Homework 5: Glory and Rainbow

Snell's law relates the angles of incidence and refraction as shown in Fig. 1. The index of refraction of a material is defined as n = c/v where c is the speed of light in a vacuum and v is the speed of light in the material. For water and visible light, we usually use n=4/3, though because of dispersion, n is slightly different for different wavelengths of light. Snell's law is $n_t \sin \theta_t = n_i \sin \theta_i$ where in Fig. 1 the specific case of $\theta_i = 60$ degrees and $\theta_t = 40.5$ degrees is shown.

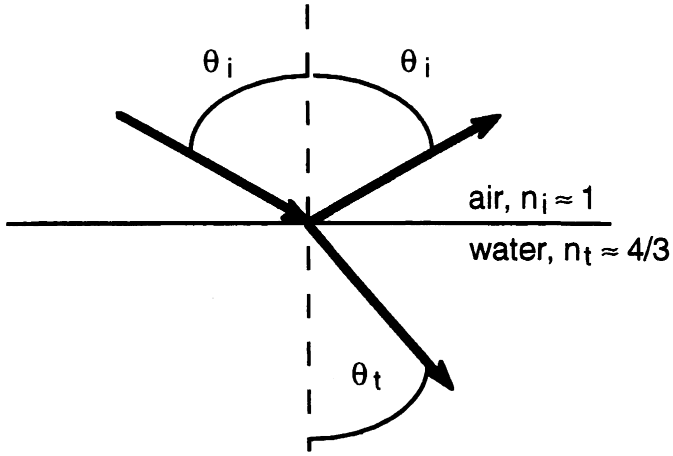

Figure 1. Path of light rays incident, reflected, and transmitted by an air-water interface.

Figure 2 shows a ray path through a cross-section of a spherical water drop. This ray paths is determined from Snell's law and it involves one reflection of a ray at the back of the drop and two transmissions through the water-air boundary. The reflection is only a partial reflection because there is a ray (not shown) which gets transmitted through as well. The scattering angle θ is defined relative to the forward direction. The impact parameter is defined as b.

The classical differential scattering cross section (SC) for a spherically symmetric scatterer is

$$\frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \frac{db}{d\theta} \right| .$$

Two singularities ($d\sigma/d\Omega \rightarrow \infty$) of SC are rainbows and the glory. These singularities of ray optics give rise to strong scattering at the rainbow angle, and in the backscattering direction (glory scattering). Diffraction (wave optics) is said to soften the singularities. Rainbows are associated with an angle of minimum deviation such that $d\theta/db \rightarrow 0$ (or equivalently $d\theta/d\phi_i \rightarrow 0$ since $b = a \sin \phi$). The glory occurs for $\sin\theta \rightarrow 0$ (that is, $\theta = \pi$ for the glory in backscattering we will consider here, but note that $\theta = 0$ corresponds to the so-called forward glory) <u>and</u> the conditions $b \neq 0$, $db/d\theta \neq 0$. Note that a ray incident and reflected from the center of the droplet in Fig. 2 is <u>not</u> a glory ray. We will show that simple ray optics provides a rudimentary understanding of the rainbow. Then just as van de Hulst did in 1947, (J. Opt. Soc. Am. **37**, 16-22), we will show that ray optics is not entirely adequate to give even a rudimentary explanation of the glory.

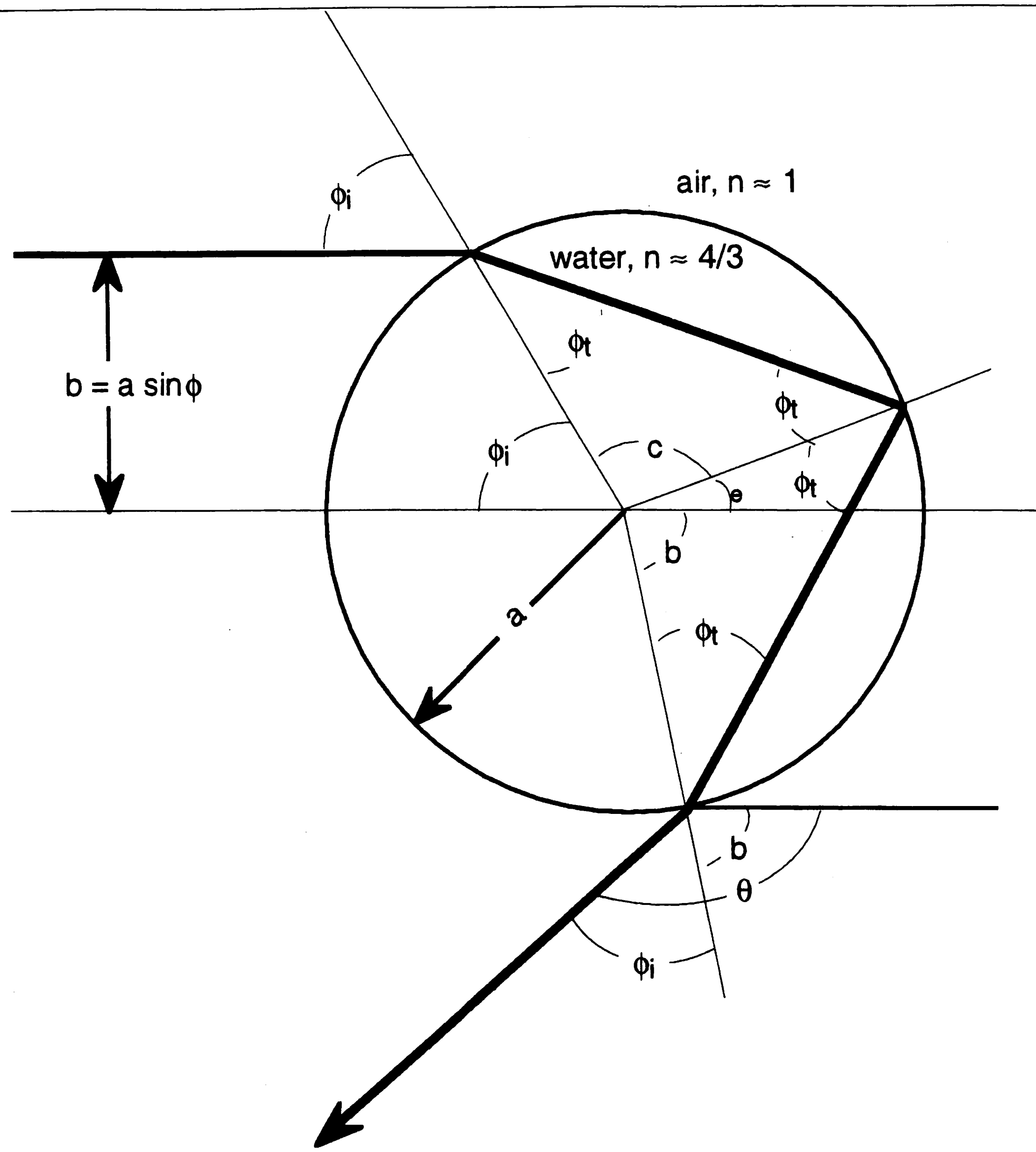

Figure 2. Ray path through a droplet, used to define the various angles.

The rainbow ray, also known as the Descartes ray, is a *ray of minimum deviation* because other rays having different angles of incident ϕ have larger scattering angles. The scattering angle of the rainbow ray, denoted by the angle θ_r, is related to the drop index of refraction by the expression

$$\theta_r = \pi + 2\, sin^{-1}\left(\sqrt{\frac{4-n^2}{3}}\right) - 4\, sin^{-1}\left(\sqrt{\frac{4-n^2}{3n^2}}\right).$$

The familiar 'colors of the rainbow' that we observe occurs because n is slightly different for different colors; hence θ_r is different as well. The initial angle ϕ_r of the rainbow ray is given by

$$\phi_r = \sin^{-1}\left(\sqrt{\frac{4-n^2}{3}}\right).$$

The impact parameter of a ray in Fig. 2 is given by b = a *sin* ϕ where a is the droplet radius. The impact parameter b_r of the rainbow ray is thus

$$b_r = a\sqrt{\frac{4-n^2}{3}}.$$

Experimental measurements of ϕ_r or of b_r and a give us a method for determining the index of refraction n. The technique of determining a particles properties from scattering (or in this case reflection and refraction) measurements is known as the inverse scattering technique.

1. Derive the expression for the rainbow angle θ_r as a function of n. To do this first work out the relationship $\theta(\phi)$. Then from the extremum condition $d\theta(\phi_r)/d\phi = 0$ which determines ϕ_r as a function of n, you get $\theta(\phi_r) = \theta_r$.

2. Calculate the rainbow angle for n = 4/3.

3. In your own words, discuss rainbows. Discuss what causes them and their main features. Assume you are writing for an enlightened undergraduate atmospheric science major who knows a little Optics.

GLORY

Consider glory rays such that $\theta = \pi$.

4. Show that 2 chord glory rays only exist for $\sqrt{2} < n < 2$, where $n = \sqrt{2}$ corresponds to $\phi_i = \pi/2$, and n = 2 corresponds to $\phi_i = 0$. Note that n = 4/3 does not fall in this range.

Thus 2 chord glory rays do not exist for water droplets and visible light such that n = 4/3. van de Hulst hypothesized ray/wave paths along the droplet surface (surface waves) to explain the glory. H. M. Nussenzveig and V. Khare (Phys. Rev. Lett. **38**, 1279-1282, 1977) applied powerful mathematics (Sommerfeld-Watson transformation) to the Mie series solution for spheres to show that the van de Hulst hypothesis is reasonable.