Ch12 supplement: Intuitive Model for Extinction and Abs. Cross sections of Particles: Anomalous Diffraction.

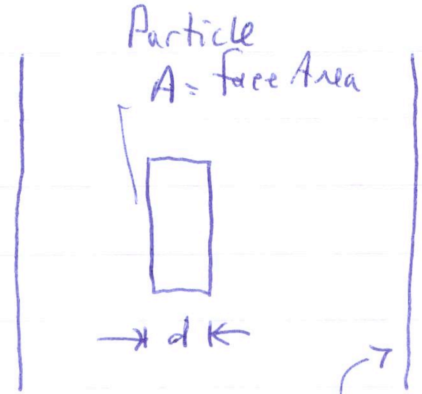
Notation: I = Irradiance or flux, $[W/m^2]$

n_r = real part of refractive index

n_i = imaginary part of refractive index.

E = Electric Field, $I \propto EE^*$ $\quad$ $*$ = complex conjugate

A_t = total area of wavefront



Plane Wave E_0

Transmitted E field, E_t

$$k = \frac{n\omega}{c} = \frac{n_r \omega}{c} + i\frac{n_i \omega}{c}$$

Looking for effects on transmitted power caused by abs and constructive interference on particle.

Total transmitted E field: $E_t = E_0 e^{ikd} \frac{A}{A_{total}}$ + (Through particle)

$k_0 = \omega/c$

$E_0 e^{ik_0 d} \frac{(A_t - A)}{A_t}$ (Around particle)

Total transmitted Irradiance: $I_t = E_t E_t^*$

$$I_t = \left| \frac{E_0 e^{ik_0 d} A}{A_t} \left[1 + \frac{(A_t - A)}{A} e^{i(k-k_0)d} \right] \right|^2$$

or

do this algebra

$$I_t = E_0^2 \left[\cancel{\frac{A^2}{A_t^2}} + \frac{2(A_t - A)A}{A_t^2} e^{-\beta d} \cos \varphi d + 1 - \frac{2A}{A_t} + \cancel{\frac{A^2}{A_t^2}} \right]$$

H.O.T. $\qquad$ keep terms A/A_t $\qquad$ H.O.T.

A^2/A_t^2 too small

$$\therefore \quad I_t = E_0^2 \left[1 - \frac{2A}{A_t} \left\{ 1 - e^{-\beta d} \cos \rho d \right\} \right]$$

$$I_t \equiv I_0 - I_0 \frac{\sigma_{ext}}{A_t}$$

$I_0 \sigma_{ext}$ is the amount of power removed from the incident beam by extinction.

$$\therefore \quad \boxed{\sigma_{ext} = 2A \left[1 - e^{-\beta d} \cos(\rho d) \right]}$$

Where $\beta \equiv n_i \omega / c$ $\quad \rho = \frac{(n_r - 1)\omega}{c}$ $\quad \omega = 2\pi f = c k_0$

or $\boxed{\beta = \frac{2\pi n_i}{\lambda_0}}$ $\quad \boxed{\rho = \frac{(n_r - 1) 2\pi}{\lambda_0}}$

Embodies interference of the forward going beam. Accounts for most of extinction for $n_r \simeq 1$. Destructive interference in forward direction means energy is going in the backward direction. Not a great approximation for $|n_r - 1| \gtrsim 0.2$

$$\boxed{\text{Beam Power} = A_t I_t = A_t I_0 - I_0 \sigma_{ext}}$$

Power removed from forward beam by extinction

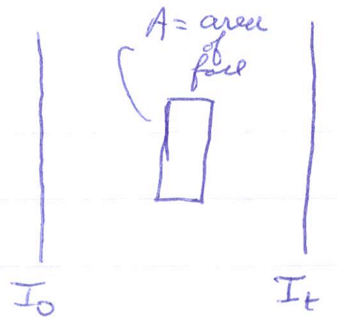
Limiting Behavior: $\rho d = 0 \Rightarrow n_r = 1$

$$\sigma_{ext} = 2A \left[1 - e^{-\beta d} \right] \approx 2A\beta d$$

σ_{ext} is due to absorption in this limit $\rightarrow$

$\Rightarrow$ Christiansen band where $n_r \simeq 1$

Absorption Model σ_{abs}



$$E_t = e^{-\beta d} e^{i k_r d} I_0^{1/2}$$
$$I_t = E_t E_t^* = e^{-2\beta d} I_0$$

$I_0 \sigma_{abs} \equiv$ power removed from forward beam by absorption.

Total transmitted power: I_t

$$I_t = \frac{A_t - A}{A_t} I_0 + I_0 e^{-2\beta d} \frac{A}{A_t}$$

around particle — through particle

= Algebra =

$$I_t = I_0 \left[1 - \left\{ \frac{A}{A_t} - \frac{A}{A_t} e^{-2\beta d} \right\} \right]$$

$$= I_0 - I_0 \frac{\sigma_{abs}}{A_t} \quad \text{where}$$

good for $|n_r - 1| \approx 0$ $\boxed{\sigma_{abs} = A\left(1 - e^{-2\beta d}\right), \quad \beta = 2\pi n_i / \lambda_0}$

Limits:

$\boxed{\beta d \gg 1 \quad \sigma_{abs} = A}$ — area regime

$\beta d \ll 1$, note $1 - e^{-x} \approx x$ when $x \ll 1$

$$\sigma_{abs} \approx 2A\beta d \approx \frac{4\pi n_i}{\lambda} V \qquad V = Ad$$

volume regime.

Motivates $d \equiv V/A$ effective diameter $= \dfrac{\text{particle volume}}{\text{particle area}}$