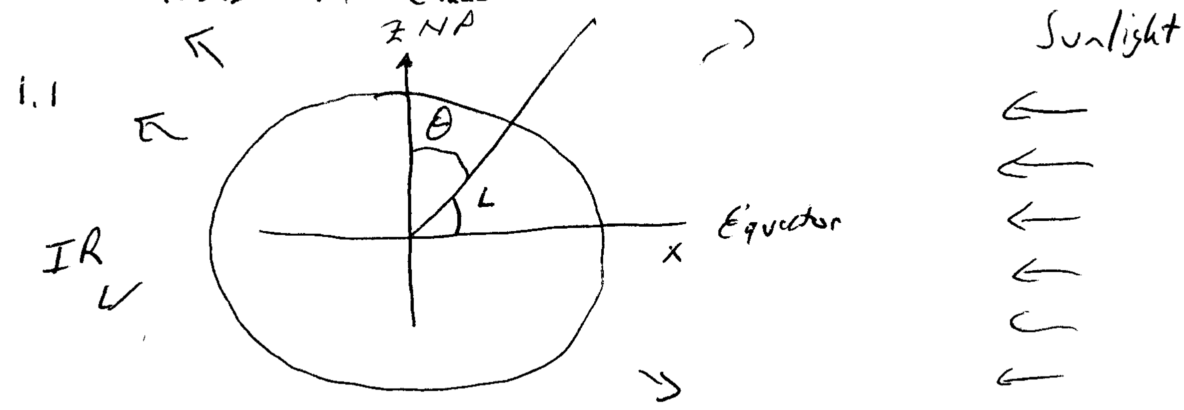
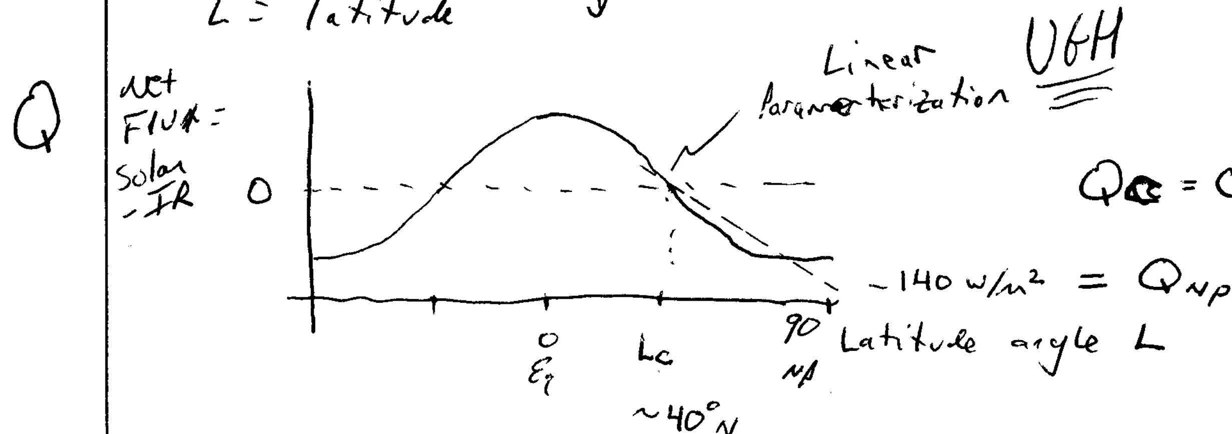
HW1 Partial Solution

Problems for class

1.1



L = latitude



A)

$$Q(L) = \frac{(Q_{NP} - Q_c)}{L_{NP} - L_c}\,(L - L_c)$$

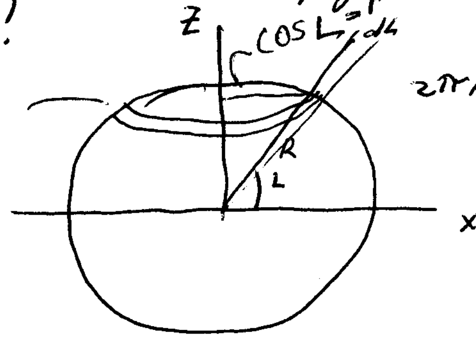
$$Q(L) = \frac{-140\,W/m^2 - 0}{90^\circ - 40^\circ}\,(L - 40^\circ)$$

$$\boxed{Q(L) = -\underbrace{2.8}_{W/m^2\cdot deg}\,\underbrace{L}_{deg} + 112 \quad W/m^2}$$

B) All of the heat transport that must make up for the north pole deficit must pass through L_c, so it is the point where $\frac{d\,H_{at}}{dL}$ is maximum.

Pg 2/5

1.1 c) How much heat per second must transfer through L_c to satisfy the deficit of Net Flux?



thickness of strip $= r\,dL$

$= R\cos L\,dL$

$2\pi R\; R\cos L\,dL = A(L)\,dL$

$\Phi = \frac{\text{heat}}{\text{sec}}$ needed to transport through L_c for the total deficit of radiation from L_c to L_{NP}.

$$\Phi = \int_{L_c}^{L_{NP}} Q(L)\,A(L)\,dL = \int_{40^\circ}^{90^\circ} (-2.8L + 112)\, 2\pi R^2 \cos L\,dL$$

Aside

Two types of integrals:

1) $\displaystyle\int_{40}^{90} \cos L\,dL = \sin L \Big]_{40}^{90} = \sin 90 - \sin 40$

How to do?

2) $I_2 = \int L\cos L\,dL$ $\quad U = L \quad dV = \cos L\,dL$

$dU = dL \quad V = \sin L$

$I_2 = L\sin L - \int \sin L\,dL$

$= L\sin L + \cos L$

Return

$R = 6373$ km

$2\pi R^2 = 2.55\times10^{14}\,m^2$

$$\therefore \frac{\Phi}{2\pi R^2} = \Big[-2.8(L\sin L + \cos L) + 112\sin L\Big]_{40}^{90}$$

$$= -2.8\cdot 90 + 112 + 2.8(40\sin 40 + \cos 40) - 112\sin 40$$

$$\boxed{|\vec{\Phi}| \cong 3.5\times10^{16}\ W}$$

D) $\rho = \dfrac{\Phi}{C} = 1.14\times10^{9}\ \dfrac{\text{Watts}}{m}$

$C = 2\pi R\cos L_c \Rightarrow \boxed{\rho \cong 1\ GW/m}$

Huge net power must pass through L_c

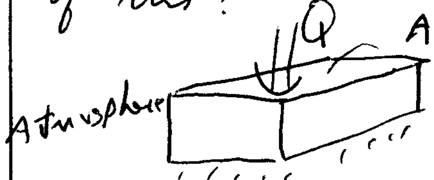
Problem 1.2

342 W/m² = average solar incident

Cloud 3% absorbed → 19% total absorbed
Gases 16% absorbed

$Q \cong 342\ \mathrm{W/m^2} * 0.19 = 65\ \mathrm{W/m^2}$ absorbed on average each day over whole planet

What is the net heating rate of the atmosphere because of this?



$\tau = 1$ day $= 86{,}400$ seconds
$M \equiv$ mass of atmosphere
$A =$ Area of earth's surface ~ same as atmosphere.

ΔT = temperature change C_p = heat capacity/mass of air @ constant pressure.

$$M C_p \Delta T = Q A \tau$$

so

$$\Delta T = \frac{Q\tau}{M/A\ C_p}$$

Now $\frac{M}{A} = \frac{Mg}{A}\frac{1}{g} = \frac{P_o}{g} =$ mass/area @ the Earth's surface, P_o = average surface pressure.

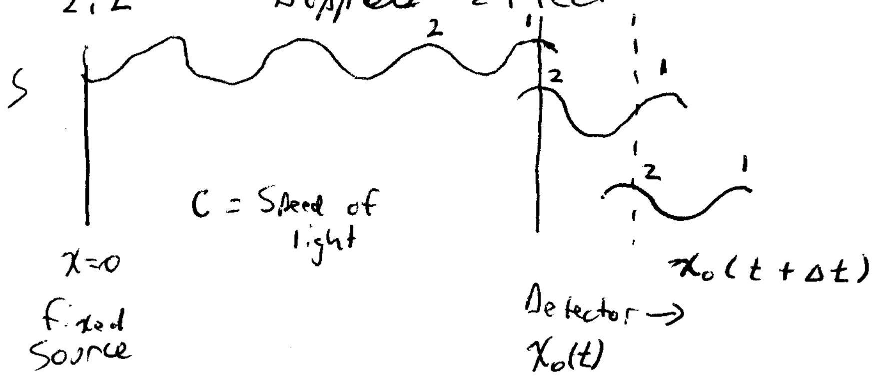
Using $Q = 65\ \frac{W}{m^2}$

$\tau = 86{,}400$ seconds/day * 1 day
$P_o = 10^5$ PA
$C_p = 1004\ \frac{J}{Kg\,K}$

$\Delta T \cong 0.55$ K/day

Average heating rate of the atmosphere because of absorbed solar radiation by clouds + gases.

2.2 Doppler's Effect



A) t_1 = time for wave 1 to go from $x=0$ to $x_o(t)$

$$t_1 = \frac{x_o(t)}{c}$$

B) t' = time for wavefront 2 to reach $x_o(t)$

$$t' = \underbrace{t_1 + \tau}_{\text{wave period}} = \frac{x_o(t)}{c} + \tau$$

$\tau = 1/\nu$ ← frequency

C) t'' = extra time the wave 2 needs to catch up with the observer now at $x_o(t+\Delta t)$ since the detector is moving →.

The detector is at $v\tau$ when wavefront 2 gets to $x_o(t)$ (farther right of $x_o(t)$)

$$\therefore \quad t'' = \frac{x_o(t+\tau) - x_o(t)}{c} = \frac{v\tau}{c}$$

$t_2 \equiv t'' + t'$ = time when wavefront 2 reaches the detector. So $t_2 - t_1 \equiv \tau'$ = Period of wave from moving detector.

Solving,

$$\tau' = \tau + \frac{v}{c}\tau = \left(1+\frac{v}{c}\right)\tau \quad > \tau$$

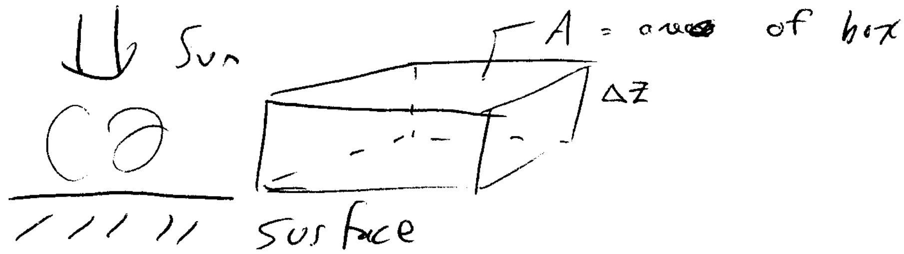
$$\boxed{\nu' = 1/\tau' = \nu\left(1+\frac{v}{c}\right)^{-1}}$$

$$(\nu' - \nu) \overset{N}{\cong} \left(1+\frac{v}{c}\right)^{-1} - 1$$

$$\boxed{\Delta\nu \approx -v/c}$$

Observer on $x_o(t)$ observes a lower frequency $\nu' \cong \nu - \frac{v}{c}$ than @ $x=0$

Problem 2.3 Atmospheric Boundary layer



Sunlight mixes the boundary layer to height ΔZ

a) $\dfrac{\text{Total solar energy absorbed}}{\text{day}} = \dfrac{3600\,s}{hr} \displaystyle\int_6^{18} F_o \frac{W}{m^2} \cos\left[\frac{\pi(t-12)}{12}\right] dt$

$\frac{W}{m^2}$ → $\frac{J}{s \cdot m^2}$

dt ↑ Units are hours!! (Need to convert to seconds)

$$Q = 3600\, F_o \frac{12}{\pi}\left(\sin\frac{\pi}{2} - \sin\frac{-\pi}{2}\right)$$

$$Q = 3600\, F_o \cdot \frac{24}{\pi} = 1.375 \times 10^7 \frac{J}{m^2}$$

b) $$m\, C_p\, \Delta T = Q A$$

$$m = \rho_{air}\, \Delta Z\, A \Rightarrow$$

$$\boxed{\Delta T = \frac{Q}{\rho_a C_p \Delta Z} = 13.7 \frac{K}{day}}$$

c) If $\Delta Z = 1\ m \Rightarrow \Delta T$ 1000 times larger.
However, if $\rho_w \approx 1000\, \rho_{air} \Rightarrow$ Same
(i.e. 1 meter of water would have the same heating rate as 1000 m air)