Generalization of complex Snell–Descartes and Fresnel laws

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Generalized Snell–Descartes and Fresnel laws are derived for harmonic inhomogeneous plane waves that are incident upon a static interface between two continuous absorbing dielectric media that are macroscopically characterized by their electric and magnetic permittivities and their conductivities. A coordinate-free formalism based on complex vector algebra is used to carry out all discussions. Surprisingly, the usual complex Snell–Descartes laws for reflection and refraction and Fresnel laws for polarization are recovered only in the special case in which the vector characterizing the direction of inhomogeneity is in the plane of incidence. In the more general case a new deflection angle between planes of incidence and refraction has to be introduced. An experiment is proposed to test this prediction. A generalized form of the TE and TM modes (with respect to the interface), which are elliptically polarized and which are called parallel electric and parallel magnetic modes, also emerges.

INTRODUCTION

The study of light propagation has been central to the emergence of ancient and modern physical sciences and has closely depended on and stimulated the development of mathematics. Let us recall the invention of the optical lunette and the development of the optical microscope at the beginning of the seventeenth century, which effectively opened new realms to human investigation. Shortly after the invention of these revolutionary devices, Descartes discovered the basic laws of geometric optics in 1637, and Huygens and Newton developed the theory of colors in 1704. Most solid foundations of modern classical optics were set up by Fresnel and Maxwell (1820 and 1864, respectively). By the end of the nineteenth century the study of light propagation in moving dielectric media culminated in the establishment of Einstein's theory of special relativity (1905). In the twentieth century our basic knowledge has been mainly completed in the relativistic case (e.g., see Ref. 1 and references therein). However, many authors, among them prestigious ones, continued to study and reveal essential details of the classical nonrelativistic laws of refraction, reflection, energy balance, and reciprocity for plane waves that are incident upon an interface. A partial but already extensive list of papers is provided.[2–23]

In this paper we are concerned with a generalization of the basic laws of refraction and reflection at a static interface, that is, generalized Snell–Descartes and Fresnel laws, which, surprisingly, we have not been able to find in the literature. Our study deals with the reflection and refraction of an arbitrarily polarized harmonic inhomogeneous plane wave (HIPW) at an interface between two homogeneous isotropic absorbing (or amplifying) dielectric media. Before proceeding with a description of HIPW properties, let us review in more detail the research recently carried out in this field.

Several important review articles have focused on different aspects of the problem of electromagnetic propagation of plane waves through an interface. We cite here the contribution of König to the *Handbuch der Physik* (Vol. XX) in 1928,[4] papers by S̃antavý[6] and Knittl[7] on the problem of reversibility, an article by Kizel[8] with 211 references, and a recent review by von Fragstein[9] that summarized a recurrent controversy on the mixed Poynting vector. Among the other papers that we cite are those of Pincherle[10] and Mahan,[11] which are very close to the present article, a quantum treatment of evanescent waves,[12] a derivation based on the equations of molecular optics,[13] and very modern analytical investigations of reciprocity[14–17] and angular momentum balance.[18] Computer[19] and even supercomputer investigations[20] have also recently included the effects that are due to diffraction in beams with finite aperture. Finally, in a very recent paper,[21] Chen and Nelson have shown how to derive Snell–Descartes and Fresnel laws without resorting to boundary conditions. This is due to the fact that their new method, called the wave-vector space method, is an asymptotic method similar to the methods used in quantum scattering. One of the strongest motivations for many recent studies has been the optics of thin films,[22] which plays an increasingly important role in modern optoelectronic technology.[23,25]

HIPW's are stationary solutions of Maxwell's equations for an unbounded medium and take the following form:

$$\mathbf{E} = \frac{1}{2}\{\mathbf{E}_0 \exp[-i(\mathbf{k}\cdot\mathbf{r} - \omega t)] + \text{c.c.}\}, \qquad (1)$$

where ω is real and $\mathbf{k}$ is the complex wave vector. The wave amplitudes may be exponentially decreasing or increasing in a direction perpendicular to the direction of propagation (in a nonabsorbing medium). As a conse-

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quence of the general formulation of our problem, we find a generalized form of the Snell–Descartes and Fresnel laws,[24] which are derived and given below. HIPW's are very natural and very useful in a number of problems, such as the description of evanescent and surface waves and also the well-known exact Sommerfeld solution[26] to the diffraction of a plane wave by a conducting half-plane. HIPW's are also often used in two-dimensional boundary-value problems.[27,28] In such cases the usefulness of HIPW's usually stems from the fact that the path of integration of a usual Fourier transform of an arbitrary bounded distribution can be modified in the complex plane by the use of the Cauchy theorem. The bounded distribution can then be viewed as a linear superposition of HIPW's.

The observability of new effects issuing from the laws derived in this paper is dependent on the possibility of generating experimentally bounded HIPW's that, in a region of space, would be close to idealization, like ordinary homogeneous plane waves. This point is discussed in the Conclusions. Generalized energy balance and reciprocity relations for HIPW will be investigated in a subsequent paper.[29] In that paper arbitrary stacks of absorbing (or amplifying) dielectric layers will be considered because of their importance for the optics of thin films. Because some of the general properties of HIPW's do not seem to be well known, we present them below in detail.

HARMONIC INHOMOGENEOUS PLANE WAVES

Maxwell's equations for the complex components of a HIPW in a homogeneous isotropic medium, characterized by its electric and magnetic permittivities, ϵ and μ, respectively, and its conductivity σ, can be written as

$$\mathbf{k}\cdot\mathbf{E}_0 = \mathbf{k}\cdot\mathbf{H}_0 = 0, \tag{2}$$

$$\mathbf{k}\wedge\mathbf{E}_0 = \omega\mu_0\mu\mathbf{H}_0, \tag{3}$$

$$\mathbf{k}\wedge\mathbf{H}_0 = -\omega\epsilon_0\tilde{\epsilon}\mathbf{E}_0, \tag{4}$$

where $\tilde{\epsilon} = \epsilon - i\sigma/(\omega\epsilon_0)$ is the complex dielectric constant. Here, the complex vector scalar product is to be interpreted as it would be for a real vector. All the usual real and complex vector identities that are extensively used throughout this paper (to keep a coordinate-free formulation[30]) are presented in Appendix A. Note the differences with respect to the frequent scalar product $\mathbf{a}\circ\mathbf{b} = \mathbf{a}^*\cdot\mathbf{b}$.

Equations (2)–(4) include the complex transversality (2) and imply the complex perpendicularity $\mathbf{E}_0\cdot\mathbf{H}_0 = 0$. They do not, however, imply that HIPW's are purely transverse waves or that $\mathbf{E}(\mathbf{r},t)$ is always perpendicular to $\mathbf{H}(\mathbf{r},t)$. Substituting Eq. (3) into Eq. (4) and using Eqs. (A4) and (2), one can find the dispersion relation

$$\mathbf{k}^2 - \omega^2\mu_0\mu\epsilon_0\tilde{\epsilon} = 0. \tag{5}$$

Relation (5) suggests a relation to disentangle the complex wave vector $\mathbf{k}$:

$$\mathbf{k} = \tilde{n}k_0\mathbf{n}, \tag{6}$$

where $\tilde{n} = \sqrt{\mu\tilde{\epsilon}}$ is the complex refractive index, $k_0 = \omega/c$, and $\mathbf{n}$ is a complex unit vector satisfying $\mathbf{n}\cdot\mathbf{n} = 1$. The vector $\mathbf{n}$ can be defined with the inhomogeneity parameter β and two real, perpendicular unit vectors[28]:

$$\mathbf{n} = \hat{n}_\parallel \cosh\beta + i\hat{n}_\perp \sinh\beta. \tag{7}$$

We restrict β to positive values to avoid redundancy (by reversing the sign of both β and $\hat{n}_\perp$). We point out here that β is a parameter whose nature is geometric and whose values may be imposed by boundary conditions at infinity or at an interface (e.g., in the case of evanescent or surface waves). Planes of constant phase are simply defined by $\mathrm{Re}(\mathbf{k})\cdot\mathbf{r} = 0$, and planes of constant amplitude are defined by $\mathrm{Im}(\mathbf{k})\cdot\mathbf{r} = 0$. It is well known[28,30] that planes of constant amplitude and constant phase coincide only when the wave is homogeneous ($\beta = 0$). It is easy to see that this case is equivalent to

$$\mathbf{k}\wedge\mathbf{k}^* = |\tilde{n}|^2k_0^2(2i\sinh\beta\cosh\beta)(\hat{n}_\perp\wedge\hat{n}_\parallel) = 0, \tag{8}$$

which, with the help of Eq. (A3), implies that

$$\mathbf{k}^*\cdot\mathbf{E}_0 = -\frac{1}{\omega\epsilon_0\tilde{\epsilon}}\mathbf{k}^*\cdot(\mathbf{k}\wedge\mathbf{H}_0) = -\frac{1}{\omega\epsilon_0\tilde{\epsilon}}(\mathbf{k}^*\wedge\mathbf{k})\cdot\mathbf{H}_0 = 0. \tag{9}$$

However, the inverse is not true, because using Eqs. (A6) and (2) we can show that

$$(\mathbf{k}\wedge\mathbf{k}^*)\cdot(\mathbf{E}_0\wedge\mathbf{E}_0^*) = -|\mathbf{k}\cdot\mathbf{E}_0^*|^2. \tag{10}$$

Similar considerations can be phrased in terms of the magnetic field $\mathbf{H}_0$ because of the symmetry of Maxwell's equations (2)–(4), but unless they are necessary for clarity we shall omit such symmetric considerations. The magnetic-field component can always be obtained from $\mathbf{E}_0$ by the use of Eq. (3).

POLARIZATION PROPERTIES OF HIPW

HIPW's have the following unusual polarization properties:

1. HIPW's are not purely transverse waves: a longitudinal component $E_{0\parallel}\hat{n}_\parallel$ is implied by the complex transversality condition (2). If we expand the electric field along a right-handed coordinate system $\{\hat{e}_1, \hat{e}_2, \hat{n}_\parallel\}$,

$$\mathbf{E}_0 = E_{01}\hat{e}_1 + E_{02}\hat{e}_2 + E_{0\parallel}\hat{n}_\parallel, \tag{11}$$

the longitudinal component is equal to

$$E_{0\parallel} = (-i\tanh\beta)(E_{01}\cos\psi + E_{02}\sin\psi), \tag{12}$$

where the direction of the inhomogeneity of the wave is specified by the angle ψ such that $\hat{n}_\perp = \hat{e}_1\cos\psi + \hat{e}_2\sin\psi$. However the field vector still oscillates in a plane and along an ellipse. Rewriting Eq. (1) as

$$\mathbf{E} = \{\mathrm{Re}(\mathbf{E}_0)\cos[\mathrm{Re}(\mathbf{k})\cdot\mathbf{r} - \omega t] + \mathrm{Im}(\mathbf{E}_0)\sin[\mathrm{Re}(\mathbf{k})\cdot\mathbf{r} - \omega t]\}\exp[-\mathrm{Im}(\mathbf{k})\cdot\mathbf{r}], \tag{13}$$

we see that the normal to the oscillation plane of the electric-field vector is given by

$$\hat{n}_E \sim \mathrm{Re}(\mathbf{E}_0)\wedge\mathrm{Im}(\mathbf{E}_0) = -\frac{1}{4i}\mathbf{E}_0\wedge\mathbf{E}_0^*. \tag{14}$$

Of course the normal remains undefined for E-linearly polarized waves precisely characterized by

$$\mathbf{E}_0 \wedge \mathbf{E}_0^* = 0. \tag{15}$$

This relation stems from the fact that in an E-linearly polarized wave the electric-field derivative is always parallel to the electric-field vector itself [$\mathbf{E} \wedge \partial\mathbf{E}/\partial t = 0$ is equivalent to Eq. (15)].

2. Interestingly enough, HIPW's cannot exhibit simultaneous linear polarization for $\mathbf{E}_0$ and $\mathbf{H}_0$.[30] To demonstrate this property let us examine the vectorial product $\mathbf{k} \wedge \mathbf{k}^*$ and substitute the following relationship [obtained from Eqs. (A4) and (2)]:

$$\mathbf{E}_0^2\mathbf{k} = -\mathbf{E}_0 \wedge (\mathbf{E}_0 \wedge \mathbf{k}). \tag{16}$$

With the help of Eqs. (3) and (A7) we find that

$$\begin{aligned}(\mathbf{E}_0^2\mathbf{E}_0^{*2})(\mathbf{k} \wedge \mathbf{k}^*) &= (\omega\mu_0\mu)^2(\mathbf{E}_0 \wedge \mathbf{H}_0) \wedge (\mathbf{E}_0^* \wedge \mathbf{H}_0^*) \\ &= (\omega\mu_0\mu)^2[(\mathbf{H}_0 \wedge \mathbf{H}_0^*) \cdot \mathbf{E}_0]\mathbf{E}_0^* \\ &\quad + [(\mathbf{E}_0 \wedge \mathbf{E}_0^*) \cdot \mathbf{H}_0]\mathbf{H}_0^*. \end{aligned} \tag{17}$$

Thus, if E and H are simultaneously linearly polarized, the right-hand side vanishes and the wave is necessarily homogeneous [cf. Eq. (8)].

3. On the other hand a circularly polarized HIPW is always simultaneously circular for E and H.[30] A circular polarization is characterized by a constant $|\mathbf{E}|$, which implies [using Eq. (1)] that

$$\mathbf{E}_0^2 = 0. \tag{18}$$

Using Eqs. (4) and (2) it is easy to see that this is also equivalent to

$$\mathbf{H}_0^2 = 0 \Leftrightarrow \mathbf{E}_0 \wedge \mathbf{H}_0 = 0, \tag{19}$$

where the last equivalence follows directly from Eq. (16). To prove the statement concerning circular polarization, it suffices to notice that $\mathbf{H}_0^2 = 0 \Leftrightarrow |\mathbf{H}| = \text{constant}$.

4. We have seen that, because of the longitudinal component, the plane of oscillation of $\mathbf{E}$ and $\mathbf{H}$ is not perpendicular to the direction of propagation $\hat{n}_\parallel$ for a HIPW. Moreover we can show that the magnetic field $\mathbf{H}$ is not in general perpendicular to the electric field unless one polarization is linear:

$$\mathbf{E} \cdot \mathbf{H} = \frac{1}{2}\,\mathrm{Re}(\mathbf{E}_0 \cdot \mathbf{H}_0^*) \sim (\tilde{n}\mathbf{n} - \tilde{n}^*\mathbf{n}^*) \cdot (\mathbf{E}_0 \wedge \mathbf{E}_0^*). \tag{20}$$

Equation (20) also shows that the fields are always perpendicular (whatever the polarization) in the usual degenerate case, in which the wave is homogeneous and the medium is nonabsorbing.

5. Finally, we characterize the polarization of a HIPW with Jones's parameterization[31]:

$$E_{01} = A\exp(i\delta)(\cos\varphi\cos\varepsilon - i\sin\varphi\sin\varepsilon), \tag{21}$$

$$E_{02} = A\exp(i\delta)(\sin\varphi\cos\varepsilon - i\cos\varphi\sin\varepsilon). \tag{22}$$

Here φ is the azimuth and ε is the ellipticity angle of the field with respect to the direct reference frame $\hat{e}_1, \hat{e}_2, \hat{n}_\parallel$. A and δ are the amplitude and the absolute phase of the field, respectively.

The complex Poynting vector for HIPW's, the Poynting theorem, and the mixed Poynting vector will be discussed in a subsequent paper on energy balance and reciprocity relations.[29]

REFLECTION AND REFRACTION OF A HIPW AT AN INTERFACE

To study the transmission of an arbitrary HIPW that is incident upon an interface between two homogeneous isotropic media, we now carefully complete the parameterization. We suppose that the interface includes the origin, its direction being specified by the real unit vector $\hat{s}$ normal to the surface. The following notations are adopted for each wave: the incident wave is denoted by the subscript i, the reflected wave is denoted by the subscript r, and the transmitted wave is denoted by the subscript t. We use a polar parameterization for the complex refractive indices of the two media $\tilde{n}_j = |\tilde{n}_j|\exp(i\eta_j)$, where $j = i, r, t$. To characterize the respective orientations of the waves and the interface we must introduce the angles of incidence, reflection, and refraction defined by

$$\cos\theta_j = \hat{s} \cdot \hat{n}_{\parallel j}, \qquad \sin\theta_j = \hat{e}_{ij} \cdot (\hat{s} \wedge \hat{n}_{\parallel j}), \tag{23}$$

where $j = i, r, t$. Let us note that this definition finally fixes the direction of the real unit vector $\hat{e}_1$ [as well as the respective right-handed coordinate system of the incident wave $\{\hat{e}_{1i}, \hat{e}_{2i}, \hat{n}_{\parallel i}\}$; see Eq. (11)]. $\hat{e}_{1i}$ is the direction perpendicular to the plane of incidence, and we also choose $\hat{e}_{1i} = \hat{e}_{1r}$. This choice is important for the description of inhomogeneity direction and the polarization. To fix the respective orientation of $\hat{e}_{1t}$ [and the respective right-handed coordinate system of the refracted wave $\{\hat{e}_{1t}, \hat{e}_{2t}, \hat{n}_{\parallel t}\}$; see Eq. (11)], a preliminary study has shown that we must introduce a new angle Δ defined by

$$\cos\Delta = \hat{e}_{1i} \cdot \hat{e}_{1t}, \qquad \sin\Delta = \hat{s} \cdot (\hat{e}_{1i} \wedge \hat{e}_{1t}). \tag{24}$$

The appearance of this angle is new in the context of this problem, and we call it the deflection angle because it is the angle between the planes of incidence and refraction. In Fig. 1 we give a pictorial representation of the angles involved.

KINEMATICS: GENERALIZED SNELL–DESCARTES LAWS

The phase-matching condition for the waves at the interface requires that

$$\mathbf{k}_i \cdot \mathbf{r}_\parallel = \mathbf{k}_r \cdot \mathbf{r}_\parallel = \mathbf{k}_t \cdot \mathbf{r}_\parallel, \tag{25}$$

where $\mathbf{r}_\parallel$ is such that $\mathbf{r}_\parallel = -\hat{s} \wedge (\hat{s} \wedge \mathbf{r}_\parallel)$. Substituting in Eq. (25) the complex decomposition of $\mathbf{k}$ in perpendicular and parallel components $\mathbf{k} = (\hat{s} \cdot \mathbf{k})\hat{s} - \hat{s} \wedge (\hat{s} \wedge \mathbf{k})$ and using Eq. (A3), we obtain

$$\begin{aligned}(\hat{s} \wedge \mathbf{k}_i) \cdot (\hat{s} \wedge \mathbf{r}_\parallel) &= (\hat{s} \wedge \mathbf{k}_r) \cdot (\hat{s} \wedge \mathbf{r}_\parallel) \\ &= (\hat{s} \wedge \mathbf{k}_t) \cdot (\hat{s} \wedge \mathbf{r}_\parallel). \end{aligned} \tag{26}$$

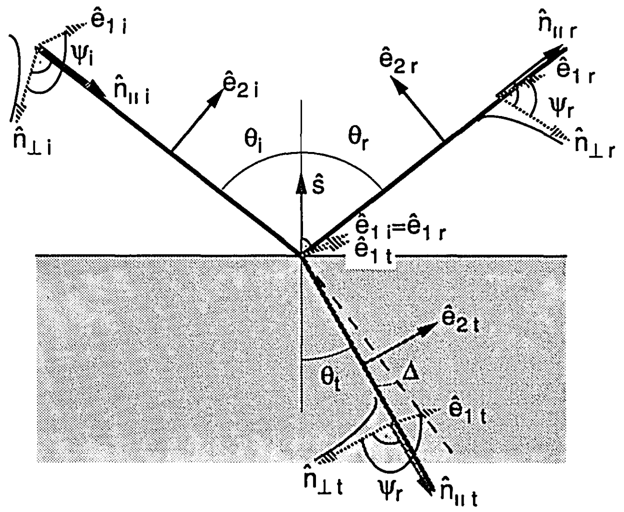

Fig. 1. Coordinate systems, vectors, and angles introduced in the text to describe the incident, reflected, and transmitted HIPW's at the interface. A figurative exponential tail has been drawn along $\hat{n}_\perp$ to represent the HIPW. Although not well represented, $\hat{n}_{\perp j}$ is always perpendicular to $\hat{n}_{\parallel j}$ and makes an angle ψ_j with $\hat{e}_{1j}$ $(j = i, r, t)$.

Because Eq. (26) is valid for any $\mathbf{r}_\parallel$ we see that the complex in-plane components of $\mathbf{k}$ must be conserved[30,32]:

$$\hat{s} \wedge \mathbf{k}_i = \hat{s} \wedge \mathbf{k}_r = \hat{s} \wedge \mathbf{k}_t . \tag{27}$$

Together with the dispersion relation (5) Eq. (27) also implies for the normal component of the reflected wave vector,

$$\hat{s} \cdot \mathbf{k}_r = -\hat{s} \cdot \mathbf{k}_i , \tag{28}$$

as expected.

The apparent simplicity of Eq. (27) hides a generalization of the complex Snell–Descartes laws. To show this generalization we project these equations on the in-plane basis $\{\hat{e}_{1i}, \hat{s} \wedge \hat{e}_{1i}\}$ and extensively use the vector identities of Appendix A and the parameterization of the HIPW's detailed above. After lengthy calculations we find for reflection

$$\cosh \beta_r \sin \theta_r = \cosh \beta_i \sin \theta_i , \tag{29}$$

$$\sinh \beta_r \sin \psi_r \cos \theta_r = \sinh \beta_i \sin \psi_i \cos \theta_i , \tag{30}$$

$$\sinh \beta_r \cos \psi_r = \sinh \beta_i \cos \psi_i . \tag{31}$$

For refraction we find

$$|\tilde{n}_t| \cosh \beta_t \sin \theta_t \cos \Delta = |\tilde{n}_i| [\cos(\eta_i - \eta_t) \cosh \beta_i \sin \theta_i + \sin(\eta_i - \eta_t) \sinh \beta_i \sin \psi_i \cos \theta_i] , \tag{32}$$

$$|\tilde{n}_t| \cosh \beta_t \sin \theta_t \sin \Delta = -|\tilde{n}_i| \sin(\eta_i - \eta_t) \sinh \beta_i \cos \psi_i , \tag{33}$$

$$|\tilde{n}_t| (\sinh \beta_t) [\cos \psi_t \sin \Delta + \sin \psi_t \cos \theta_t \cos \Delta] = |\tilde{n}_i| [-\sin(\eta_i - \eta_t) \cosh \beta_i \sin \theta_i + \cos(\eta_i - \eta_t) \sinh \beta_i \sin \psi_i \cos \theta_i] , \tag{34}$$

$$|\tilde{n}_t| (\sinh \beta_t) (\cos \psi_t \cos \Delta - \sin \psi_t \cos \theta_t \sin \Delta) = |\tilde{n}_i| \cos(\eta_i - \eta_t) \sinh \beta_i \cos \psi_i . \tag{35}$$

The generalized Snell–Descartes laws shown in Eqs. (29)–(31) for reflection and in Eqs. (32)–(35) for refraction differ considerably from the well-known complex Snell–Descartes laws:

$$\sin \tilde{\theta}_r = \sin \tilde{\theta}_i , \tag{36}$$

$$\tilde{n}_t \sin \tilde{\theta}_t = \tilde{n}_i \sin \tilde{\theta}_i , \tag{37}$$

$$\tilde{\theta}_j = \theta_j - i\beta_j , \qquad j = i, r, t . \tag{38}$$

These complex laws are recovered only in the special case

$$\psi_i = \frac{\pi}{2} \Rightarrow \psi_t = -\psi_r = \frac{\pi}{2} . \tag{39}$$

In vectorial form this condition may also be expressed as

$$\hat{s} \cdot (\mathbf{n} \wedge \mathbf{n}^*) = 0 \quad \text{or} \quad \hat{s} \cdot (\hat{n}_\parallel \wedge \hat{n}_\perp) = 0 . \tag{40}$$

The simplified form of the Snell–Descartes laws in this case is due to the high symmetry resulting from the invariance under lateral spatial translations. Another special case would be given by $\hat{s} \cdot \hat{n}_\perp = 0$, but apparently it does not lead to great simplifications.

A number of other consequences and special cases of the general laws (29)–(31) and (32)–(35) are worth discussing:

(a) We note for the first time the appearance of the deflection angle Δ, which is the angle between the planes of incidence and refraction on each side of the interface.

(b) The reflection laws (29)–(31) determine uniquely the alternative reflection laws

$$\theta_r = \pi - \theta_i , \tag{41}$$

$$\psi_r = -\psi_i , \tag{42}$$

$$\beta_r = \beta_i . \tag{43}$$

The proof is as follows: We can substitute Eqs. (29) and (31) into Eq. (30) by the use of trigonometric and hyperbolic sum rules. Solving the resulting second-order equation by division with the trivial solution $\sinh^2 \beta_r = \sinh^2 \beta_i$ yields

$$\sinh^2 \beta_r = \sinh^2 \beta_i \sin^2 \theta_i \cos^2 \psi_i - \cos^2 \theta_i . \tag{44}$$

Because all quantities are real this equation implies that $\beta_r \leq \beta_i$. But, because Eqs. (29)—(31) are symmetric with respect to the interchange $i \leftrightarrow r$, a symmetric relation $\beta_i \leq \beta_r$ also holds. Therefore, β_r is equal to β_i, which implies Eqs. (41)–(43). A last remark is that the unusual law $\theta_r = \pi - \theta_i$ (instead of $\theta_r = -\theta_i$) comes from our definition (23).

(c) We investigate the conditions for the occurrence of the unusual case $\Delta \neq 0$. From Eq. (33) we deduce that this unusual case occurs only when the right-hand side of Eq. (33) is nonzero. Therefore, for purely inhomogeneous waves, when Eq. (39) fails, and when the arguments of the complex refractive indices of the two media are different, the deflection angle does not vanish.

(d) Finally, Eqs. (34) and (35) give us a hint about the possible ways to generate purely inhomogeneous waves from a homogeneous incident wave. If $\beta_i = 0$, we can find with a little algebra that

$$|\tilde{n}_t|\sinh \beta_t \sin \psi_t \cos \theta_t = -|\tilde{n}_i|\sin(\eta_i - \eta_t)(\sin \theta_i \cos \Delta). \tag{45}$$

This means that generally $\beta_t \neq 0$ only if $(\eta_i - \eta_t) \neq 0$. The special cases are when the transmitted wave is evanescent ($\theta_t = \pi/2$) or when $\psi_t = 0$ or $\Delta = \pi/2$.

Numerically the direct solution of the generalized laws (32)–(35) in terms of the parameters of the transmitted wave involves finding the unique solution of four simultaneous nonlinear equations. However, the multivaluedness of the inverse trigonometric functions and the many different conditions for signs and bounds of the unknown variables makes direct solution complicated. We preferred to obtain the transmitted parameters by disentangling the transmitted wave vector, which can be simply calculated using the dispersion relation (5) and the conservation of the tangential components (27). The deflection angle can then be obtained by forming the scalar products involved in its definition (24). We checked that the generalized laws (32)–(35) were satisfied by this numerical solution.

DYNAMICS: GENERALIZED POLARIZATIONS AND GENERALIZED FRESNEL LAWS

At the interface the integral form of Maxwell's equations allows us to derive the continuity or discontinuity equations for the fields between the two media:

$$\hat{s} \wedge \mathbf{E}_i + \hat{s} \wedge \mathbf{E}_r - \hat{s} \wedge \mathbf{E}_t = 0, \tag{46}$$

$$\mu_i \hat{s} \cdot (\mathbf{H}_i + \mathbf{H}_r) - \mu_t \hat{s} \cdot (\mathbf{H}_t) = 0, \tag{47}$$

$$\hat{s} \wedge \mathbf{H}_i + \hat{s} \wedge \mathbf{H}_r - \hat{s} \wedge \mathbf{H}_t = 0, \tag{48}$$

$$\epsilon_i \hat{s} \cdot (\mathbf{E}_i + \mathbf{E}_r) - \epsilon_t \hat{s} \cdot (\mathbf{E}_t) = \frac{\tilde{\Sigma}}{2}. \tag{49}$$

In the derivation of Eqs. (46)–(49) we used the fact that the surface current density vanishes if the quantities ϵ, μ, and σ remain constant infinitely close to the interface.[32] As a consequence, the parallel component of $\mathbf{H}$ remains continuous. On the other hand the surface charge density $\tilde{\Sigma}$, which was retained in Eq. (49), does not vanish whenever

$$\epsilon_i \sigma_t - \epsilon_t \sigma_i \neq 0. \tag{50}$$

This is easily checked by examining the equation for the conservation of charge and Ohm's law on each side of the interface (see, for example, Ref. 32, p. 555).

Generalized TE and TM polarizations with respect to the interface can be defined only by use of complex orthogonal unit vectors,

$$\mathbf{q}_1 = \frac{\hat{s} \wedge \mathbf{n}}{[(\hat{s} \wedge \mathbf{n})^2]^{1/2}}, \qquad \mathbf{q}_2 = n \wedge \mathbf{q}_1, \tag{51}$$

satisfying $\mathbf{q}_1{}^2 = \mathbf{q}_2{}^2 = 1$, $\mathbf{q}_1 \cdot \mathbf{q}_2 = 0$ (but $\mathbf{q}_1 \cdot \mathbf{q}_2{}^* \neq 0$), and $\mathbf{q}_1 = (\mathbf{q}_2 \wedge \hat{s})/(\hat{s} \cdot \mathbf{n})$. The projections $(\mathbf{E}_{\mathrm{PE}}, \mathbf{E}_{\mathrm{PM}})$ and $(\mathbf{H}_{\mathrm{PM}}, \mathbf{H}_{\mathrm{PE}})$ of the electric and magnetic fields $\mathbf{E}_0$ and $\mathbf{H}_0$ along $(\mathbf{q}_1, \mathbf{q}_2)$, respectively, are defined by

$$\mathbf{E}_{\mathrm{PE}} = E_{\mathrm{PE}} \mathbf{q}_1, \qquad \mathbf{H}_{\mathrm{PE}} = H_{\mathrm{PE}} \mathbf{q}_2, \tag{52}$$

$$\mathbf{E}_{\mathrm{PM}} = E_{\mathrm{PM}} \mathbf{q}_2, \qquad \mathbf{H}_{\mathrm{PM}} = H_{\mathrm{PM}} \mathbf{q}_1 \tag{53}$$

and satisfy independently Maxwell's equations (2)–(4) and are not coupled by the interface. We can also write the complex amplitudes as

$$E_{\mathrm{PE}} = \mathbf{q}_1 \cdot \mathbf{E}_0, \qquad H_{\mathrm{PE}} = \mathbf{q}_2 \cdot \mathbf{H}_0, \tag{54}$$

$$E_{\mathrm{PM}} = \mathbf{q}_2 \cdot \mathbf{E}_0, \qquad H_{\mathrm{PM}} = \mathbf{q}_1 \cdot \mathbf{H}_0. \tag{55}$$

Because these generalized modes possess a longitudinal component we cannot meaningfully use the designation generalized TE modes. We prefer to call them parallel electric (PE) modes because the polarization ellipse of the electric field $\mathbf{E}_{\mathrm{PE}} = E_{\mathrm{PE}} \mathbf{q}_1$ always remain parallel to the interface plane ($\hat{s} \cdot \mathbf{E}_{\mathrm{PE}} = 0$). Similar considerations hold of course for the parallel magnetic (PM) modes. Using Jones's parameterization we can easily see that these modes are in general elliptically polarized, e.g.,

$$\mathbf{E}_{\mathrm{PE}} = A_{\mathrm{PE}} \exp(i\delta_{\mathrm{PE}})[\mathbf{e}_{\mathrm{PE}}(-i \tanh \beta)(\hat{n}_\perp \cdot \mathbf{e}_{\mathrm{PE}})\hat{n}_\parallel], \tag{56}$$

where

$$\mathbf{e}_{\mathrm{PE}} = \frac{\hat{e}_1 + \chi_{\mathrm{PE}} \hat{e}_2}{(1 + |\chi_{\mathrm{PE}}|^2)^{1/2}}, \tag{57}$$

$$\chi_{\mathrm{PE}} = \frac{i \tanh \beta \cos \theta \cos \psi}{\sin \theta - i \tanh \beta \cos \theta \sin \psi}. \tag{58}$$

Solving the boundary conditions (46)–(49) we derive the generalized Fresnel laws for PE-polarized HIPW's:

$$E_{\mathrm{PE}r} = r_{\mathrm{PE}} E_{\mathrm{PE}i} \quad \text{with} \quad r_{\mathrm{PE}} = \left(\frac{\hat{s} \cdot \mathbf{k}_i/\mu_i - \hat{s} \cdot \mathbf{k}_t/\mu_t}{\hat{s} \cdot \mathbf{k}_i/\mu_i + \hat{s} \cdot \mathbf{k}_t/\mu_t}\right), \tag{59}$$

$$E_{\mathrm{PE}t} = t_{\mathrm{PE}} E_{\mathrm{PE}i} \quad \text{with} \quad t_{\mathrm{PE}} = \left(\frac{2\hat{s} \cdot \mathbf{k}_i/\mu_i}{\hat{s} \cdot \mathbf{k}_i/\mu_i + \hat{s} \cdot \mathbf{k}_t/\mu_t}\right). \tag{60}$$

For PM-polarized HIPW's,

$$H_{\mathrm{PM}r} = r_{\mathrm{PM}} H_{\mathrm{PM}i} \quad \text{with} \quad r_{\mathrm{PM}} = \left(\frac{\hat{s} \cdot \mathbf{k}_i/\tilde{\epsilon}_i - \hat{s} \cdot \mathbf{k}_t/\tilde{\epsilon}_t}{\hat{s} \cdot \mathbf{k}_i/\tilde{\epsilon}_i + \hat{s} \cdot \mathbf{k}_t/\tilde{\epsilon}_t}\right), \tag{61}$$

$$H_{\mathrm{PM}t} = t_{\mathrm{PM}} H_{\mathrm{PM}i} \quad \text{with} \quad t_{\mathrm{PM}} = \left(\frac{2\hat{s} \cdot \mathbf{k}_i/\tilde{\epsilon}_i}{\hat{s} \cdot \mathbf{k}_i/\tilde{\epsilon}_i + \hat{s} \cdot \mathbf{k}_t/\tilde{\epsilon}_t}\right). \tag{62}$$

We have kept these laws in condensed form because developing the complex scalar products in terms of the parameterization is cumbersome. Because most often one deals with electric fields,[28] we note here that Eq. (61) holds also for E_{PM} because $E_{\mathrm{PM}} = -(\tilde{n}/\omega\epsilon_0\tilde{\epsilon})H_{\mathrm{PM}}$ and that Eq. (62) can be replaced by the following nonsymmetrical expression:

$$E_{\mathrm{PM}t} = \left[\frac{2\tilde{n}_t(\hat{s} \cdot \mathbf{k}_i)}{\tilde{n}_i(\tilde{\epsilon}_t/\tilde{\epsilon}_i)(\hat{s} \cdot \mathbf{k}_i) + \tilde{n}_i(\hat{s} \cdot \mathbf{k}_t)}\right] E_{\mathrm{PM}i}. \tag{63}$$

From Eq. (49) the surface charge density at the interface can be evaluated. We note that PE waves do not generate a surface charge density. We have checked that the expressions of Fresnel coefficients for PM waves r_{PM} and t_{PM} were compatible with the continuity equation for charges at the interface cited above [relation (50)].

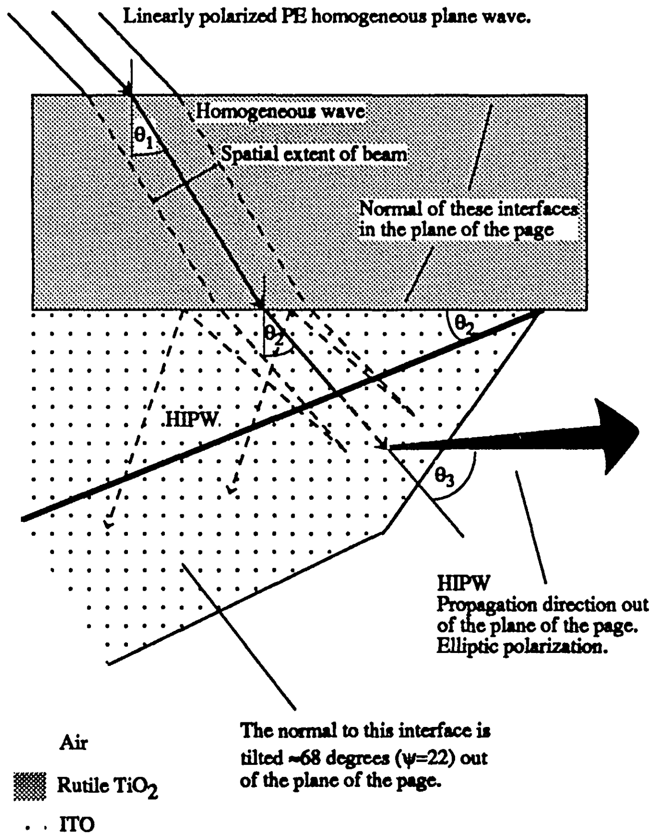

Fig. 2. Proposed experiment to test the generalized Snell–Descartes and Fresnel laws in an ITO wedge on a rutile slab. A homogeneous plane wave is incident upon the rutile slab, coupled onto the rutile–ITO interface, where the HIPW is generated. The orientation of the ITO–air final interface is such that $\psi \neq \pi/2$; therefore only generalized laws are valid for that interface. Note that the different angles and parameters are indicated in Table 1 and are not to scale in this figure.

To conclude this section let us mention first that the transmission of an arbitrarily polarized wave can be calculated using the principle of linear superposition in PE and PM modes with respect to this interface. Second, the Fresnel coefficients r_{PE}, t_{PE}, r_{PM}, t_{PM} given by Eqs. (59)–(62), respectively, also play an important role for the formulation of energy balance and reciprocity relations, which are extensively discussed in a subsequent paper.[29]

PROPOSAL FOR AN EXPERIMENT

An experimental verification of the generalized laws can be made under various circumstances. In Fig. 2 we propose a case that could be realized in practice to generate a HIPW and demonstrate a deflection. Equation (33) indicates that such a deflection occurs only under restricted conditions, that is, when the right-hand side of Eq. (33) is nonzero [e.g., at an interface between an absorbing and a transparent medium, when Eq. (39) fails]. Therefore we might expect to observe a deflection at the second interface of an absorbing wedge for particular orientations of an incoming ordinary plane wave. The implicit assumption here is that the lateral spatial extent of the incoming beam is so small that a superposition of the multiply reflected waves in the wedge at the second interface is avoided. In Fig. 2 the different media are air, rutile (TiO_2), indium tin oxide ($ITO:S_nO_2/In_2O_3$), and air, which have the following refractive indices at a wavelength of 633 nm: 1, 2.6, 2, and 1, respectively. Only the ITO wedge is slightly absorbing; its absorption coefficient $\alpha = -2k_0 \operatorname{Im}(\tilde{n})$ is equal to 0.214 μm^{-1}. The role of the rutile–ITO interface is to create the HIPW, and the role of the ITO–air interface (which is tilted with respect to the normal out of the plane of the page) is to show the peculiarities of the generalized laws. The calculated angles (angles of incidence, reflection, refraction, deflection, etc.) are displayed in Table 1. Near normal incidence the deflection angle Δ has an appreciable magnitude of 27°, and the wave has a slightly elliptical polarization.

CONCLUSIONS

We have derived a generalization of complex Snell–Descartes and Fresnel laws for the most general kind of plane wave, that is, the HIPW. The polarization properties of HIPW's and the form of generalized PE and PM modes have been determined. The theory does not include anisotropic materials (birefringence) or partial polarization states. It includes arbitrary values of all the wave quantities specified by the parameterization angles θ_i, θ_r, θ_t, ψ_i, ψ_r, ψ_t, Δ; the plane-wave inhomogeneities β_i, β_r, β_t; and homogeneous isotropic dissipative media, characterized by the complex refractive indices $\tilde{n}_i$, $\tilde{n}_t$. The theory can describe numerous known phenomena, such as total internal reflection, polarization by reflection (Brewster), metallic-type reflection, refraction of metals, the skin effect, and surface waves at the interface. New phenomena are predicted by the present theory, which cannot be described by standard reflection and refraction laws, when the incident wave includes an inhomogeneity in the transverse direction [when Eq. (39) fails]. In particular a deflection of the HIPW by the interface [characterized by the deflection angle Δ, Eqs. (24)] is predicted when the condition (50) is satisfied. These effects should be observable in the limited region of space where edge diffraction

Table 1. Angles and Parameters of the HIPW and of Each of the Layers in the Proposed Experiment[a]

Layer	n	α (μm^{-1})	θ_i	ψ	β	ϕ	ϵ	Δ
Rutile	2.6	0	47	90	0	0	0	0
ITO with respect to first interface	2.0	0.214	71.92	−90	1.57×10^{-2}	0	0	0
ITO with respect to second interface	2.0	0.214	0.01	22	1.57×10^{-2}	0	0	0
Air	1.0	0	1.86×10^{-2}	−5.16	3.13×10^{-2}	27.16	2.4×10^{-3}	27.16

[a]The symbols are defined in the text, the material parameters corespond to a wavelength of 633 nm, and the angles are expressed in degrees.

effects that are due to the finite lateral spatial extension of the incoming beam are negligible. To the best of our knowledge, these new phenomena do not seem to have been noticed before,[27,28,33] because in all usual problems starting from ordinary homogeneous plane waves that are incident upon an arbitrary set of parallel interfaces Eq. (39) remains valid. An experiment has been proposed to demonstrate the generalized laws and the deflection. Finally, it should also be mentioned that once a HIPW is generated the standard complex Snell–Descartes and Fresnel laws fail also at an interface between pure dielectrics even if the wave is not deflected. It is not yet clear whether such effects could be usefully exploited in optical devices. Thin-film technology might one day face layers with nonparallel interfaces, where the phenomena discussed here might become important. Further generalizations for static or moving interfaces between birefringent or optically active media should be possible in the classical[30] and the relativistic cases.[1]

APPENDIX A. COMPLEX VECTORIAL ALGEBRA

Because complex vector identities are not usually well known and play an important role in this series of papers, we present the main results for the two kinds of scalar product:

Let **a**, **b**, **c**, **d** be four arbitrary complex vectors. In this paper the scalar product and the vectorial product are defined as

$$\mathbf{a}\cdot\mathbf{b}=\sum_{i=1}^{3}a_ib_i=\mathbf{b}\cdot\mathbf{a}, \tag{A1}$$

$$\mathbf{a}\wedge\mathbf{b}=\sum_{i=1}^{3}\epsilon_{ijk}a_jb_k=-\mathbf{b}\wedge\mathbf{a}, \tag{A2}$$

respectively, where ϵ_{ijk} is the fully antisymmetric tensor of rank two. In this case all the usual vector identities that hold for real vectors are valid:

$$\mathbf{a}\cdot(\mathbf{b}\wedge\mathbf{c})=\mathbf{c}\cdot(\mathbf{a}\wedge\mathbf{b})=\mathbf{b}\cdot(\mathbf{c}\wedge\mathbf{a}), \tag{A3}$$

$$\mathbf{a}\wedge(\mathbf{b}\wedge\mathbf{c})=(\mathbf{a}\cdot\mathbf{c})\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{c}, \tag{A4}$$

$$\mathbf{a}\wedge(\mathbf{b}\wedge\mathbf{c})+\mathbf{b}\wedge(\mathbf{c}\wedge\mathbf{a})+\mathbf{c}\wedge(\mathbf{a}\wedge\mathbf{b})=0, \tag{A5}$$

$$(\mathbf{a}\wedge\mathbf{b})\cdot(\mathbf{c}\wedge\mathbf{d})=(\mathbf{c}\cdot\mathbf{a})(\mathbf{d}\cdot\mathbf{b})-(\mathbf{d}\cdot\mathbf{a})(\mathbf{c}\cdot\mathbf{b}), \tag{A6}$$

$$\begin{aligned}(\mathbf{a}\wedge\mathbf{b})\wedge(\mathbf{c}\wedge\mathbf{d})&=[(\mathbf{c}\wedge\mathbf{d})\cdot\mathbf{a}]\mathbf{b}-[(\mathbf{c}\wedge\mathbf{d})\cdot\mathbf{b}]\mathbf{a}\\&=[(\mathbf{b}\wedge\mathbf{d})\cdot\mathbf{a}]\mathbf{c}-[(\mathbf{b}\wedge\mathbf{c})\cdot\mathbf{a}]\mathbf{d}.\end{aligned} \tag{A7}$$

A last very useful identity allows us to perform the decomposition of an arbitrary real or complex vector **x** along three noncollinear vectors:

$$\begin{aligned}[(\mathbf{b}\wedge\mathbf{c})\cdot\mathbf{a}]\mathbf{x}&=[(\mathbf{b}\wedge\mathbf{c})\cdot\mathbf{x}]\mathbf{a}+[(\mathbf{c}\wedge\mathbf{a})\cdot\mathbf{x}]\mathbf{b}\\&\quad+[(\mathbf{a}\wedge\mathbf{b})\cdot\mathbf{x}]\mathbf{c}.\end{aligned} \tag{A8}$$

On the other hand the scalar product for complex vectors is sometimes defined as

$$\mathbf{a}\circ\mathbf{b}=\sum_{i=1}^{3}a_i^*b_i=\mathbf{a}^*\cdot\mathbf{b} \tag{A9}$$

because the real norm of a complex vector is then properly given by the scalar product of the vector with itself. This definition is somewhat awkward because most identities (A3)–(A8) do not remain valid. A simple way that we found to solve this problem is to redefine the vectorial product:

$$\mathbf{a}\overset{\circ}{\wedge}\mathbf{b}=\sum_{i=1}^{3}\epsilon_{ijk}a_j^*b_k^*=\mathbf{a}^*\wedge\mathbf{b}^*=-\mathbf{b}\overset{\circ}{\wedge}\mathbf{a}. \tag{A10}$$

The identities (A3)–(A8) can now be used. It must, however, be emphasized that the order of the arguments in Eqs. (A3)–(A8) has been carefully chosen and must now be absolutely respected because the scalar product is not commutative anymore: $\mathbf{a}\circ\mathbf{b}\neq\mathbf{b}\circ\mathbf{a}$.

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