierra Nevada–western Great Basin lake and river basins (Figure 2). The reconstruction extends from 300 BC to AD 2001, and shows wet and dry periods with durations ranging from 1 to 82 years (Figure 3). The reference level for the process is zero, since reconstructed values are 'normalized' (i.e. expressed in standard deviation units).

Magnitudes and durations were computed for all positive and negative episodes, each with 374 observations. An observation here is a vector with two coordinates: absolute value of magnitude and duration. Magnitudes and durations of positive events will be denoted by X and N respectively, and magnitudes and durations of negative events will be denoted by Y and M. The BEG models are fit to the joint distributions of both
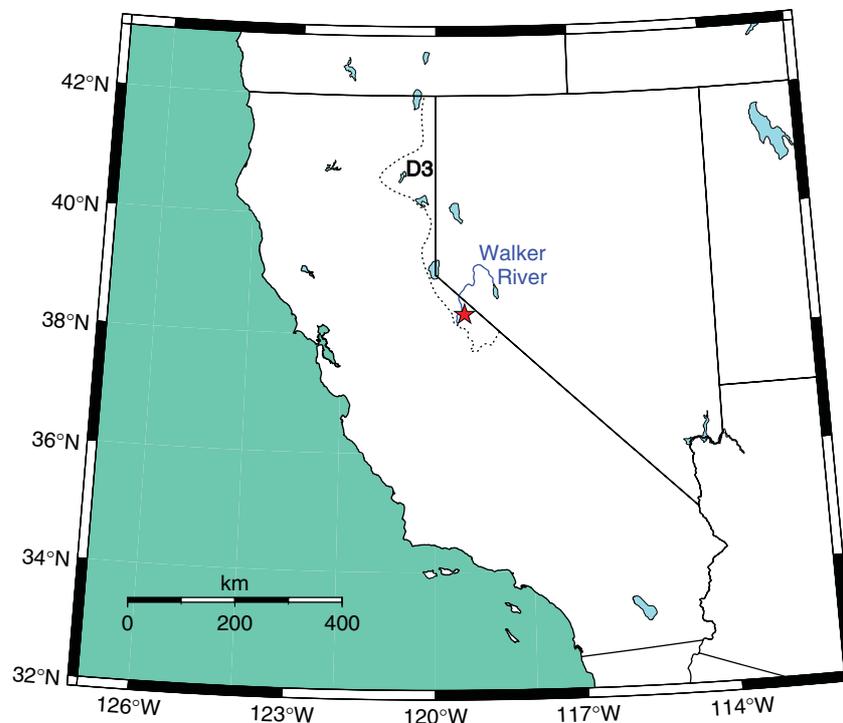

Figure 2. Map of the Walker River, the tree-ring collections sites (star), and the boundary of California Climate Division 3 (D3, dotted line). This figure is available in colour online at www.interscience.wiley.com/ijoc
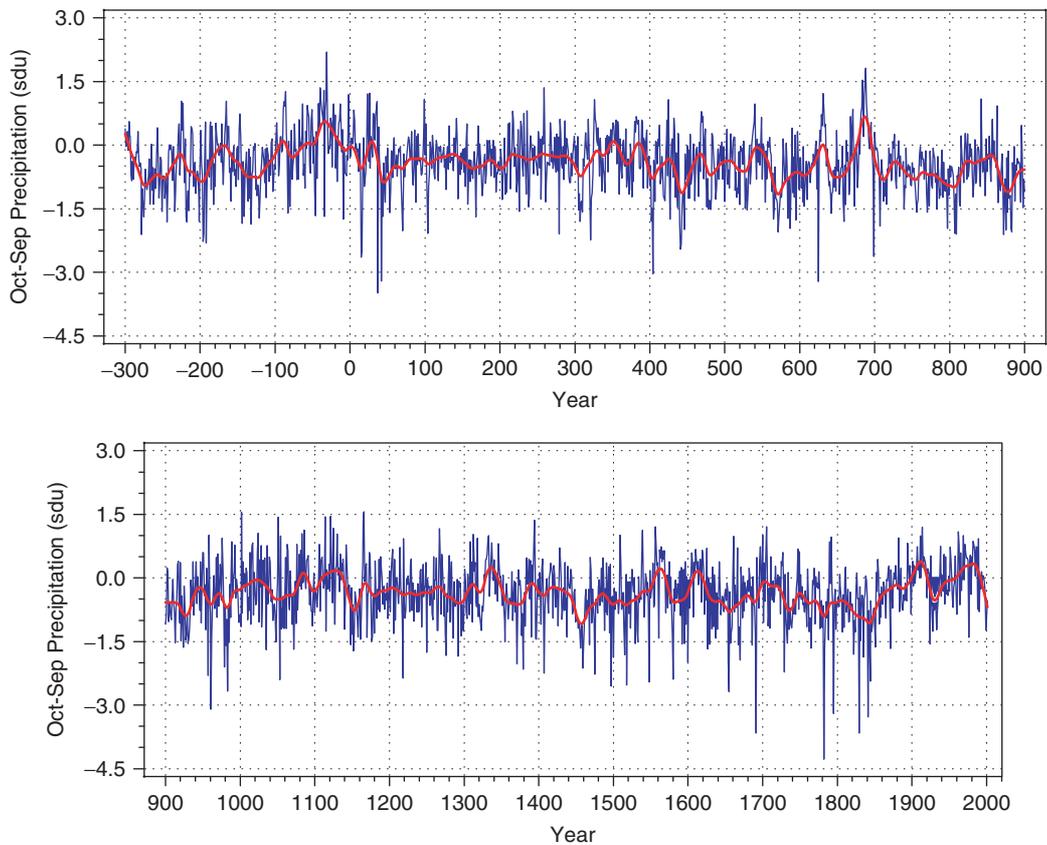
Figure 3. Tree-ring reconstruction of water-year total precipitation (sdu: standard deviation units) in California Climate Division 3. A 30-year cubic smoothing spline (Cook and Peters, 1981) is overlaid (heavy line) on the annual records (thin line). The 1926–36 drought, which corresponds to the 'Dust Bowl' period, is indicated by shading below the zero reference level. This figure is available in colour online at www.interscience.wiley.com/ijoc

Table I. Calibration (CAL) and validation (VAL) statistics for the tree-ring reconstruction of water-year precipitation. Two 50 year periods were alternatively used for calibration and validation. All tests are significant at the 95% confidence level (Holmes, 1994)

	CAL	VAL	CAL	VAL
Time period	1895–1944	1945–94	1945–94	1895–1944
Variance explained R_a^2 (%)	46.2	34.3	34.3	46.2
Cross-product t test	3.56	2.08	2.84	3.77
Linear correlation test	0.69	0.60	0.60	0.69
Sign-product test	16	11	13	17
Reduction of error test	0.47	0.35	0.34	0.48

(X, N) and (Y, M) data. Assuming that the (X_i, N_i) and (Y_i, M_i) are, respectively, i.i.d. $\mathcal{BEG}(\beta_+, p_+)$ and $\mathcal{BEG}(\beta_-, p_-)$, their parameters were estimated using the maximum likelihood method, resulting in $\hat{p}_+ = 0.58$, $\hat{\beta}_+ = 2.40$, $\hat{p}_- = 0.23$, and $\hat{\beta}_- = 1.42$.

Marginal distributions are evaluated considering that, in the model, the X_i values are i.i.d. $\mathcal{EXP}(\beta_+ p_+)$ with an estimated mean of $1/(\hat{\beta}_+ \hat{p}_+) = 0.72$, whereas the N_i values are i.i.d. $\mathcal{GEO}(p_+)$ with an estimated parameter $\hat{p}_+ = 0.58$. Furthermore, the Y_i values are i.i.d. $\mathcal{EXP}(\beta_- p_-)$ with an estimated mean of $1/(\hat{\beta}_- \hat{p}_-) = 3.14$,

and the M_i values are i.i.d. $\mathcal{GEO}(p_-)$ with an estimated parameter $\hat{p}_- = 0.23$. One may note the difference between estimated parameters for dry and wet episodes, which could indicate that large-scale climatic processes affecting the eastern Sierra Nevada–western Great Basin are not symmetrical for droughts and floods. On the other hand, since the influence of water availability on tree growth diminishes as soil moisture increases, tree-ring records are known to represent dry periods more accurately than wet ones (Cook *et al.*, 1999; Woodhouse, 2003; Knapp *et al.*, 2004).

Histograms of the X_i and Y_i values overlaid with the corresponding estimated exponential PDFs (Figure 4) show that the magnitude data are in close agreement with the exponential model. Notice that the vertical axis of a histogram is dimensionless: the area under a PDF or the area of a histogram bar equals the probability (relative frequency) of an observation falling within a given interval of values, so it is standard practice not to label the vertical axis of histograms or PDF graphs. Theoretical and empirical cumulative density functions (CDFs) of durations N_i and M_i (Figure 5), as well as empirical and model relative frequencies (see Tables II and III), also indicate a tight match. The reason for showing a fit of the magnitudes using PDFs and a fit of durations using CDFs is that magnitudes are continuous random variables, but durations are discrete random variables.

The goodness-of-fit of the bivariate model was tested by considering the conditional distributions of X given $N = n$ and of Y given $M = n$, which according to our model should be gamma with shape parameter n and scale parameter $\hat{\beta}$. The fit of the model to the data is illustrated by episodes of durations $n = 1, 2$. Four empirical datasets, two for positive and two for negative episodes, were obtained by selecting the X_i values
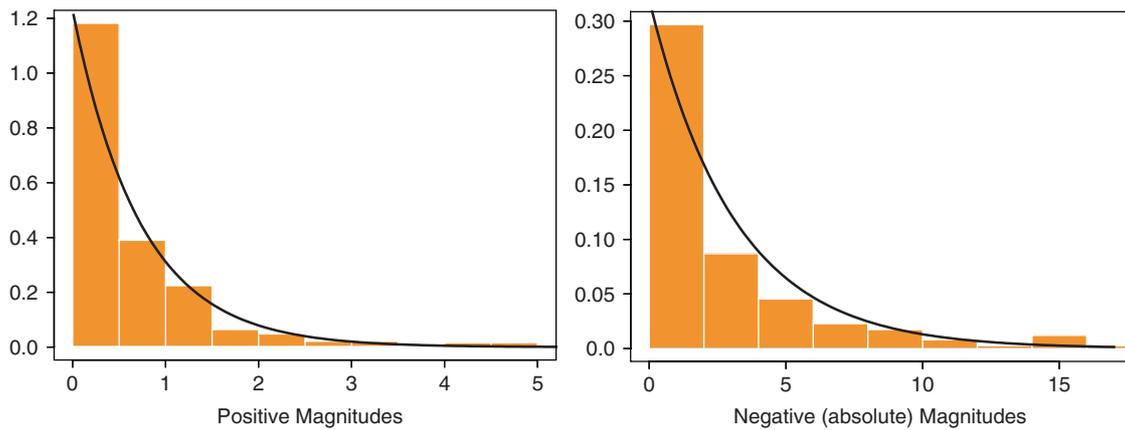

Figure 4. Probability histogram and fitted exponential distribution for positive and negative magnitudes of reconstructed hydroclimatic episodes. Note the difference in the axis limits between the two graphs

Table II. Estimated distribution parameters for duration of positive episodes N

N (years)	1	2	3	4	5	6	7	8+
Frequency	239	84	21	12	7	3	2	6
Relative frequency	0.639	0.225	0.056	0.032	0.019	0.008	0.005	0.016
Geometric probability	0.580	0.244	0.102	0.043	0.018	0.008	0.003	0.002

Table III. Estimated distribution parameters for duration of negative episodes M

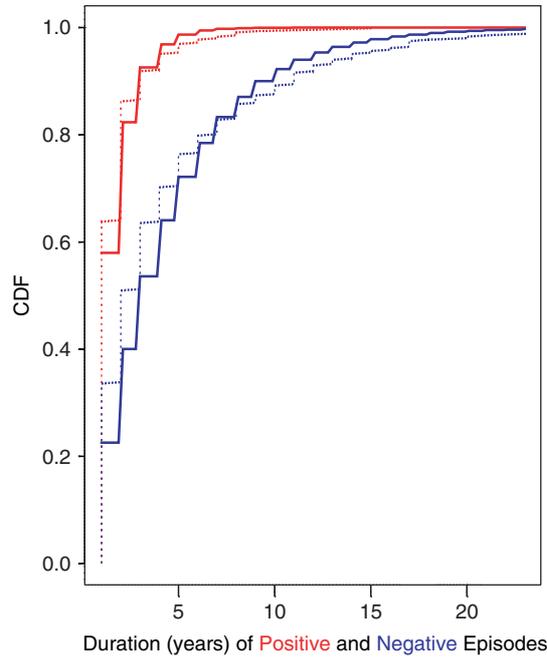| M (years)	1	2	3	4	5	6	7	8+
Frequency	126	65	47	25	23	13	11	64
Relative frequency	0.337	0.174	0.126	0.067	0.061	0.035	0.029	0.171
Geometric probability	0.226	0.175	0.135	0.105	0.081	0.063	0.049	0.166

Figure 5. Empirical (solid line) and hypothesized (dashed line) geometric cumulative density function (CDF) of duration for positive (red lines) and negative (blue lines) episodes in the reconstructed hydroclimatic record
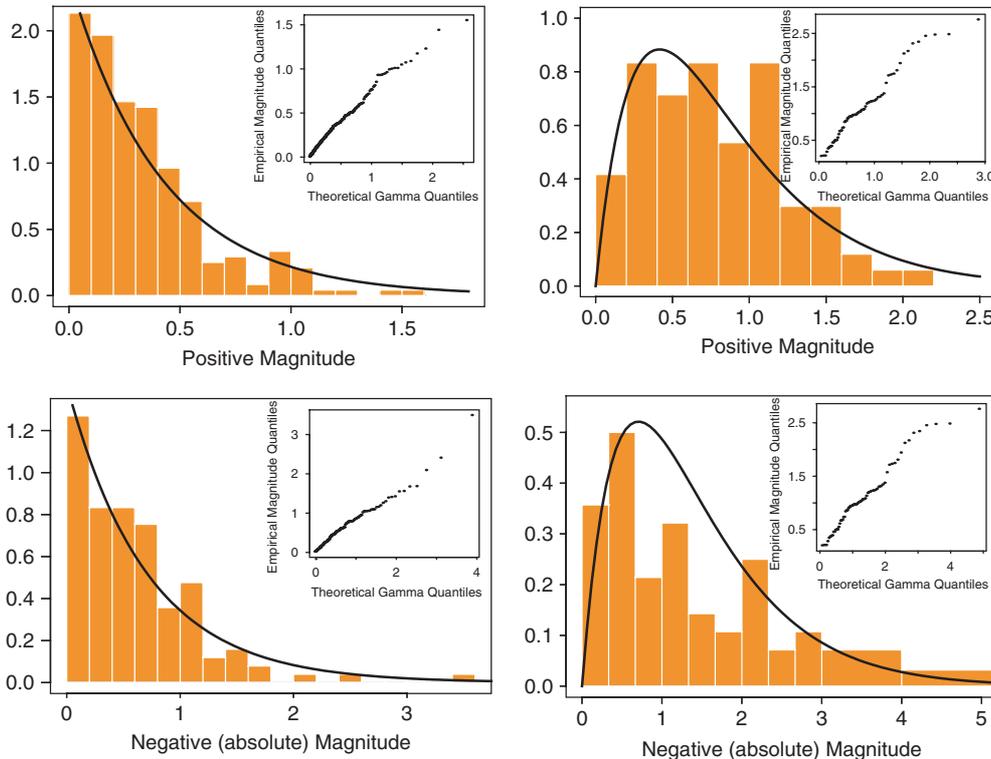

Figure 6. Conditional distribution of magnitude given duration n for positive (upper panels) and negative (lower panels) episodes in the water-year precipitation reconstruction. The data histogram is overlaid with the model gamma density, and a probability plot of the data is shown against the theoretical gamma model. Left panels show results obtained for $n = 1$, and right panels show results obtained for $n = 2$
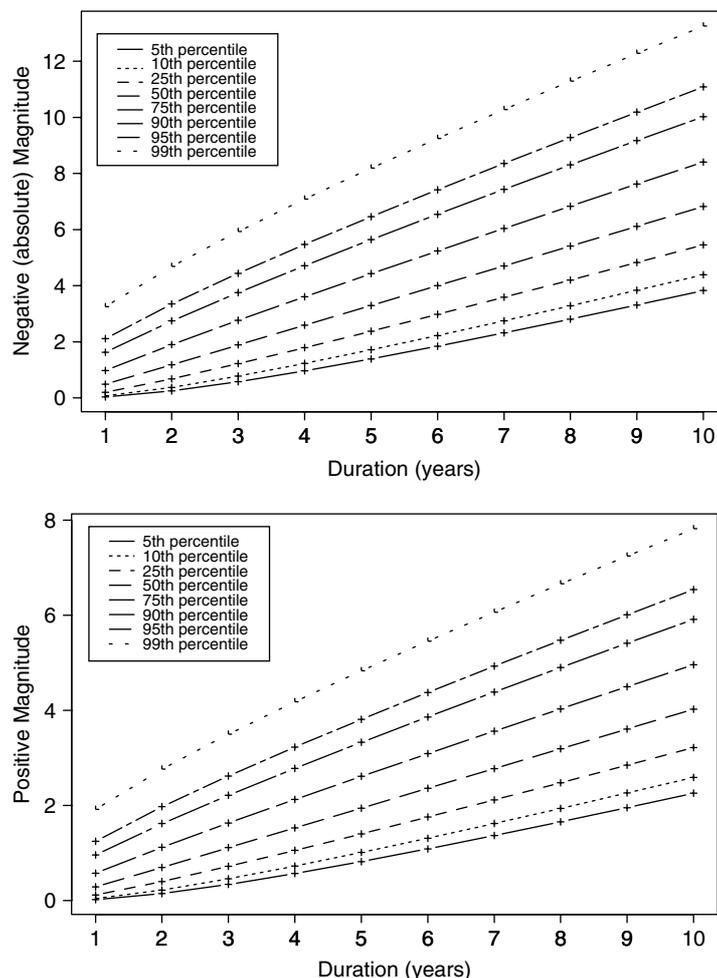
Figure 7. Conditional distributions of positive and negative magnitudes given durations, represented by selected quantiles, for the reconstruction of water-year precipitation

(or Y_i values) corresponding to $N_i = 1, 2$ (or $M_i = 1, 2$). For every dataset, a histogram was constructed and overlaid with the estimated gamma $\mathcal{G}(n, \hat{\beta})$ density, and a probability plot of the gamma model was drawn, showing a good fit in all cases (Figure 6). Note that the scale parameter $\hat{\beta}_+$ (or $\hat{\beta}_-$) was obtained from the original series X_i values (or Y_i values), not from the plotted magnitude data, so it was estimated using general model properties, rather than the particular conditional samples, and this confirms the model goodness-of-fit. In conclusion, the BEG model captures the marginal distributions and, more importantly, the conditional distributions of the reconstructed climatic episodes. Based on this analysis, it is also possible to calculate the conditional distributions of positive and negative magnitudes given durations (Figure 7).

As an example of the information that can be derived from this modelling approach, one can consider the period from 1926 to 1936 (Figure 3), an 11 year drought that corresponds to the well known 'Dust Bowl' period. From the estimated model parameters, the chance of a drought with the same or longer duration is 7.7%, and the chance of a drought with the same or greater magnitude is 8.3%. Conditional probabilities are much higher, i.e. a drought of that magnitude has a 61.7% chance of lasting for 11 years or longer, and a drought that lasts 11 years has a 46.0% chance of having an equal or greater magnitude. In addition, because of the bivariate model, we can estimate a 5.6% chance of witnessing a drought that has both the same or longer duration and the same or greater magnitude. This type of information provides a way to place the 'Dust

Bowl' episode in a better perspective, as such numerical probability statements can be used for science-based management and policy decisions.

Additional examples of model applications can be provided using wet (i.e. positive) episodes from the same reconstruction. For instance, our reconstructed time series includes a pronounced wet period from 1905 to 1919, a 15 year episode that corresponds to the well-known 'pluvial' of the early 1900s (Fye *et al.*, 2003). The chance of a pluvial with the same or longer duration is 0.0005%, four orders of magnitude smaller than the chance of a drought ≥ 11 years. A pluvial of greater magnitude has a chance of 0.02%, about 400 times less than the chance of a 'Dust Bowl'-size drought. Conditional probabilities reflect the rareness of such a prolonged wet period, with an 8% chance that a pluvial of that magnitude can last 15 years or longer, and a 46.5% chance that a 15 year pluvial will have a greater magnitude. From the bivariate model, one can estimate a 0.0003% chance of a wet episode that is both longer and greater. The estimated probabilities for the 'pluvial' of the early 20th century, when compared with those for the 'Dust Bowl' period, clearly hint at the management and policy-making implications of our new methodology.

4. CONCLUSIONS

Once the BEG model was applied to the multi-century long precipitation series, it allowed computing the likelihood of dry or wet episodes of a given duration and magnitude. The fit of exponential and geometric models to the marginal distributions of magnitudes and durations had already been documented in the literature, so goodness-of-fit analysis was limited to graphical and numerical comparisons. It should be emphasized that the BEG model may not provide the best fit to the data. Indeed, other distributions that allow for a greater number of free parameters could generate a closer empirical fit. What we argue is that we have identified a theoretical, stochastic mechanism capable of generating the probability distributions that were empirically found to describe duration/magnitude data in several studies of climatic episodes. The BEG framework provides a parametric modelling approach to understand the co-variability of duration and magnitude. Our next step would be to include an additional variable that is commonly used to describe climatic episodes, i.e. the peak value, or absolute maximum, and to model the joint trivariate distribution of duration, magnitude, and peak, as well as the bivariate properties of peak and duration, and of peak and magnitude.

River basins straddling the eastern Sierra Nevada and western Great Basin regions have often been the centre of heated debate on issues concerning water rights and water supply versus demand (Powers, 2002; Singletary, 2002; Remick, 2003). For instance, in the Walker River watershed, land jurisdiction is under two states (California and Nevada), five counties (one in California, four in Nevada), two National Forests, the Bureau of Land Management, the Department of Defense, and three Native American Reservations. Allocation of water is constrained by the need for sustaining farmers and ranchers, recreational activities, fish and wildlife species, and urban settlements. Such complex land jurisdiction, combined with the characteristic imbalance in western American watersheds between where water originates and where it is used, has made these basins extremely difficult areas for water policy decisions, especially in times of drought. According to the US Drought Monitor long-term indicator (a blend of drought-related impacts that respond to precipitation on time scales ranging from several months to a few years, such as reservoir stores, irrigated agriculture, groundwater levels, and well water depth), most of the Great Basin was in a long-term drought during 2004, with northwest Nevada being the most impacted area. Comparative studies using the BEG distribution and other approaches should provide improved estimates of occurrence and return probabilities for climatic episodes, thereby testing the adequacy of water management strategies currently in place, refining science-based management decisions, and possibly reducing future conflicts among stakeholders.

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