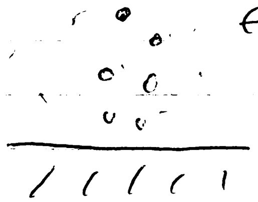
Problem 1.4 Greenhouse Gas effect on climate + radiative forcing.

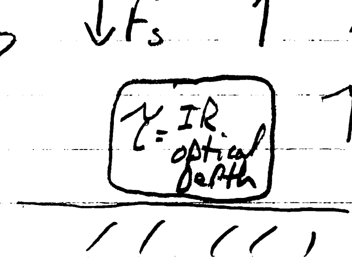


← add more GHG, say CO_2,

Atmosphere

What is the net radiative forcing effect for an instantaneous addition of a GHG to the atmosphere?

Solution:



$\downarrow \bar{F}_s$ $\uparrow$ Net IR out

Net Solar radiation absorbed

$\boxed{\tau = \text{IR optical depth}}$ Υ_{eff} = IR transmissivity of atmosphere.

$T_{surface}$

Now then, $N \equiv$ net radiative forcing

$$= \underline{\text{In}} - \text{out} = \text{Solar} - \text{IR}$$

$$N = \bar{F}_s(1-\bar{\rho}) - \sigma \Upsilon_{eff} T_s^4$$

On long term, $\underline{N=0}$, i.e. Solar & IR balance.

But we make a sudden change of Υ_{eff} and throw the system out of balance.

For GHG, $\Upsilon_{eff} = e^{-\tau} = e^{-(\tau_{CO_2} + \tau_{H_2O} + \tau_{O_3} + \ldots)}$

compact notation $\Longrightarrow$ $\Upsilon_{eff} = e^{-\sum_{i=1}^{N} \tau_i}$.

We can isolate mathematically the contribution from the ith gas as

$$\Upsilon_{eff} = e^{-\tau_i} e^{-\sum_{j \neq i} \tau_j}, \qquad (1)$$

Now let's perturb climate by letting

$$\tau_i = \tau_i + \Delta\tau_i$$

increased optical depth of the ith gas, like adding more CO_2.

From Eq (1),

$$\mathcal{T}_{eff}(\tau_i+\Delta\tau_i) = \underline{e^{-(\tau_i+\Delta\tau_i)}}\ \underline{e^{-\sum_{j\neq i}\tau_j}}$$

IR transmissivity when we add $\Delta\tau_i$ to optical depth

$$= \underline{\mathcal{T}_{eff}}\, e^{-\Delta\tau_i}$$

$$= \mathcal{T}_{eff}(\tau_i)\, e^{-\Delta\tau_i} . \quad (2)$$

Therefore the change in $\mathcal{T}_{eff}$ is

$$\Delta\mathcal{T}_{eff} \equiv \mathcal{T}_{eff}(\tau_i+\Delta\tau_i) - \mathcal{T}_{eff}(\tau_i)$$

$$= \mathcal{T}_{eff}\left(e^{-\Delta\tau_i} - 1\right) . \quad \text{from (2).}$$

Aside: when $x \ll 1$, $e^x \sim 1 + x + x^2/2! + \ldots$

Therefore we can use $e^{-\Delta\tau_i} \approx 1 - \Delta\tau_i$ for $\Delta\tau_i \ll 1$.

Solving,

$$\boxed{\Delta\mathcal{T}_{eff} = -\mathcal{T}_{eff}\,\Delta\tau_i} \quad (3)$$

Now let's make use of it.

Net radiative forcing:

$$N = \bar{F}_s(1-\bar{\rho}) - \sigma \tau_{eff} T_s^4 \,.$$

Instantaneously perturb N by changing τ_{eff}, e.g. by adding more GHG to the atmosphere.

$$\Delta N = -\sigma \Delta \tau_{eff} T_s^4 \qquad . \qquad (4)$$

Using Eq(3) in Eq(4),

$$\Delta N = \underbrace{\sigma \tau_{eff} T_s^4}_{\text{By definition } = F_{TOA}} \Delta \tau_i \quad \text{or} \,,$$

(a)

$$\boxed{\Delta N = F_{TOA} \, \Delta \tau_i} \qquad (5)$$

Interpretation: If $\Delta \tau_i > 0$, we have added GHG to the atmosphere. Then $\Delta N > 0$. Because

$$N = \bar{F}_s(1-\bar{\rho}) - \tau_{eff} \sigma T_s^4 \,,$$

One way to balance again ($N=0$) is to make T_s go up. As τ_{eff} goes down the surface temperature must go up to remain in balance.

(B) Calculate the necessary $\Delta\tau_i$ (change of optical depth for absorption) for a radiative forcing of $\Delta N = 4\ W/m^2$ associated with doubling CO_2.

Solution: Some preliminaries:

In the unperturbed state,

$$\bar{F}_S(1-\bar{\rho}) = \gamma_{eff}\,\sigma T_S^4 \Rightarrow$$

$$\gamma_{eff} = e^{-\Sigma\tau_i} = \frac{\bar{F}_S(1-\bar{\rho})}{\sigma T_S^4} .$$

Then: $\Sigma\tau_i = -\ln\left[\frac{\bar{F}_S\,(1-\bar{\rho})}{\sigma T_S^4}\right]$. Using #'s,

$$\Sigma\tau_i = -\ln\left[\frac{340\,W/m^2\,(1-0.3)}{5.67\times10^{-8}\,W/m^2K^4\ 288^4K^4}\right] = 0.494 \quad (6)$$

Next, from (5),

$$\Delta\tau_i = \frac{\Delta N}{F_{TOA}} = \frac{\Delta N}{\bar{F}_S(1-\bar{\rho})} = \frac{4\ W/m^2}{342\,W/m^2(1-0.3)}$$

$$\Delta\tau_i = 1.67\times10^{-2} \quad (7)$$

Combining (6) and (7),

$$\frac{\Delta\tau_i}{\Sigma\tau_i} = \frac{1.67\times10^{-2}}{0.494} = 3.4\times10^{-2} = 3.4\,\%$$

Therefore a doubling of CO_2 induced warming requires an increase of the optical depth of 3.4%. From Eq. (3) we can see that if γ_{eff} is small, $\Delta\gamma_{eff}$ is small, i.e. it takes more of an optical depth change to affect IR opacity.