Homework 7. ATMOS 749 Radiation Transfer. Multiple Scattering.

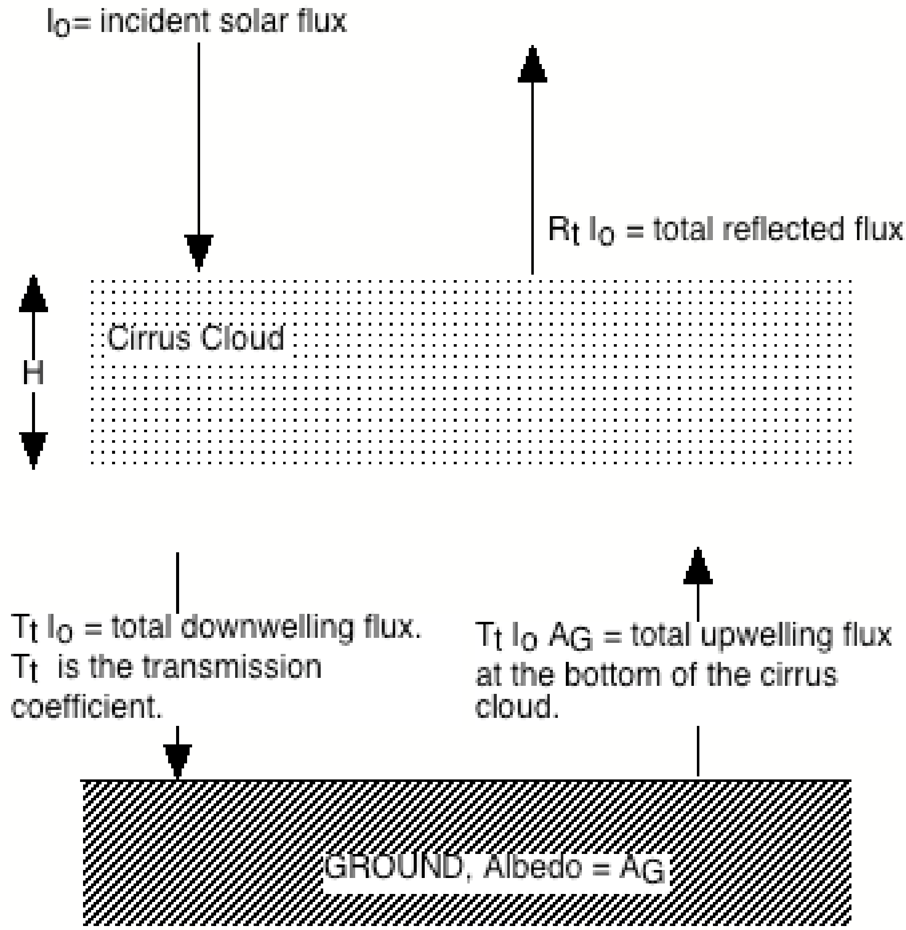

Figure 1. Schematic arrangement of the cirrus cloud.

The following properties apply to the cirrus cloud. The cloud is composed of a population of ice crystals coming in only two sizes, small and large. The purpose of this problem is to investigate the effect of small ice crystals on the transmission and reflection of sunlight by the cloud. In practice, the measurement of small ice crystals is difficult, and the purpose of this problem is to find out if their influence is important enough to make the measurement of their numbers worthwhile.

The ice water content,

$$IWC = 20\ \text{mg/m}^3 = IWC_s + IWC_L = \alpha\, IWC + (1 - \alpha)\, IWC,$$

where $IWC_s = \alpha\, IWC$ and $IWC_L = (1 - \alpha)\, IWC$, and where $0 \le \alpha \le 1$ determines the fraction of total ice water content associated with small and large ice crystals.

For convenience, we consider spherical particles of diameters D_s = 10 μm, and D_L = 100 μm. The number of small crystals per unit volume, N_s is determined from the fraction of ice water content associated with them,

$N_s \frac{\pi D_s^3}{6} \rho_{ice} = IWC_s$, where the density of ice is ρ_{ice} = 900 kg/m^3, and the number of large crystals per unit volume is similarly determined.

The scattering optical depth in the large (enough) particle limit is simply twice the projected area of all crystals,

$$\tau_{sca} = \tau_{sca}^s + \tau_{sca}^L = 2N_s \frac{\pi D_s^2}{4} H + 2N_L \frac{\pi D_L^2}{4} H \ , \qquad (1)$$

where H = 2 km is the cloud thickness. The asymmetry parameter for small and large crystals is taken to be g_s = 0.7 and g_L = 0.8, so that the asymmetry parameter for the combination is

$$g = \frac{g_s \tau_{sca}^s + g_L \tau_{sca}^L}{\tau_{sca}} \ . \qquad (2)$$

PROBLEM 1.
Derive equation 2.

PROBLEM 2.
Compute and plot the cirrus albedo, R_t, and the downwelling coefficient, T_t, as a function of the cirrus mass fraction, α, due to small ice crystals, and interpret your result. Describe how you did the calculation (software used). Your result probably will look like the following;

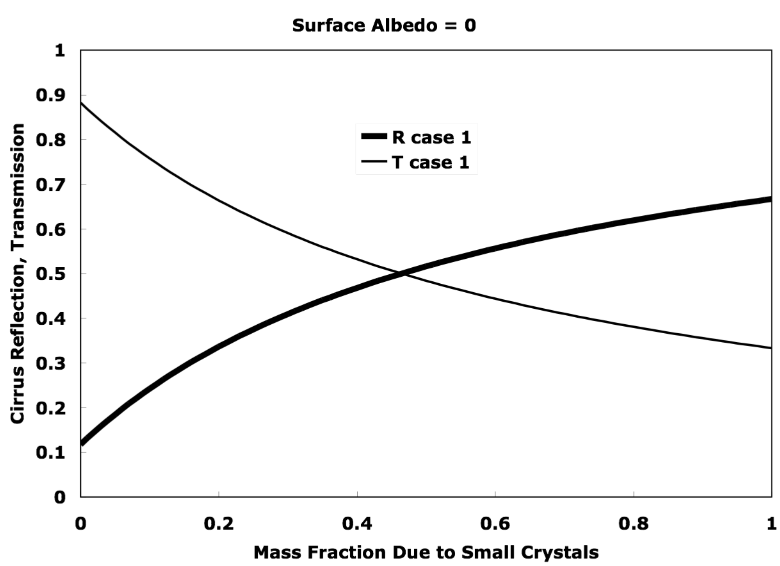

Figure 2. Cirrus albedo (thick curve) and transmission.

PROBLEM 3.

In class, we considered multiple reflections between the ground and the cloud, and showed that

$$R_t = R + \frac{T^2 A_G}{1 - A_G R} \qquad T_t = \frac{T}{1 - A_G R} \quad , \tag{3 a,b}$$

where R and T are the layer reflection coefficients obtained by considering A_G=0, and are given in Eqs. (14) and (15) of Bohren's 1987 article. Eqs. (3 a,b) can also be derived by considering both a downwelling, and upwelling flux as a boundary condition at the lower part of the cirrus, as shown in Figure 1. Using this approach, you can also show that

$$R_t = \frac{\tau^*(1 - A_G) + 2A_G}{2 + \tau^*(1 - A_G)} \qquad T_t = \frac{2}{2 + \tau^*(1 - A_G)} \quad , \tag{4 a,b}$$

if you would like to use this form. The definition of τ^* is given in the article. Overlay the plot in problem 2 with reflection and transmission calculated with the cirrus cloud to be over a ground having albedo $A_G = 0.3$. Intrepret these results. NOTE: Eq. 3 is general for all ω while Eq. 4 applies only to the case where ω=1.

PROBLEM 4.

Consider now the more general problem of non-conservative scattering by the cirrus cloud, e.g. the single scattering albedo, ω, of the cloud now differs from unity and the absorption coefficient is non-zero. By assuming A_G = 0 in Figure 1, it can be shown that the cirrus reflectance R, and transmittance T, are given by

$$R = \frac{\omega(1-g)\,\sinh(K\tau)/K}{\{2 - \omega(1+g)\}\sinh(K\tau)/K + 2\cosh(K\tau)}$$

$$T = \frac{2}{\{2 - \omega(1+g)\}\sinh(K\tau)/K + 2\cosh(K\tau)} \,, \tag{5 a,b}$$

where

$$K \equiv [(1-\omega)(1-\omega g)]^{1/2}$$

and where τ is the total optical depth of the layer, $\tau=\tau_{sca}+\tau_{abs}$. Check this result by taking the limit ω=1 in Eqs. (5), and comparing with the conservative case, Eqs. (14) and (15) in Bohren's article. Also, in the limit of no scattering, ω = 0, $\tau=\tau_{abs}$, show that R=0 and that $T=\exp(-\tau_{abs})$, i.e. that Beer's law applies when scattering can be ignored. Then plot R and T for ω=0.99 and ω=0.90 and overlay these with the results from problem 2. Interpret this plot.