Sets up Problem 3

Sep 11
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Notes on cross sections, extinction, absorption, scattering, optical depth, and irradiance.

Sunlight, laser,

$\sigma_{ext} \equiv$ Extinction Cross Section

σ_{sca}

σ_{abs}

// Beam of Light

Particle or Gas, etc

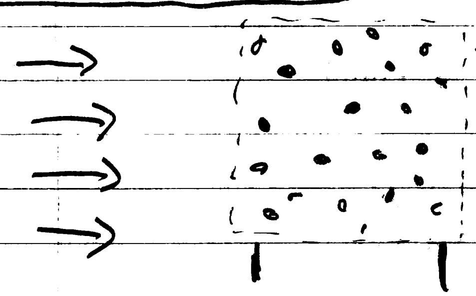
$I_0 \sigma_{ext}$ = Power removed by extinction.

$I_0 \equiv$ Irradiance $\left[\frac{W}{m^2}\right]$

[Cross Sections]

$$\underset{\text{extinction}}{\sigma_{ext}} = \underset{\text{absorption}}{\sigma_{abs}} + \underset{\text{scattering}}{\sigma_{sca}}$$

$[cm^2/\text{molecule}]$ for gases



A = area $[m^2]$

x $\quad x+dx$

$$A\,[\overbrace{I(x+dx) - I(x)}^{dI}] = -I(x)\, n\, A\, dx\, \sigma$$

Power lost [Watts]

(can add scattered Light in later)

n $\frac{\text{molecules}}{\text{Volume}}$

will do Particles Later

Write as

$$dI = -I\, n\sigma\, dx$$

$$\int_{I_0}^{I(x)} \frac{dI}{I} = -\int_0^x n\sigma\, dx' = \ln \frac{I(x)}{I_0}$$

For $n\sigma$ ~ constant with position,

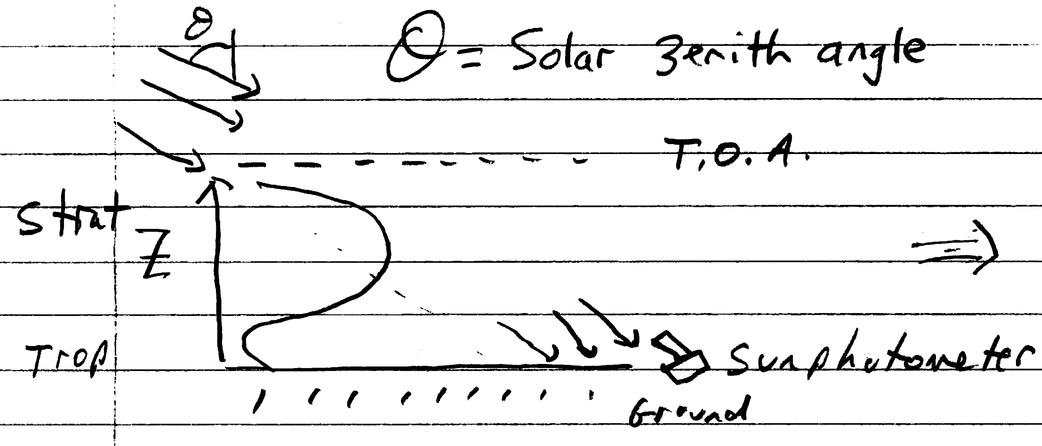
$$\boxed{I(x) = I_0\, e^{-n\sigma x} \equiv I_0 e^{-\tau}.}$$

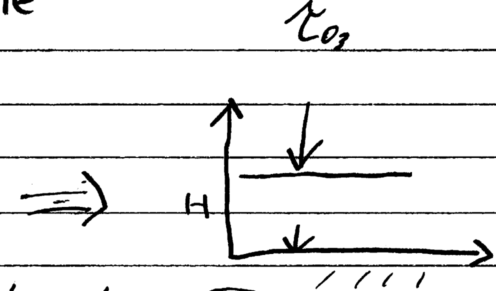
$$\tau \equiv \int_0^x n\sigma\, dx' = \tau(x). \quad \tau = n\sigma x \text{ special case.}$$

Optical Depth

Case 1 Ozone absorption of Sunlight

θ = Solar zenith angle





$n(z) \equiv \frac{\#\ O_3\ \text{molecules}}{\text{Volume}}$

$$\boxed{\int_0^\infty n(z)\,dz \equiv \frac{P_0}{kT_0} H}$$

$\frac{P_0}{kT_0}$ = # / Volume at STP

$$\bar{\tau}_{O_3} = \frac{\tau_{O_3}}{\cos\theta} = m\,\tau_{O_3}$$

τ_{O_3} = OD for $\theta = 0$, Sun overhead

$$\tau_{O_3} = \int_0^\infty n_{O_3}\,\sigma_{O_3}\,dz$$

$$\tau_{O_3} \equiv \sigma_{O_3}\, H_{O_3}\, \frac{P_0}{kT_0}$$

$\frac{P_0}{kT_0}$: n_0 at STP from IGL.

H_{O_3}: Thickness of ozone layer if all O_3 was compressed to STP.

Typical #: $H = 300$ Dobson Units = 300 DU = 3 mm

$P_0 = 101325\ \text{Pa}$ $\quad T_0 = 273.15\ \text{K}$. $\quad n_0 = 2.69\times10^{25}\ \frac{}{\text{Volume}}$

Then

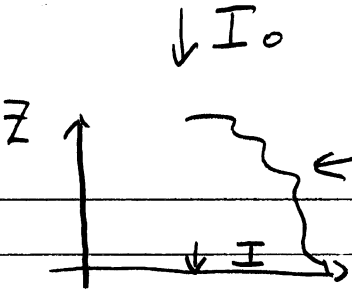
$$\tau_{O_3} = \sigma_{O_3}\left[\frac{\text{cm}^2}{\text{Molecule}}\right] \times 10^{-4}\,\frac{\text{m}^2}{\text{cm}^2} \times H\,[\text{DU}] * 10^{-5}\,\frac{\text{m}}{\text{DU}}\, n_0$$

τ, σ depend on λ

$$\boxed{\tau_{O_3} = \sigma_{O_3}\left[\frac{\text{cm}^2}{\text{Molecule}}\right] H_{O_3}\,[\text{DU}] * 2.7\times10^{16}}$$

Use sunphotometer to measure τ, look up σ, get H!

Case 2: Water Vapor



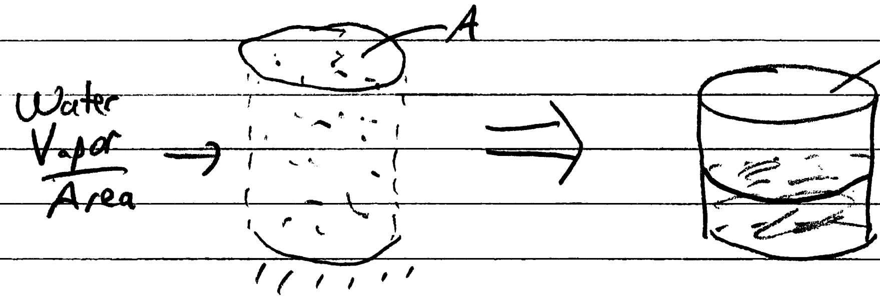
← T_{Dew}, RH, T (use to get $n_{H_2O}(z) = \frac{\# \text{ } H_2O \text{ molecules}}{\text{Volume}}$)

$$\tau_{H_2O} = \int_0^\infty \underbrace{\sigma_{H_2O}(\lambda, z)}_{\frac{\text{cross section}}{\text{Molecule}}} \underbrace{n_{H_2O}(z)}_{\frac{\text{molecules}}{\text{Volume}}} \underbrace{dz}_{\text{Path length}}$$

Drop H_2O Subscript

Aside: Molecule path $\equiv \int_0^\infty n(z)\,dz = \frac{\text{Average \# molecules}}{\text{Area}}$

Define Precipitable Water:



ρ = water density

$H \equiv$ Precipitable Water (mm)

Conserve Molecules

$$A \int_0^\infty n(z)\,dz = \frac{\rho A H * 10^{-3} \frac{m}{mm}}{M_w [kg/mole]} * N_a \frac{\text{Molecules}}{\text{mole}}$$

$M_w \approx 0.018$ kg/mole

$\rho = 1000$ kg/m^3

$$\boxed{\tau_{H_2O} = \sigma\left[\frac{cm^2}{\text{molecule}}\right] H[mm] * 3.345 \times 10^{21}}$$

Notes:

1.) Water Vapor and liquid water have DIFFERENT!! σ values.

2) H can be obtained from Sunphotometer τ and look up σ.

H also from soundings!

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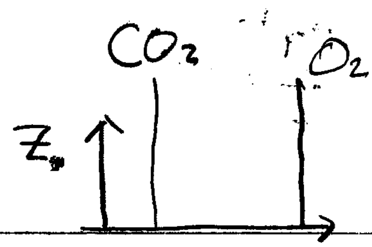
Case 3. Well mixed gases:



e.g. $C_{O_2} \cong 21\%$ $C_{CO_2} = 385 \text{ PPM} = 385 \times 10^{-6}$

$$\tau_i(\lambda) \cong \sigma_i(\lambda) \int_0^\infty n_i(z)\,dz \quad i\text{th gas}$$

Ideal gas law: $\underset{\text{total gases}}{PV = NkT}$, $n \equiv \frac{N}{V} = \frac{P}{kT}$

$$n = \sum_{i=1} n_i = \frac{1}{kT} \sum_i P_i = P/kT$$

P_i = Partial Pressure ith gas.

Note: $\frac{n_i}{n} \equiv C_i$ = concentration by volume of the ith gas.

so $C_i = P_i/P$, $n_i = n\,C_i$

Assemble:

$$\tau_i = \sigma_i \int_0^\infty n_i(z)\,dz = \sigma_i \left[\int_0^\infty n(z)\,dz\right] C_i$$

Pull C_i out because $\sim$ constant

$\int_0^\infty n(z)\,dz$ = total # molecules / Volume

$$\tau_i = \sigma_i C_i \int_0^\infty \underbrace{\frac{P(z)}{kT(z)}}_{n(z)}\,dz$$

$n(z)$ because of I.G.L.

Use ...

$$dP = -\rho g\,dz \longrightarrow dz = -\frac{1}{\rho g}\,dP$$

Hydrostatic Equation

Case 3 Continued

$$\rho = n\left[\frac{\text{Molecules}}{\text{Volume}}\right] M_w\left[\frac{\text{Kg}}{\text{Mole}}\right] \frac{1}{N_a}\left[\frac{\text{Mole}}{\text{Molecules}}\right]$$

$n = \frac{P}{kT}$ IGL

N_a: Avogadro's #

So

$$\rho = \frac{P M_w}{N_a k T} = \frac{P M_w}{R_o T} \qquad R_o = 8.3143 \frac{J}{\text{Mole } K}$$

Then

$$\tau_i = \sigma_i C_i \int_0^\infty \frac{P(z)}{kT(z)}\, dz$$

$$\tau_i = -\sigma_i C_i \int_{P_o}^{0} \frac{P(z)}{kT(z)} \frac{dP}{\rho g}$$

$$\boxed{\tau_i = \sigma_i C_i \int_0^{P_o} \frac{N_a}{M_w g}\, dP = \frac{\sigma_i C_i N_a P_o}{M_w g}}$$

Numbers

$$\tau_i = \frac{C_i[\text{fraction}]\ \sigma_i\left[\frac{\text{cm}^2}{\text{Molecule}}\right] * 10^{-4}\frac{\text{m}^2}{\text{cm}^2}\ N_a\ P_o[\text{mb}]\ \frac{100\,\text{Pa}}{\text{mb}}}{0.029\ \text{Kg/mole} * 9.8\frac{\text{m}}{\text{s}^2}}$$

$$\boxed{\tau_i = C_i[\text{fraction}]\ \sigma_i\left[\frac{\text{cm}^2}{\text{Molecule}}\right] P_o[\text{mb}]\ 2.12 \times 10^{22}}$$

e.g. O_2, $C_i \approx 0.21$ $\qquad P_o \approx 860$ mb Reno

CO_2, $C_i = 385 \times 10^{-6}$