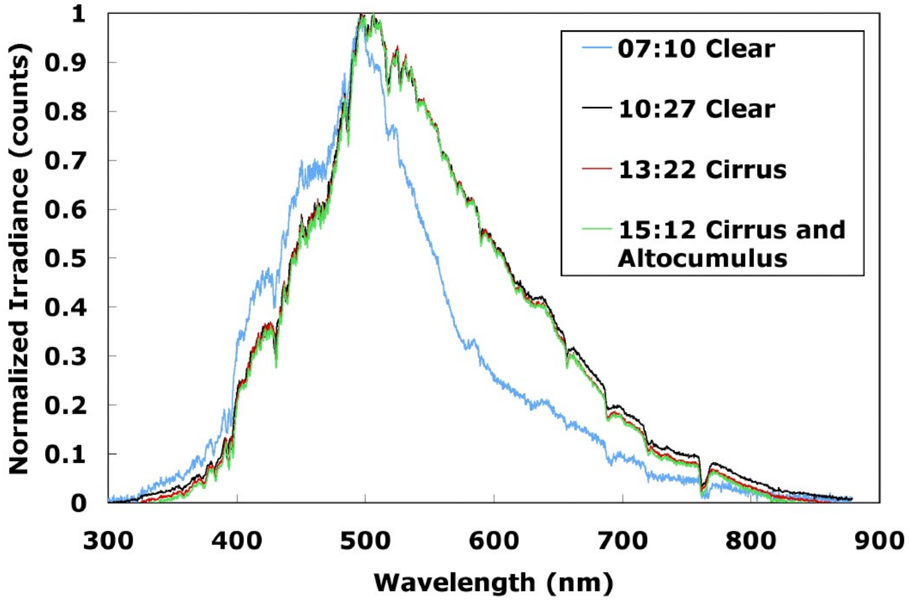
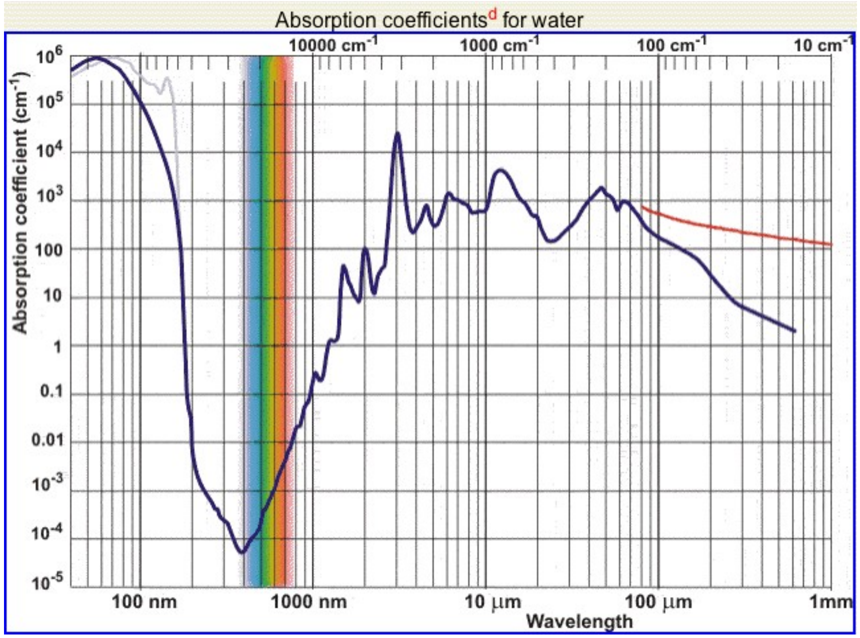
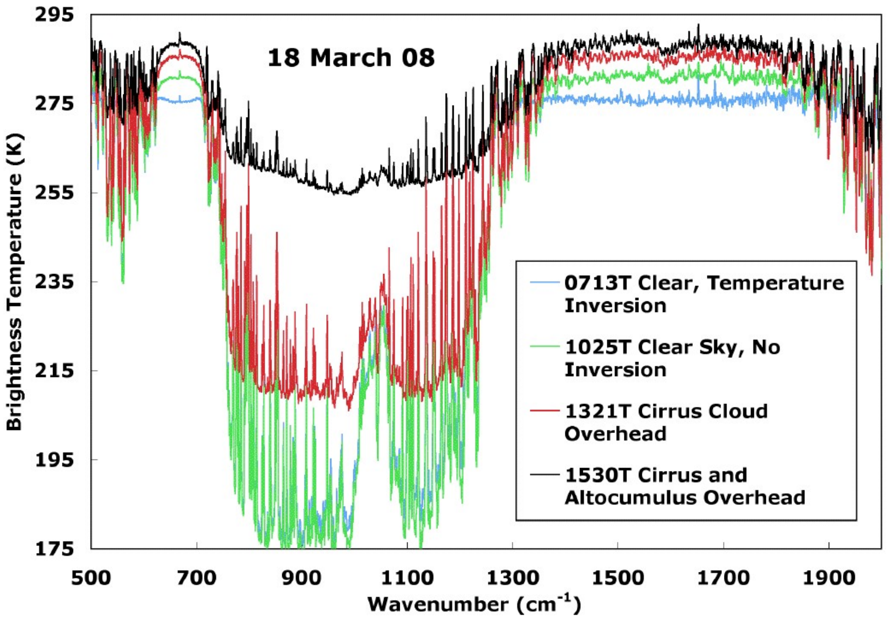
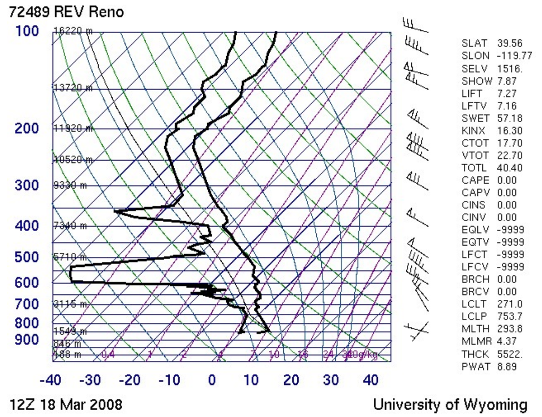
Chapter 4: Atmospheric Radiation Transfer

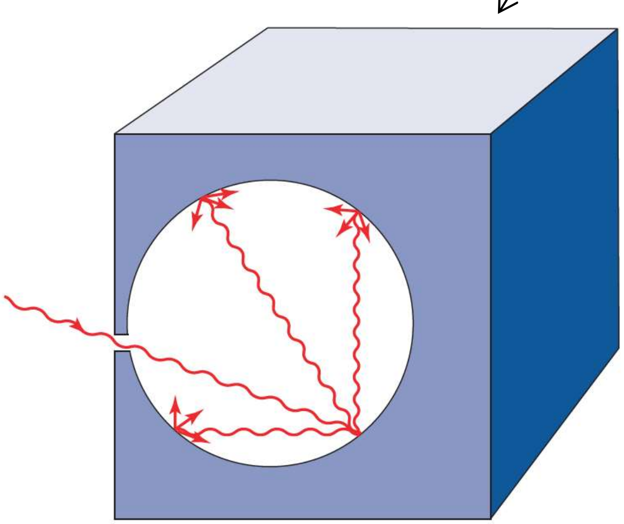


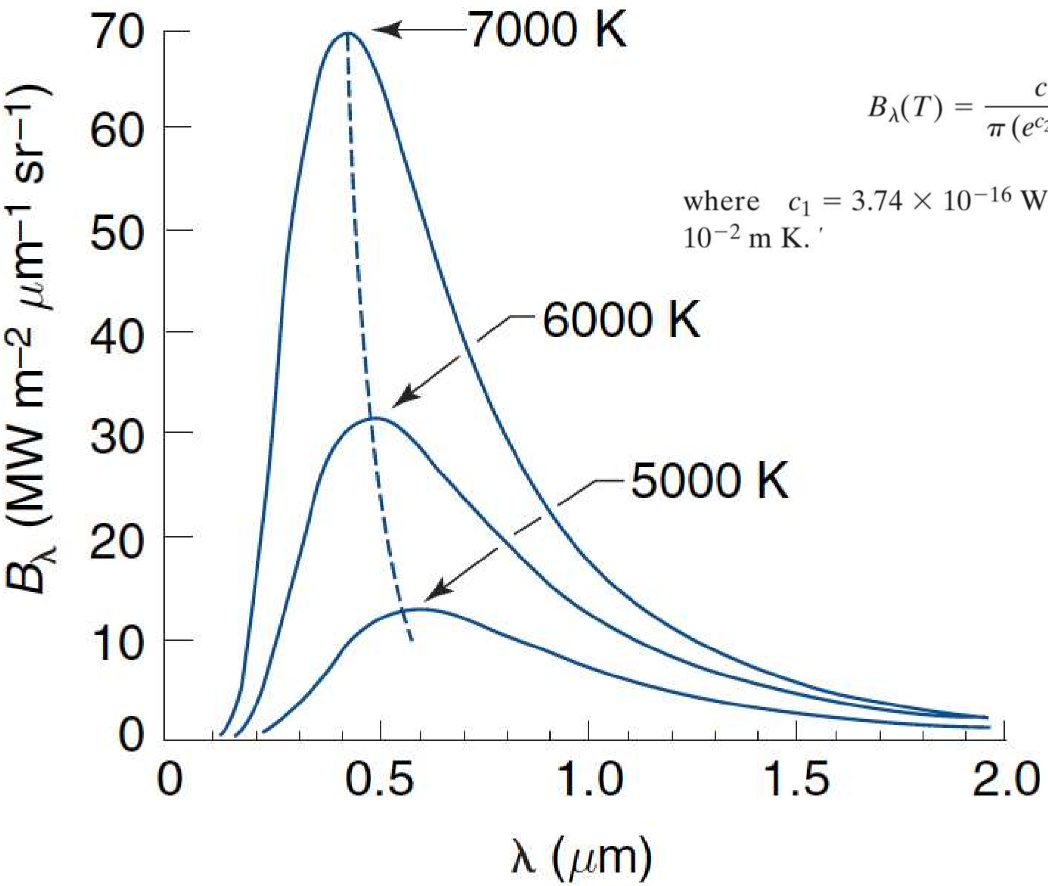






Planck's Blackbody Function for Different Temperatures





$$B_\lambda(T) = \frac{c_1\lambda^{-5}}{\pi\,(e^{c_2/\lambda T} - 1)}$$

where $c_1 = 3.74 \times 10^{-16}$ W m^2 and $c_2 = 1.45 \times 10^{-2}$ m K.'

$$B(T_B,\nu) = 2x10^{11}\, hc^2\nu^3 \frac{1}{e^{100\, hc\nu/kT} - 1} \qquad \frac{mW}{m^2\ Sr\ cm^{-1}}$$

Wavenumber distribution

Peak obeys Wien's displacement 'law' $\lambda_m = \frac{2897}{T}$

Wavelength (μm)
T (K)

Total irradiance $F = \sigma T^4$ Stefan-Boltzmann 'law'

Human, as we see him; as we would see him with infrared sensitive eyes. (from Wikipedia).

VISIBLE

Wien’s Displacement Law:

λ_{max} (microns) = 2897 / T (Kelvin)

T_{sun}=5780 K, λ_{max} = 0.50 um = 500 nm
T_{room} = 300 K, λ_{max} = 9.7 microns.
$T_{sunCenter}$ = 15,000,000 K, λ_{max}=1.9 Å

INFRARED

note the cool nose, ears, and glasses, and that the trash bag is nearly transparent in the IR.

Example Problem

IR Emission from Clotheless Person!

Looking at the World With Infrared Eyes...

How much energy would this person lose in 1 day?

Solution: The body surface area is about

$$BSA(m^2) = \frac{[\text{Mass (Kg)}]^{0.425}\,[\text{height (cm)}]^{0.725}}{139.2}$$

$$BSA = \frac{90^{0.425}\;180^{0.725}}{139.2} = 2.1\,m^2$$

$$\left\{\begin{matrix}\text{Energy Lost}\\ \text{Per Day}\end{matrix}\right\} = \sigma T^4 \left(\frac{W}{m^2}\right) * BSA\,(m^2) * (24 \times 3600)\,\frac{\text{Seconds}}{\text{Day}}$$

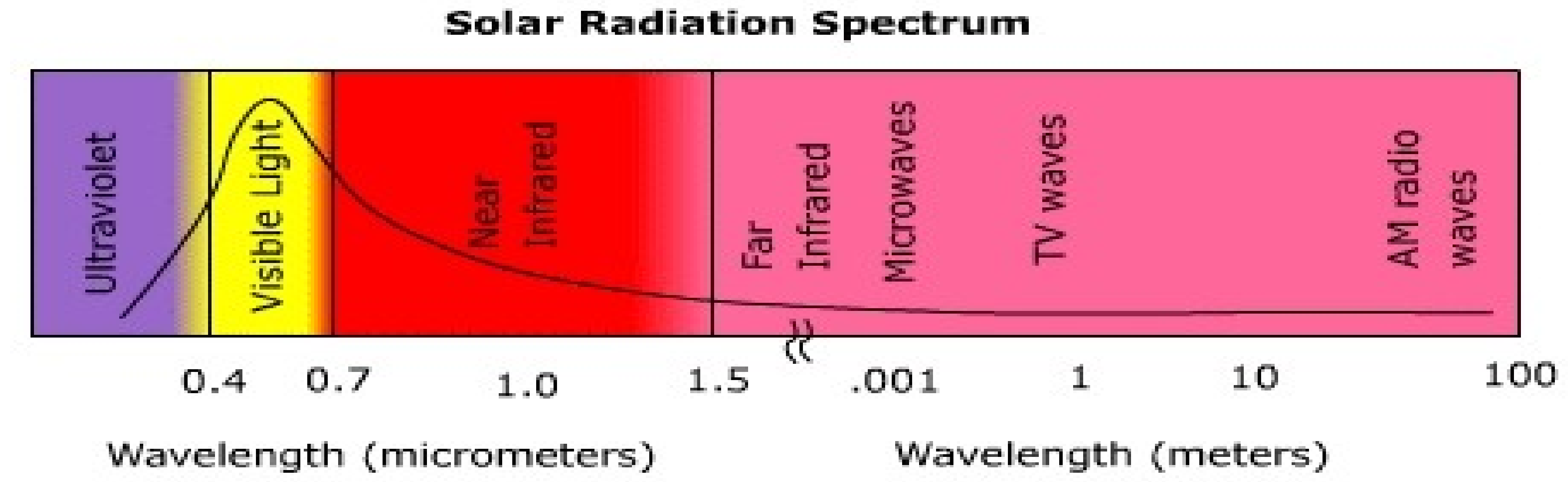
$$= 5.67\times10^{-8} \cdot [305.4]^4 * 2.1 * 86{,}400 \;\frac{\text{Joules}}{\text{Day}}$$

$$= 89{,}493{,}651 \text{ Joules / Day}$$

$$\approx 21{,}000 \text{ kcals / day } !!!$$

Why don't I have to eat 21,000 food calories/day?

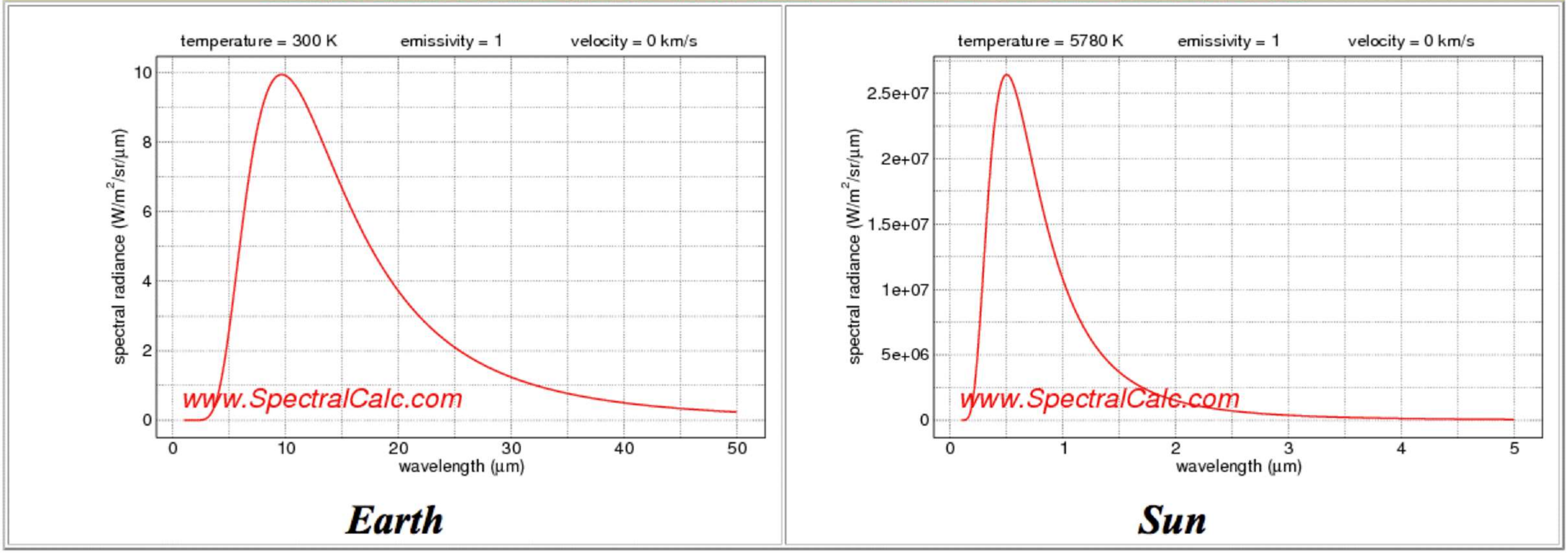
Spectrum of Solar Radiation FluxBl



- The sun emits 41% of its radiation in the visible spectrum,
- 9% in the ultraviolet spectrum
- 50% in the near infrared spectrum

Earth and Sun Radiation from their ‘surfaces’

Blackbody radiation curves for longwave (terrestrial) radiation on the left, and shortwave (solar) radiation on the right.



Astronomical Radiation Balance

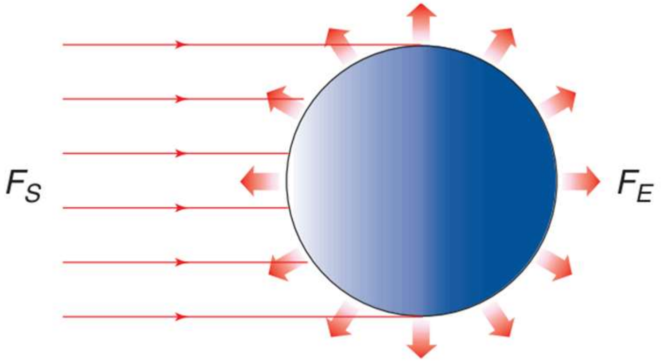


Fig. 4.8 Radiation balance of the Earth. Parallel beam solar radiation incident on the Earth's orbit, indicated by the thin red arrows, is intercepted over an area πR_E^2 and outgoing (blackbody) terrestrial radiation, indicated by the wide red arrows, is emitted over the area $4\pi R_E^2$.

From the Stefan–Boltzmann law (4.12)

$$F_E = \sigma T_E^4 = \frac{(1 - A)F_s}{4} = \frac{(1 - 0.30) \times 1368}{4}$$

$$= 239.4 \text{ W m}^{-2}$$

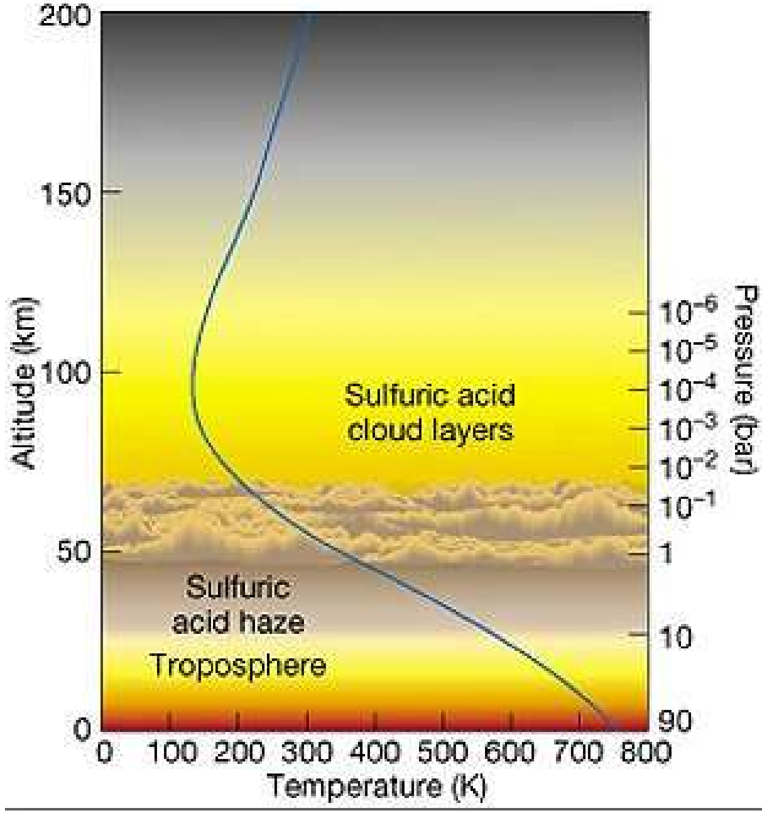
Solving for T_E, we obtain

$$T_E = \sqrt[4]{\frac{F_E}{\sigma}} = \left(\frac{239.4}{5.67 \times 10^{-8}}\right)^{1/4} = 255 \text{ K}$$

Planet	**Distance from sun**[a]	**F_s (W m^{-2})**	**A**	**T_E (K)**
Mercury	0.39	8994	0.06	439
Venus	0.72	2639	0.78	225
Earth	1.00	1368	0.30	255
Mars	1.52	592	0.17	216
Jupiter	5.18	51	0.45	105

[a] Astronomical units are multiples of Earth–Sun distance.

Venus: What's going on here?



Venus Atmosphere

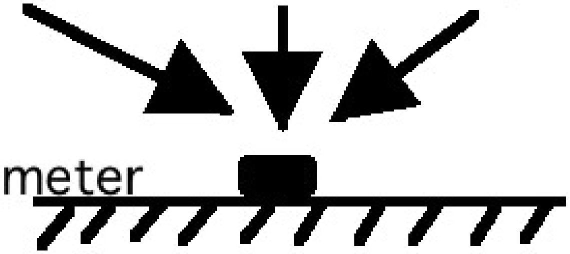
Surface pressure: 92 bars
Surface density: ~65. kg/m^3
Scale height: 15.9 km
Total mass of atmosphere: ~4.8 x 10^{20} kg
Average temperature: 737 K (464 C)
Diurnal temperature range: ~0
Wind speeds: 0.3 to 1.0 m/s (surface)
Mean molecular weight: 43.45
Atmospheric composition (near surface, by volume):
  Major: 96.5% Carbon Dioxide (CO_2), 3.5% Nitrogen (N_2)
  Minor (ppm): Sulfur Dioxide (SO_2) - 150; Argon (Ar) - 70; Water (H_2O) - 20;
  Carbon Monoxide (CO) - 17; Helium (He) - 12; Neon (Ne) - 7

$$Emitted\ \ Irradiance = Emissivity * \sigma T^4 = \varepsilon * \sigma T^4$$

$$Note: 0 \le \varepsilon \le 1$$

Effective Atmospheric Emissivity ε:

IR radiation from atmosphere



$$\varepsilon \equiv \frac{F_{IR}^{\downarrow}}{\sigma T_0^4},$$

IR radiometer

where

$F_{IR}^{\downarrow}$ is the measured downwelling infrared irradiance,

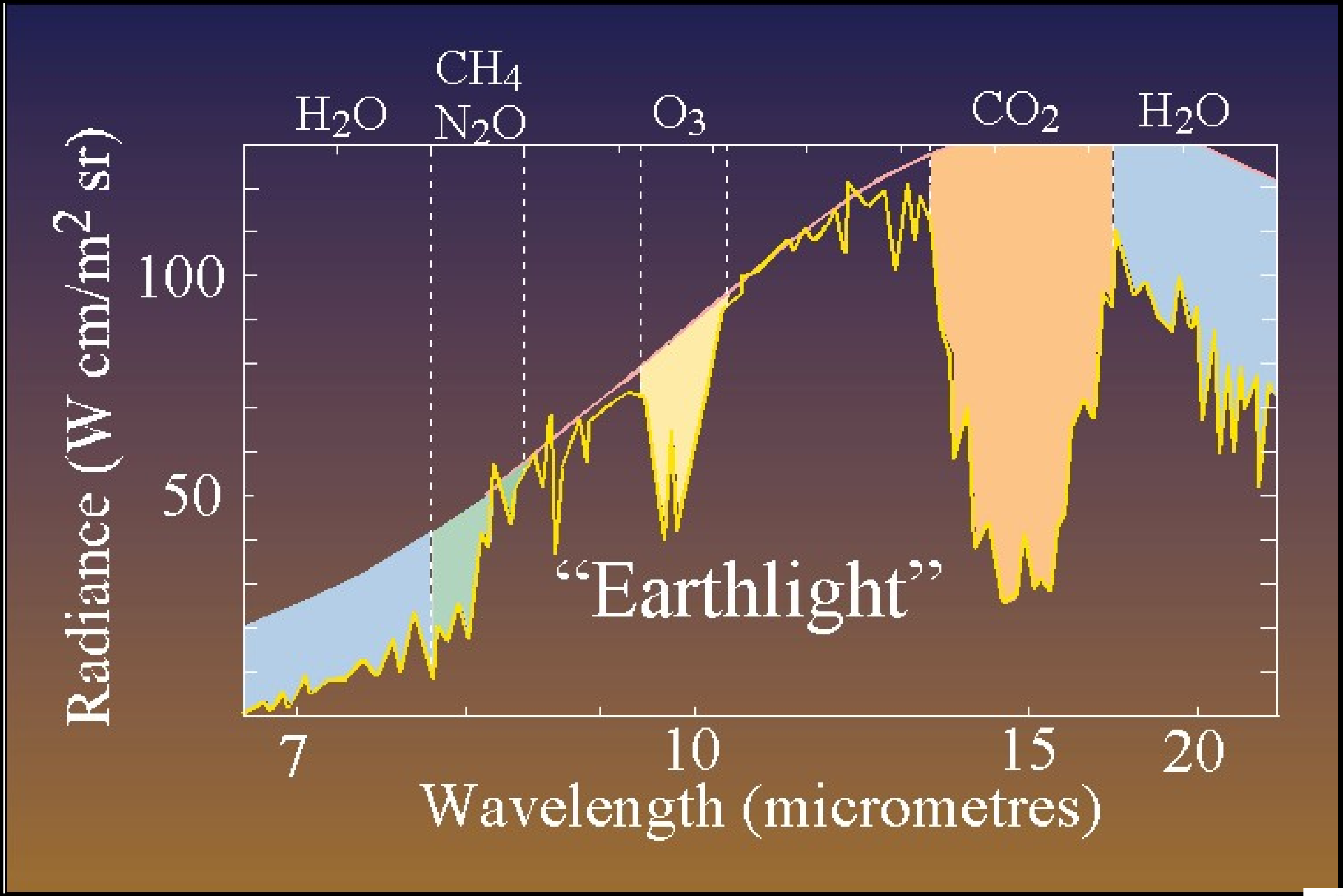
T_0 = surface temperature,

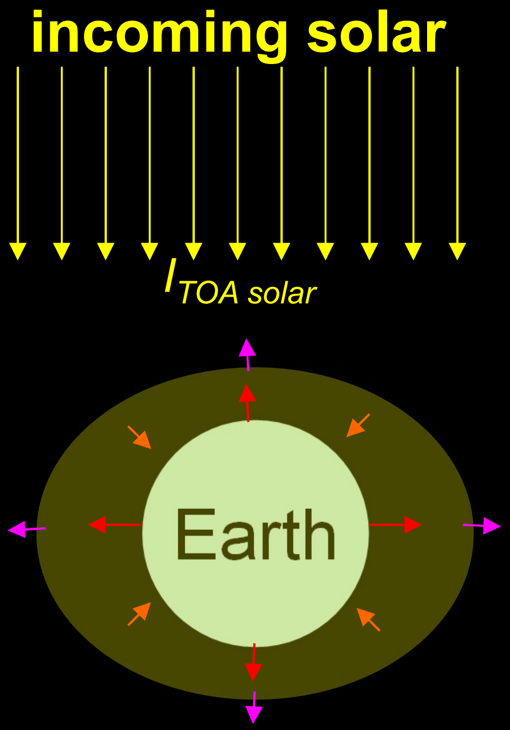
σ = Stefan Boltzmann constant = $5.67x10^{-8} \dfrac{W}{m^2\ K^4}$.

Parameterization for ε to try:

$\varepsilon \approx 0.67\ e^{0.08}$ where e in mb is vapor pressure from homework 7.

Earth Light: Spectrum of Outgoing Infrared Radiation





IR radiation emitted by the Earth (red), the atmosphere (orange), and transmitted to space after absorption by the atmosphere (pink).

$$\pi r^2\, I_{TOA\ solar}(1 - Albedo) = 4\pi r^2\, I_{earthly\ IR}$$

$$I_{earthly\ IR} = \sigma T^4$$

$$T_{earth\ from\ Space} = \left(\frac{I_{TOA\ solar}(1 - Albedo)}{4\,\sigma}\right)^{\frac{1}{4}}$$

Yellow: Spectrum of infrared light going to space.

Pink Curve: Spectrum of light emitted by the surface.

Greenhouse gases reduce the amount of infrared radiation from the surface that gets to space.

From http://www.lib.utah.edu/services/prog/gould/1998/Figure_5.gif

Radiance and Irradiance

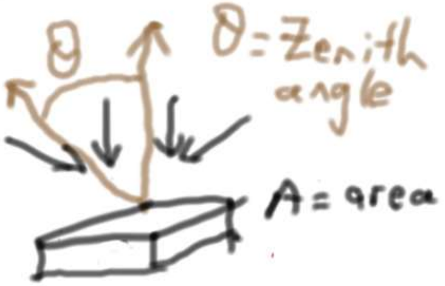
[redacted]

Radiance detector: Rotate to get radiation as a function of θ & φ

Units: $\frac{\text{Watts}}{m^2\, Sr\, \mu m}$

Symbol: R

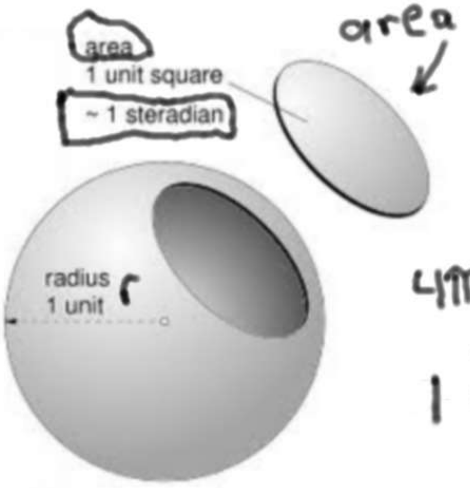
Sr = Steradian ⇒ angle of acceptance for the detector.
Solid Angle!



Irradiance detector: Integrate over all θ & φ and

Units: $\frac{\text{Watts}}{m^2\, \mu m}$

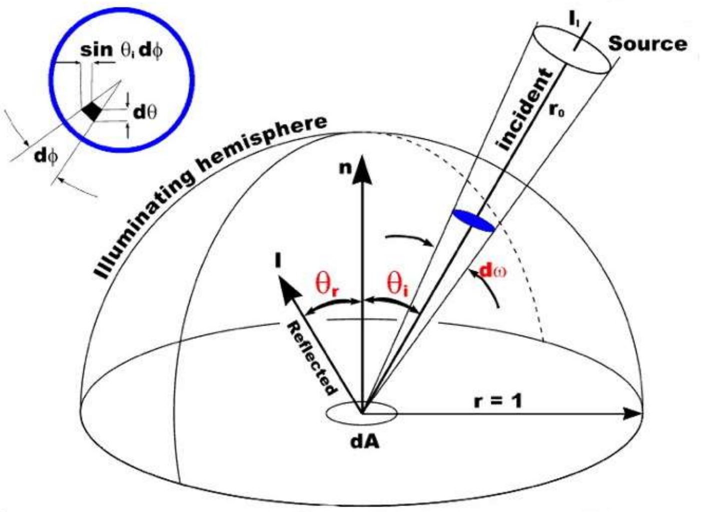
Symbol: I



Solid Angle = $\frac{\text{area}}{r^2}$

4π Steradians in full sphere. 1 Steradian shown

Angle measure in 3-D



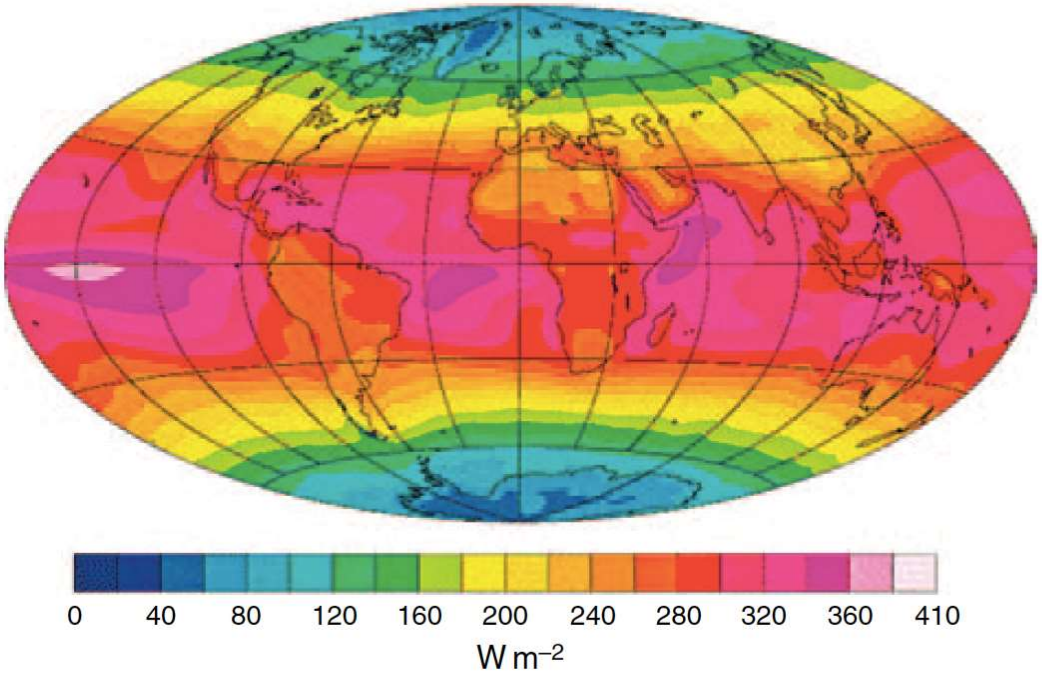
Relationship:

$$I = \int\limits_{\substack{0\\ \theta}}^{\pi/2} \int\limits_{\substack{0\\ \phi}}^{2\pi} R(\theta,\phi)\cos\theta\, \underbrace{\sin\theta\, d\theta\, d\phi}_{\substack{d(\text{Solid Angle})\\ d\omega}} = \pi R$$

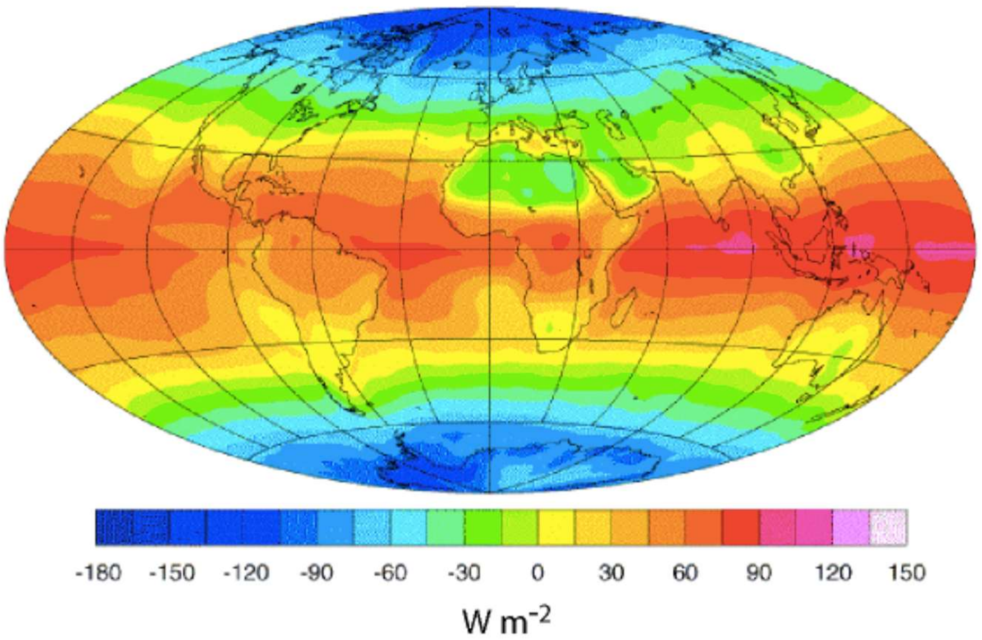
when R ≈ constant (Lambertian Source)

Global Radiation Balance at Top of Atmosphere: 1 year of data from the NASA Earth Radiation Budget Experiment

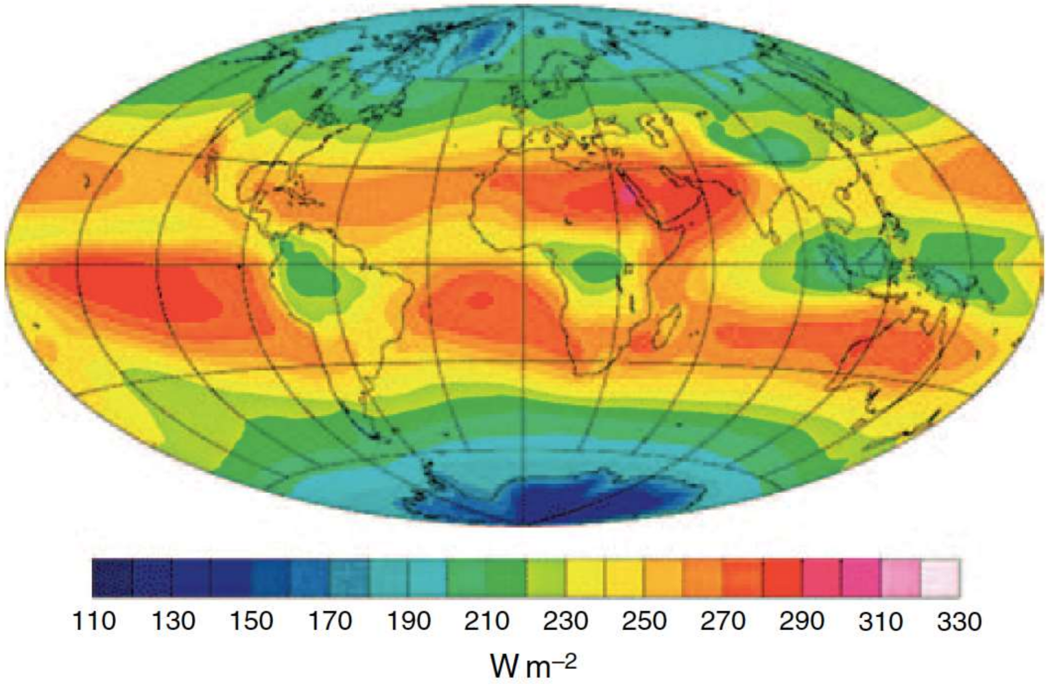
Absorbed Solar Radiation



Net Radiation

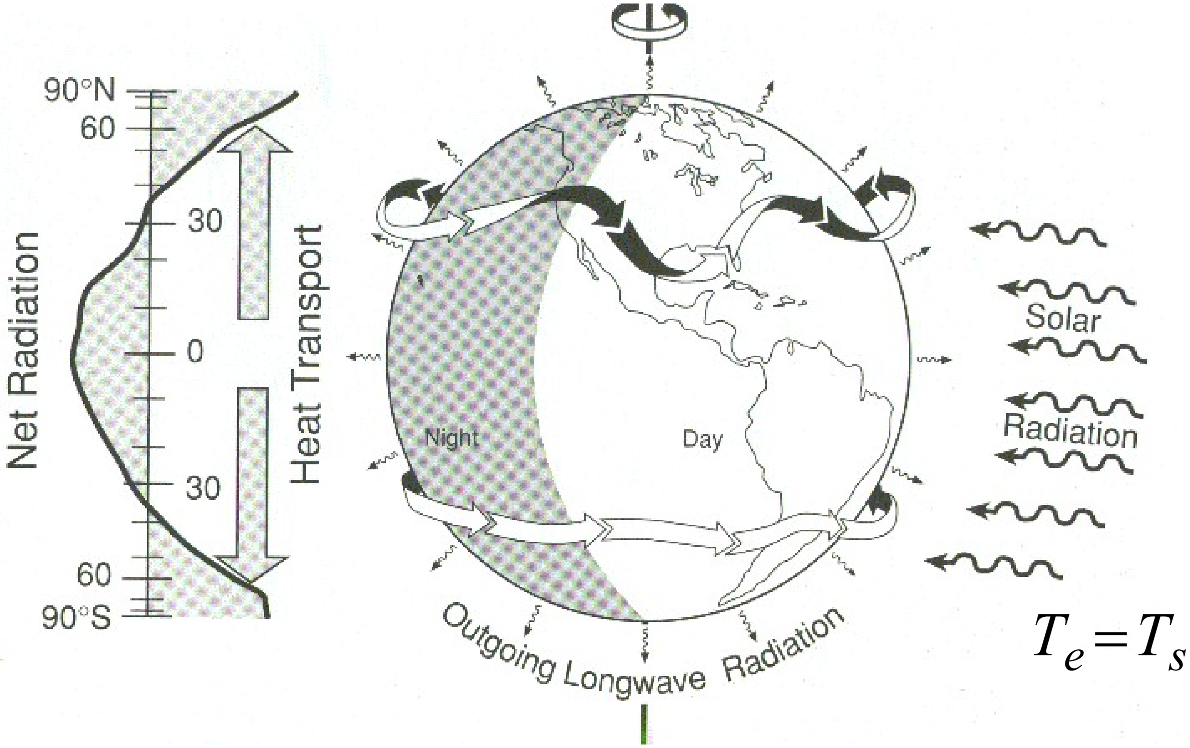


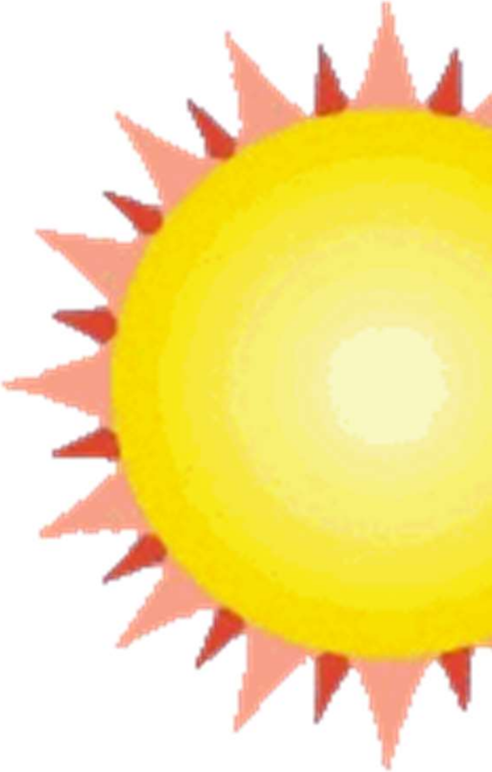
Outgoing Longwave Radiation



Notes:

- Absorbed solar mostly near equator.
- IR emitted strong near Sahara desert.
- Low outgoing IR (green patches) in central Africa, South America, etc, are due to persistent cumulonimbus.
- Net radiation=Solar Absorbed – Outgoing IR; surplus near the equator, deficit over Sahara desert and polar regions.



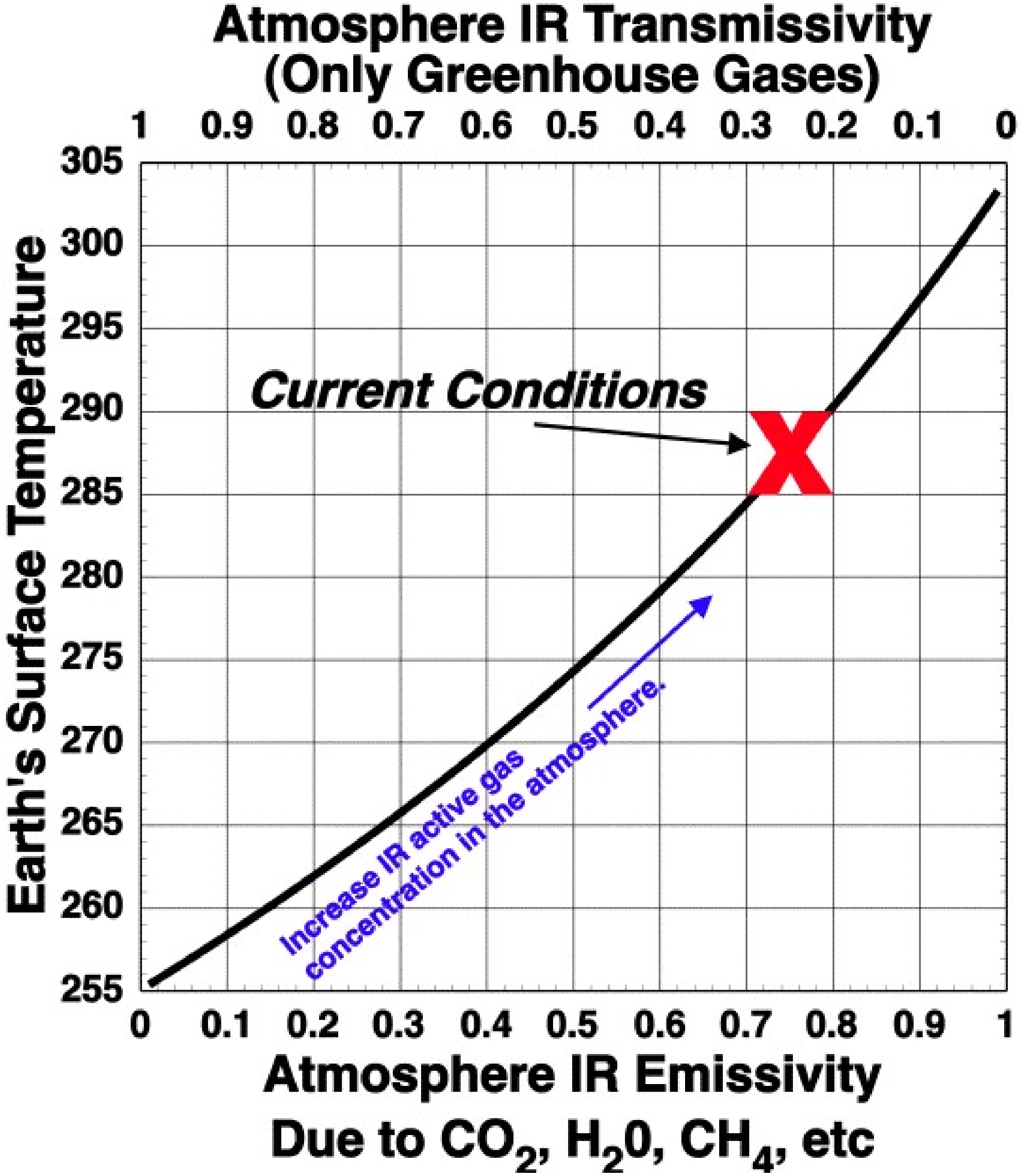


$$T_e = T_s \left(\frac{R_s{}^2}{R_{se}{}^2} \frac{(1-a)}{2(1+t)} \right)^{1/4}$$

$$t = 0,\ T_e = 303\,K\ (Greenhouse\,Max)$$
$$t = 1,\ T_e = 255\,K\ (No\,Atmosphere)$$
$$t = 0.2,\ T_e = 289\,K\ (Just\quad Right)$$

T_e	Earth's radiative temperature
T_s	Sun's radiative temperature
R_s	Sun's radius
R_{se}	Sun to Earth distance
a	Earth's surface solar reflectance
t	IR transmittance of Earth's atmosphere.

Simple Surface Temperature Calculation Assuming Solar Absorption only at the surface, IR emission by the atmosphere and Earth's surface, and IR absorption by the Atmosphere.



$$T_e = \sqrt[1/4]{\frac{S_0(1 - SW\ albedo)}{4\sigma(1 - \varepsilon / 2)}}$$

$\varepsilon = Infrared\ Emissivity$

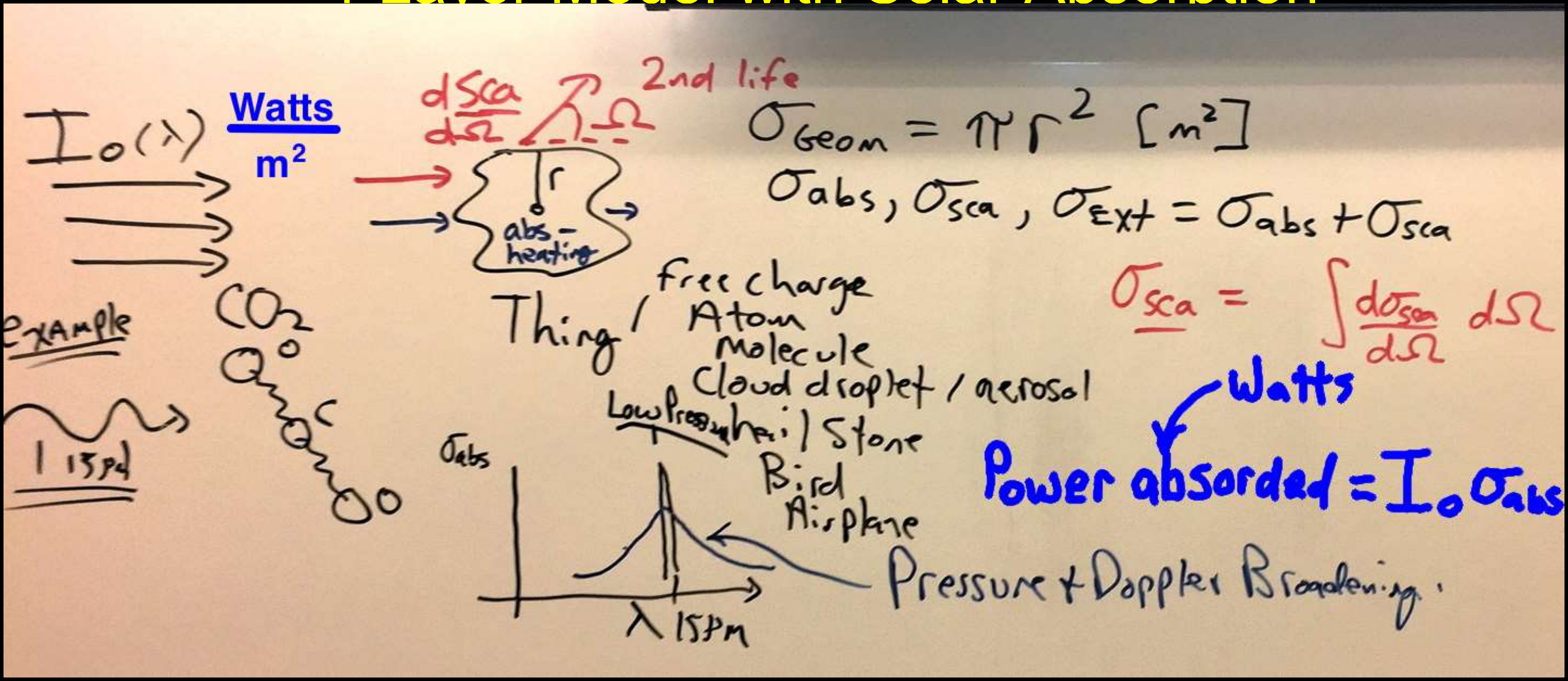
$$T_e = \sqrt[1/4]{\frac{2S_0(1 - SW\ albedo)}{4\sigma(1 + \tau)}}$$

$\tau = Infrared\ Transmissivity$

$\varepsilon = 1 - \tau$

S_0 = 1376 W/m^2=Solar Irradiance at the TOA and σ=Stefan-Boltzmann constant

1 Layer Model with Solar Absorption



$I_o(\lambda)$ **Watts / m²**

$\frac{dSca}{d\Omega}$ 2nd life

r

abs – heating

example

CO_2

O C O

1 15μm

$\sigma_{Geom} = \pi r^2$ [m²]

σ_{abs}, σ_{sca}, $\sigma_{Ext} = \sigma_{abs} + \sigma_{sca}$

$\sigma_{sca} = \int \frac{d\sigma_{sca}}{d\Omega} d\Omega$

Thing / free charge
Atom
Molecule
Cloud droplet / aerosol
hail Stone
Bird
Airplane

Low Pressure

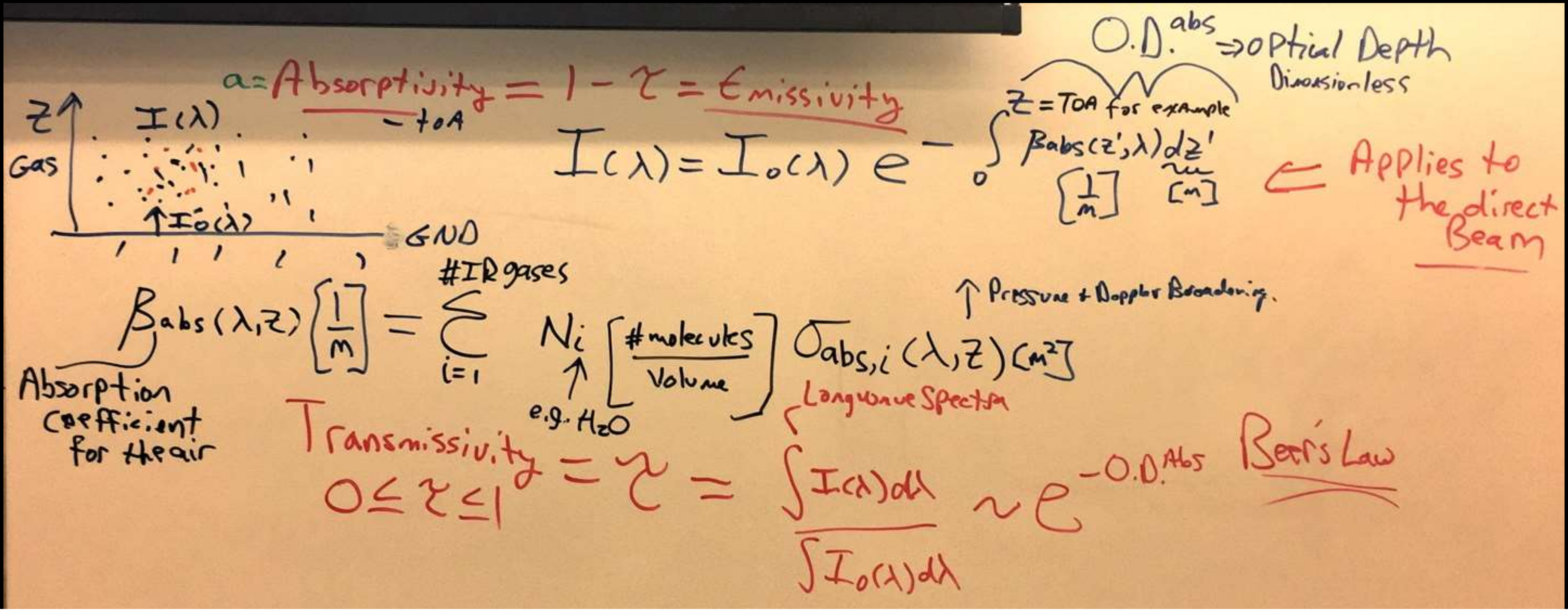
σ_{abs}

λ 15μm

Pressure + Doppler Broadening.

Watts

Power absorded = $I_o \sigma_{abs}$



a = Absorptivity = 1 − τ = Emissivity

z, Gas, $I(\lambda)$, toA, $I_o(\lambda)$, GND

O.D.^{abs} ⇒ Optical Depth
Dimensionless

z = TOA for example

$$I(\lambda) = I_o(\lambda)\, e^{-\int_0^{z} \beta_{abs}(z',\lambda)\, dz'}$$

$\left[\frac{1}{m}\right]$ [m]

⇐ Applies to the direct Beam

$$\beta_{abs}(\lambda,z)\left[\frac{1}{m}\right] = \sum_{i=1}^{\#IR\ gases} N_i \left[\frac{\#\text{molecules}}{\text{Volume}}\right] \sigma_{abs,i}(\lambda,z)\,[m^2]$$

Absorption coefficient for the air

e.g. H_2O

Pressure + Doppler Broadening.

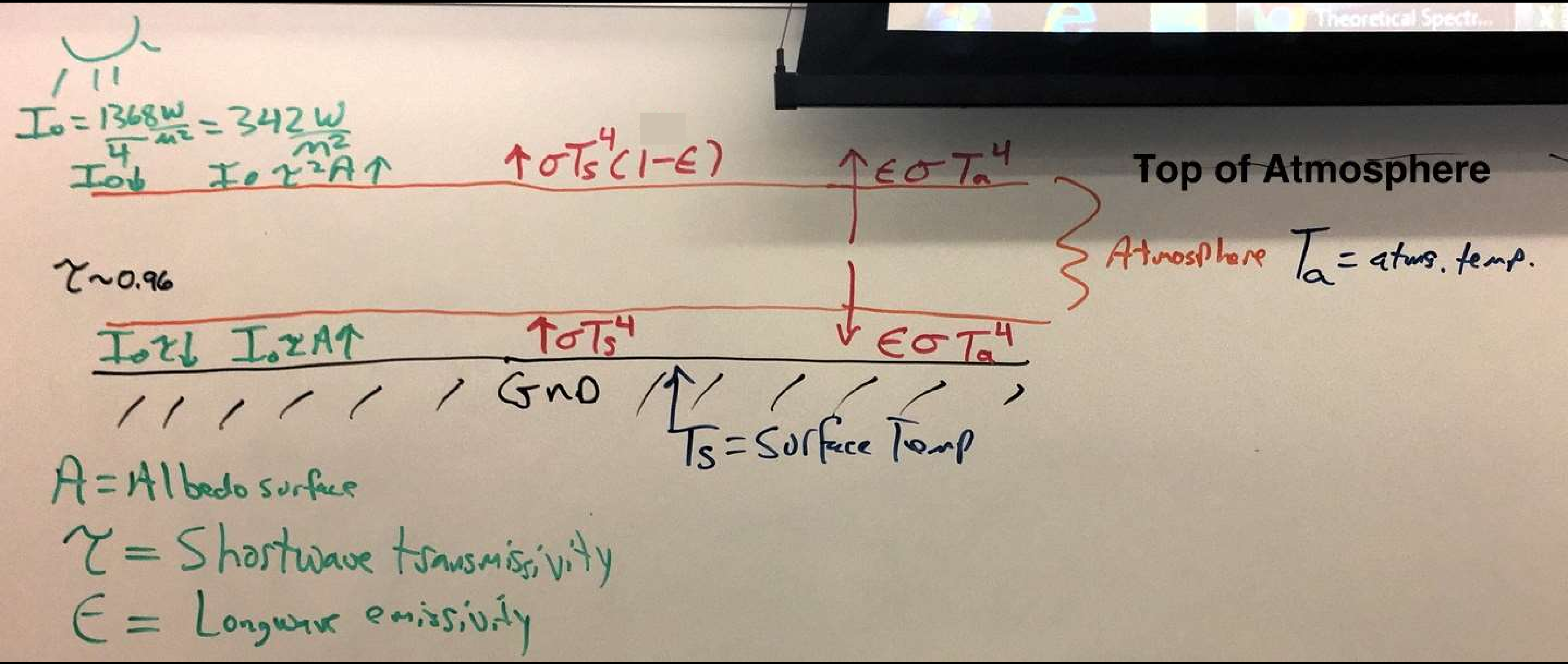
Longwave Spectra

$$\text{Transmissivity} = \tau = \frac{\int I(\lambda)\,d\lambda}{\int I_o(\lambda)\,d\lambda} \sim e^{-O.D.^{Abs}}$$

$0 \le \tau \le 1$

Beer's Law

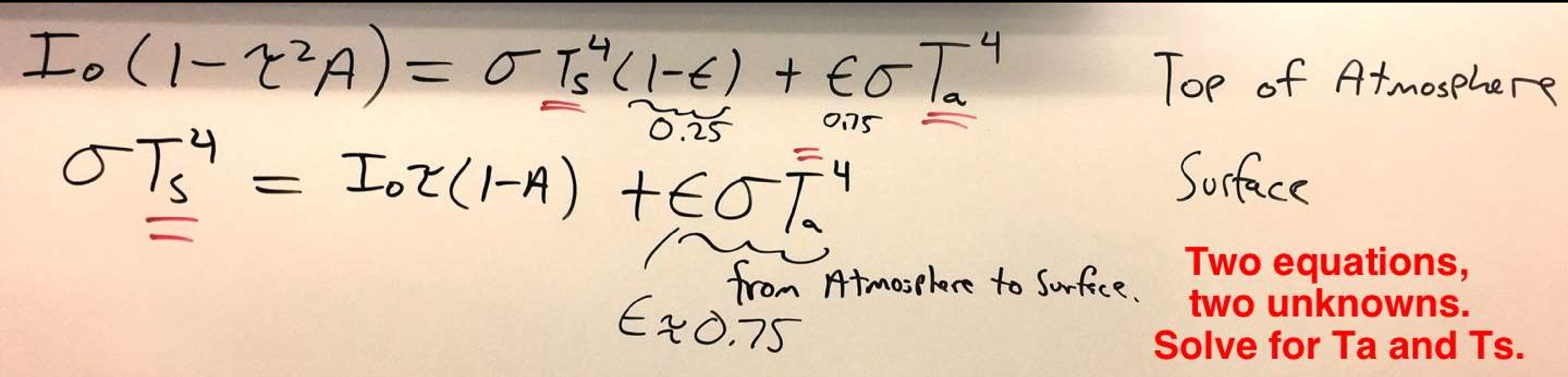
1 Layer Model with Solar Absorption Continued



A = Albedo surface

τ = Shortwave transmissivity

ϵ = Longwave emissivity



$$I_o(1-\tau^2 A) = \sigma \underline{\underline{T_s}}^4 \underbrace{(1-\epsilon)}_{0.25} + \underset{0.75}{\epsilon}\sigma \underline{\underline{T_a}}^4$$

Top of Atmosphere

$$\sigma \underline{\underline{T_s}}^4 = I_o\tau(1-A) + \underbrace{\epsilon\sigma \overset{=}{T_a}^4}_{\text{from Atmosphere to Surface.}}$$

Surface

$\epsilon \approx 0.75$

Two equations, two unknowns. Solve for Ta and Ts.

Surface Temperature

$$T_s = \sqrt[1/4]{\frac{I_0}{\sigma(2-\varepsilon)}(1+\tau)(1-A\tau)}$$

Atmosphere Temperature

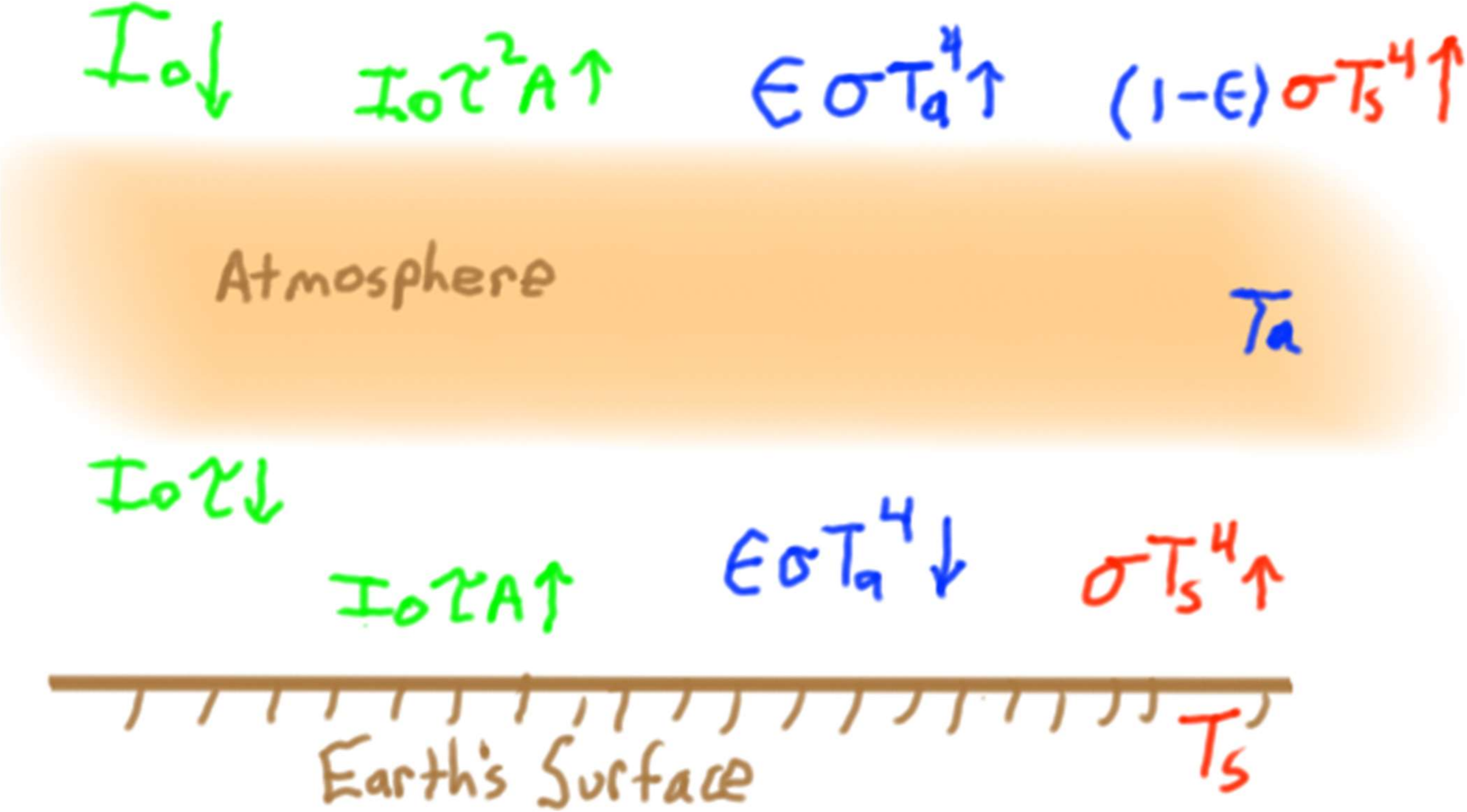
$$T_a = \sqrt[1/4]{\frac{I_0}{\sigma\varepsilon(2-\varepsilon)}\left[(1-\tau)(1+A\tau)+\varepsilon\,\tau(1-A)\right]}$$

NOTES:

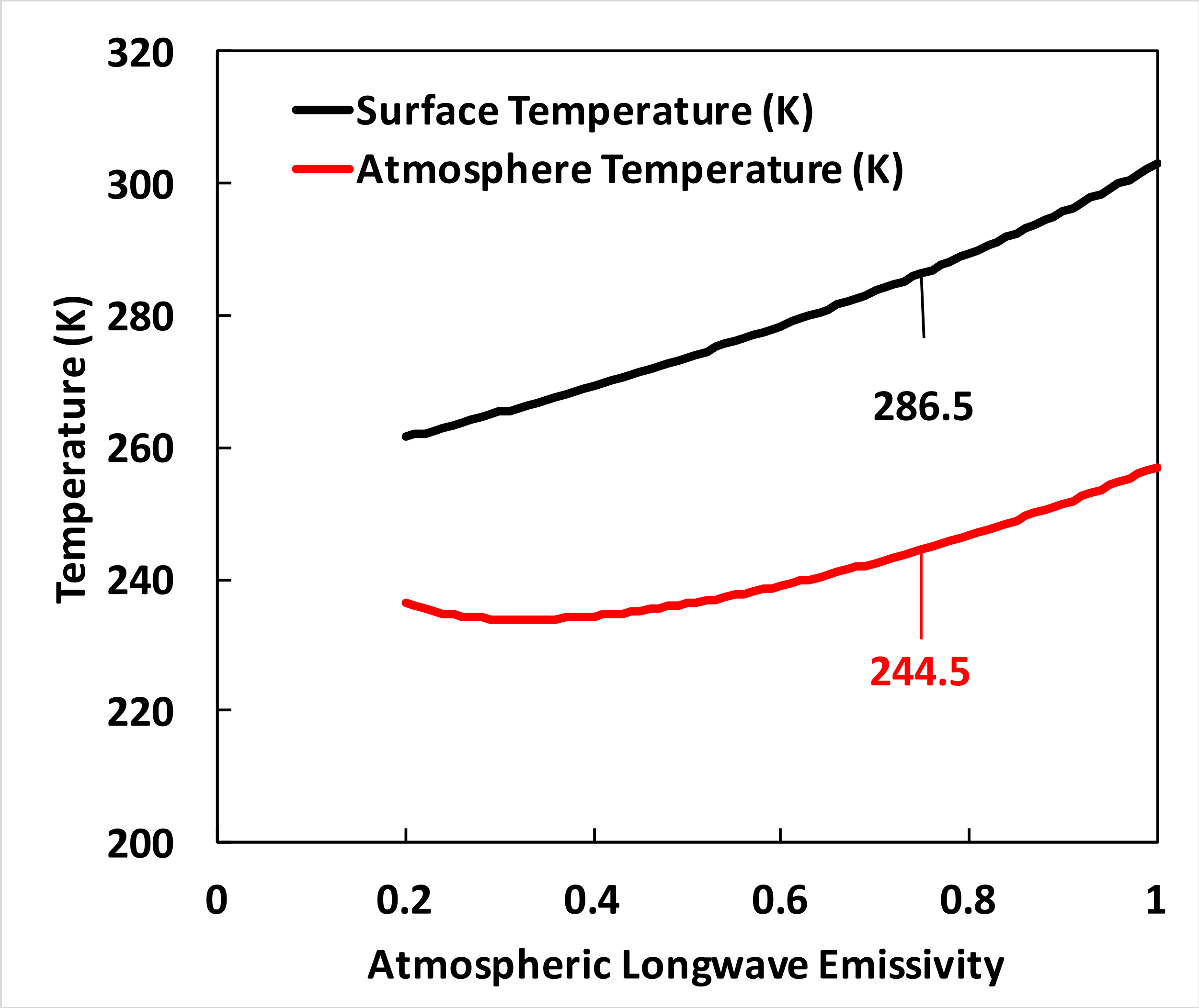
τ is solar transmission

ε is long wave emissivity

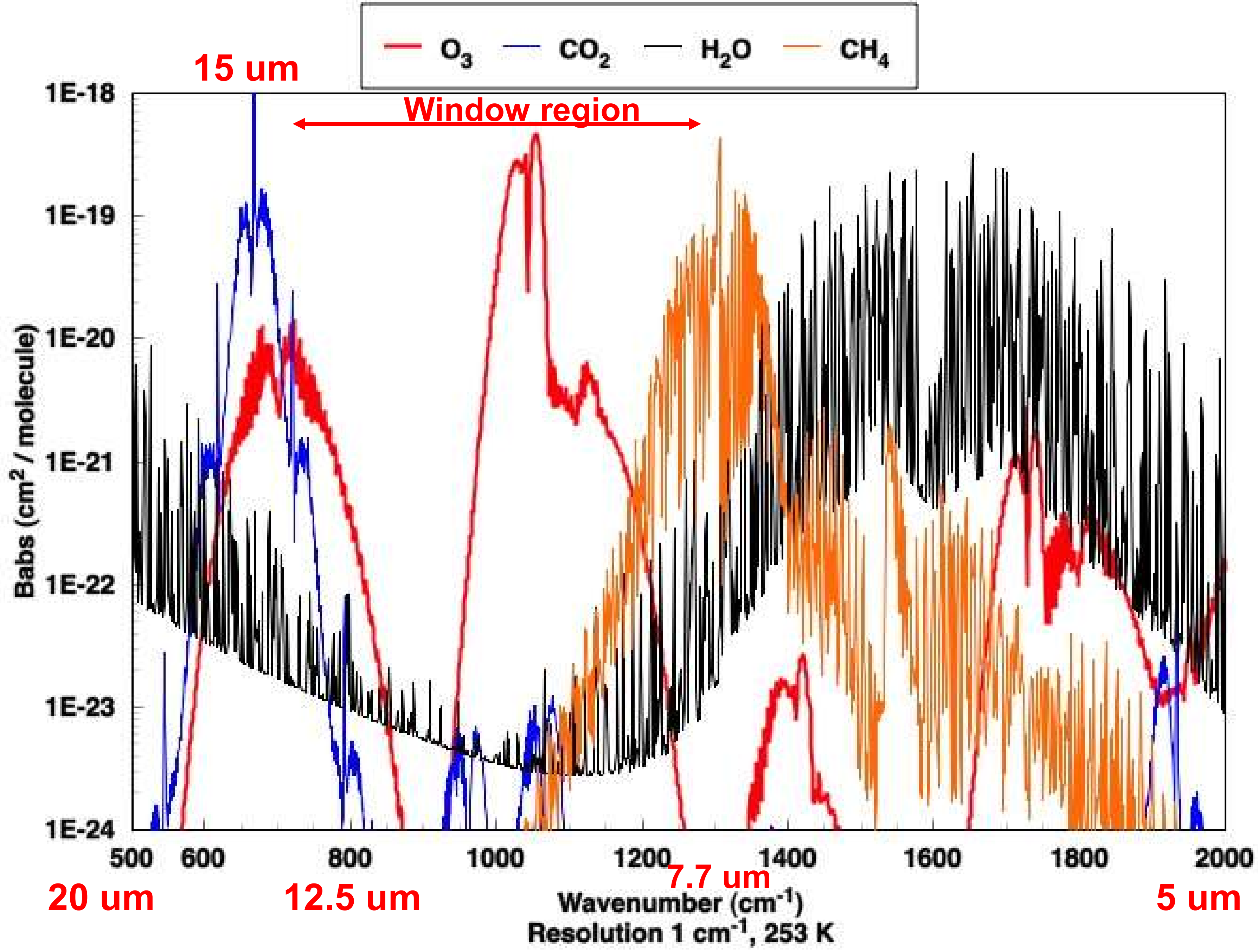
I_0=1368/4 W/m^2=342 W/m^2



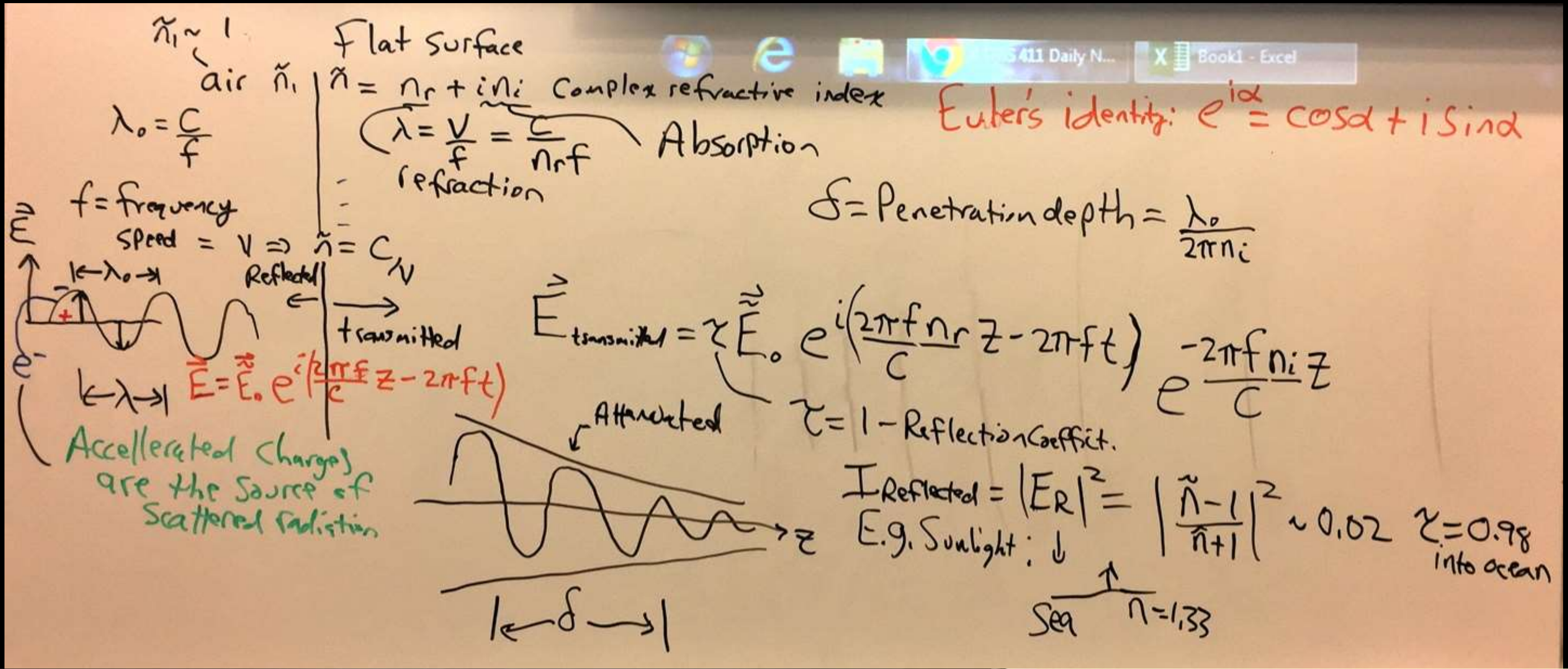
1 Layer Atmosphere Model
Shortwave Albedo = 0.3 Shortwave Transmissivity = 0.96



Absorption by Infrared Active Gases (Greenhouse Gases)



Scattering and Absorption by Gases, Aerosol, and Hydrometeors



$\tilde{n}_1 \sim 1$

air $\tilde{n}_1$

Flat surface

$\tilde{n} = n_r + i n_i$ Complex refractive index

$\lambda_0 = \frac{c}{f}$

$\lambda = \frac{v}{f} = \frac{c}{n_r f}$ refraction

Absorption

f = frequency

speed = $v \Rightarrow \tilde{n} = c/v$

Euler's identity: $e^{i\alpha} = \cos\alpha + i\sin\alpha$

δ = Penetration depth = $\frac{\lambda_0}{2\pi n_i}$

$\vec{E} = \vec{\tilde{E}}_0 e^{i\left(\frac{2\pi f}{c} z - 2\pi f t\right)}$

$\vec{E}_{transmitted} = \tau \vec{\tilde{E}}_0 e^{i\left(\frac{2\pi f n_r}{c} z - 2\pi f t\right)} e^{-\frac{2\pi f n_i}{c} z}$

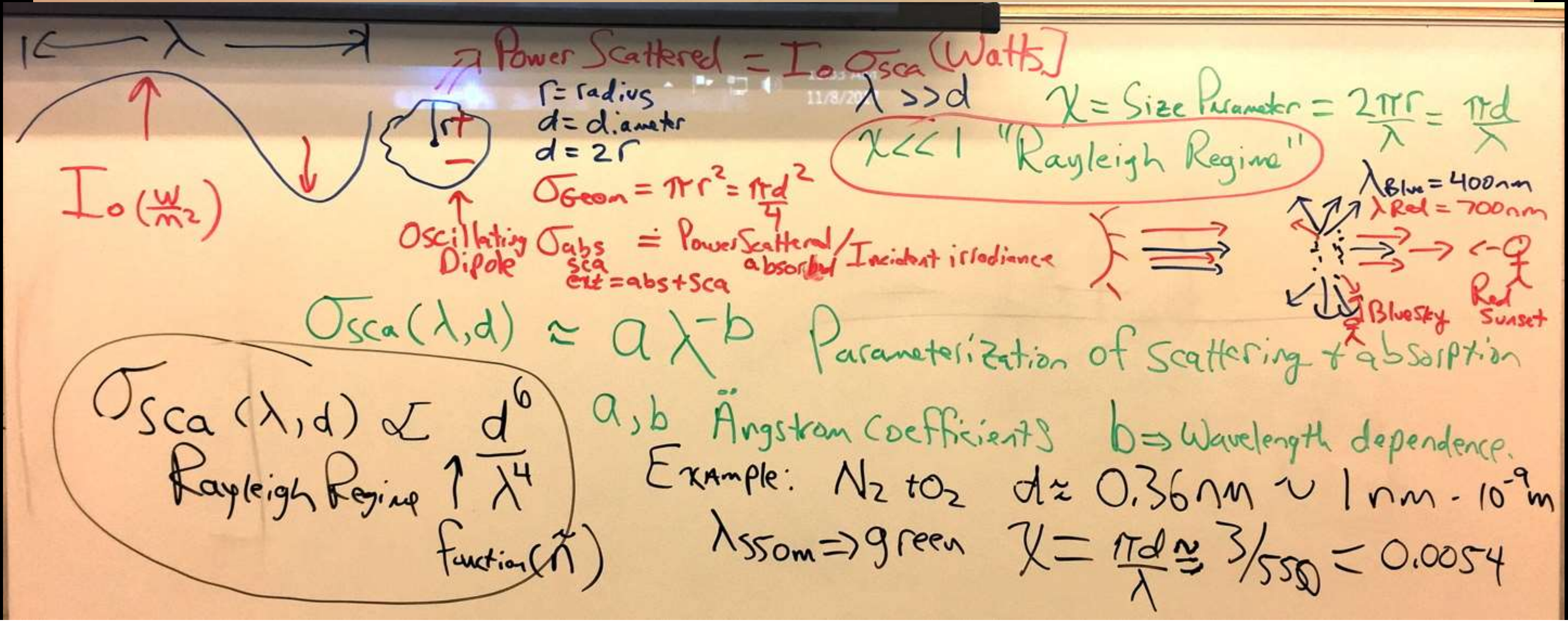
$\tau = 1 -$ Reflection Coeffict.

Accellerated charges are the source of Scattered radiation

$I_{Reflected} = |E_R|^2 = \left|\frac{\tilde{n}-1}{\tilde{n}+1}\right|^2 \sim 0.02$ $\tau = 0.98$ into ocean

E.g. Sunlight: ↓

Sea $n = 1.33$



Power Scattered $= I_0 \sigma_{sca}$ [Watts]

r = radius

d = diameter

$d = 2r$

$I_0 \left(\frac{W}{m^2}\right)$

Oscillating Dipole

$\sigma_{Geom} = \pi r^2 = \frac{\pi d^2}{4}$

σ_{abs}, σ_{sca}, σ_{ext} $\doteq$ Power Scattered/absorbed / Incident irradiance

ext = abs + sca

$\lambda >> d$

χ = Size Parameter $= \frac{2\pi r}{\lambda} = \frac{\pi d}{\lambda}$

$\chi << 1$ "Rayleigh Regime"

$\lambda_{Blue} = 400nm$

$\lambda_{Red} = 700nm$

$\sigma_{sca}(\lambda, d) \simeq a\lambda^{-b}$ Parameterization of Scattering + absorption

$\sigma_{sca}(\lambda, d) \propto \frac{d^6}{\lambda^4}$

Rayleigh Regime ↑

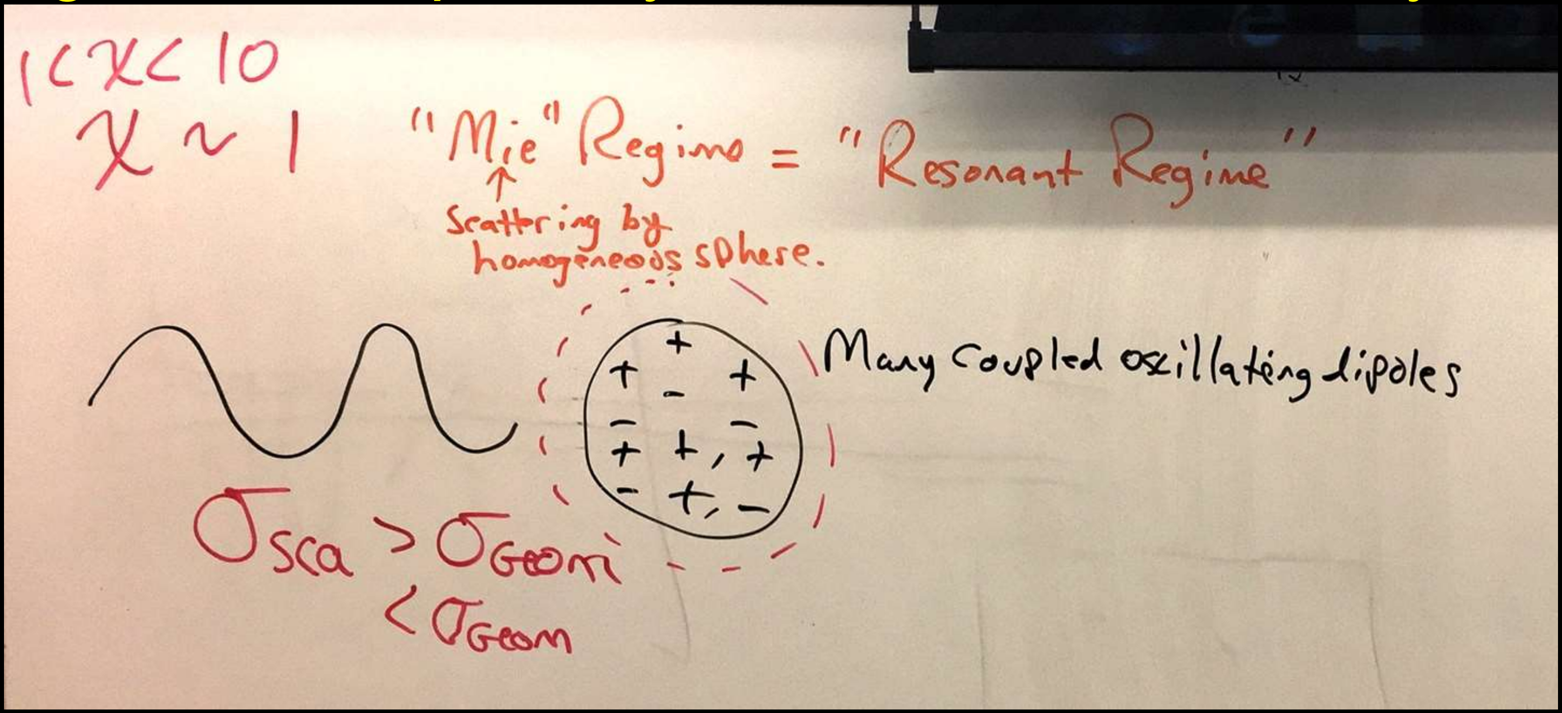
function($\tilde{n}$)

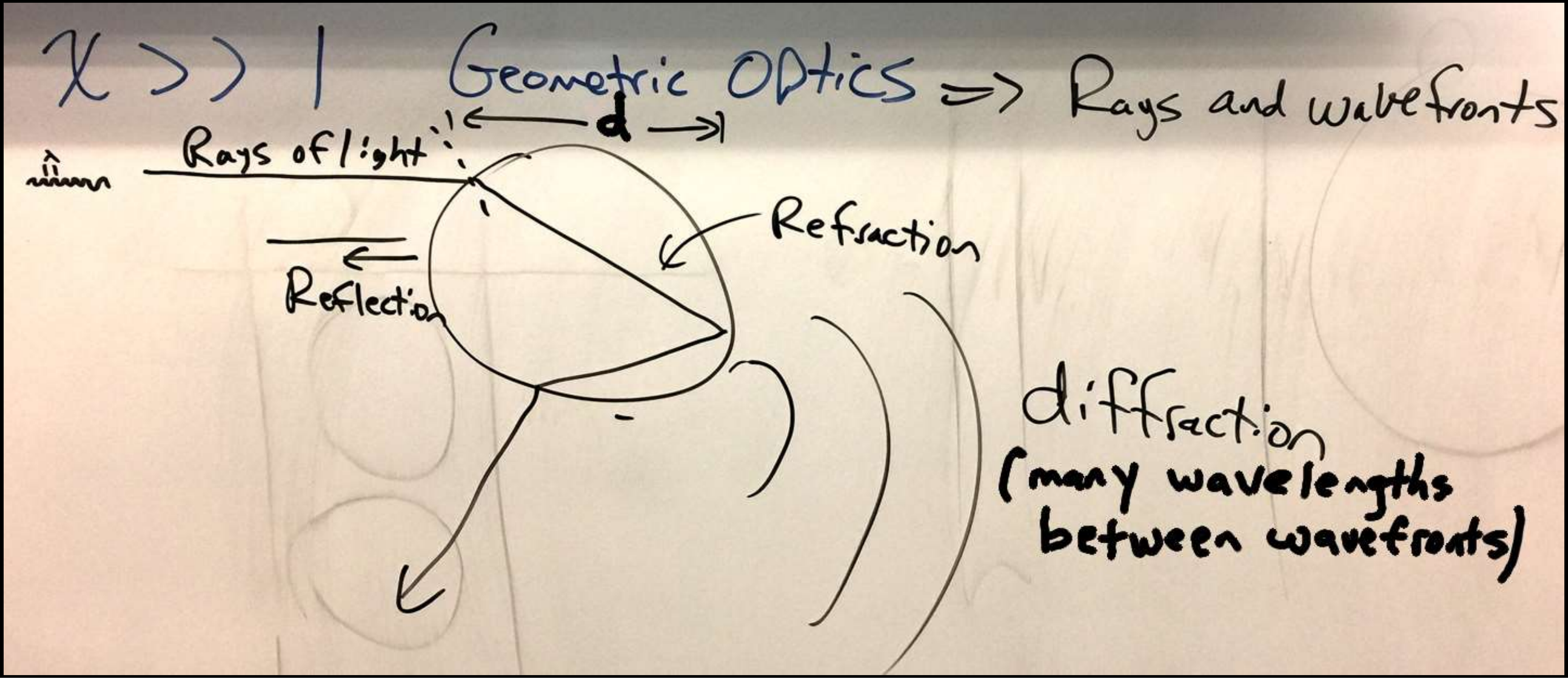
a, b Ängstrom Coefficients b ⇒ Wavelength dependence.

Example: N_2 + O_2 $d \simeq 0.36nm \sim 1nm \cdot 10^{-9}m$

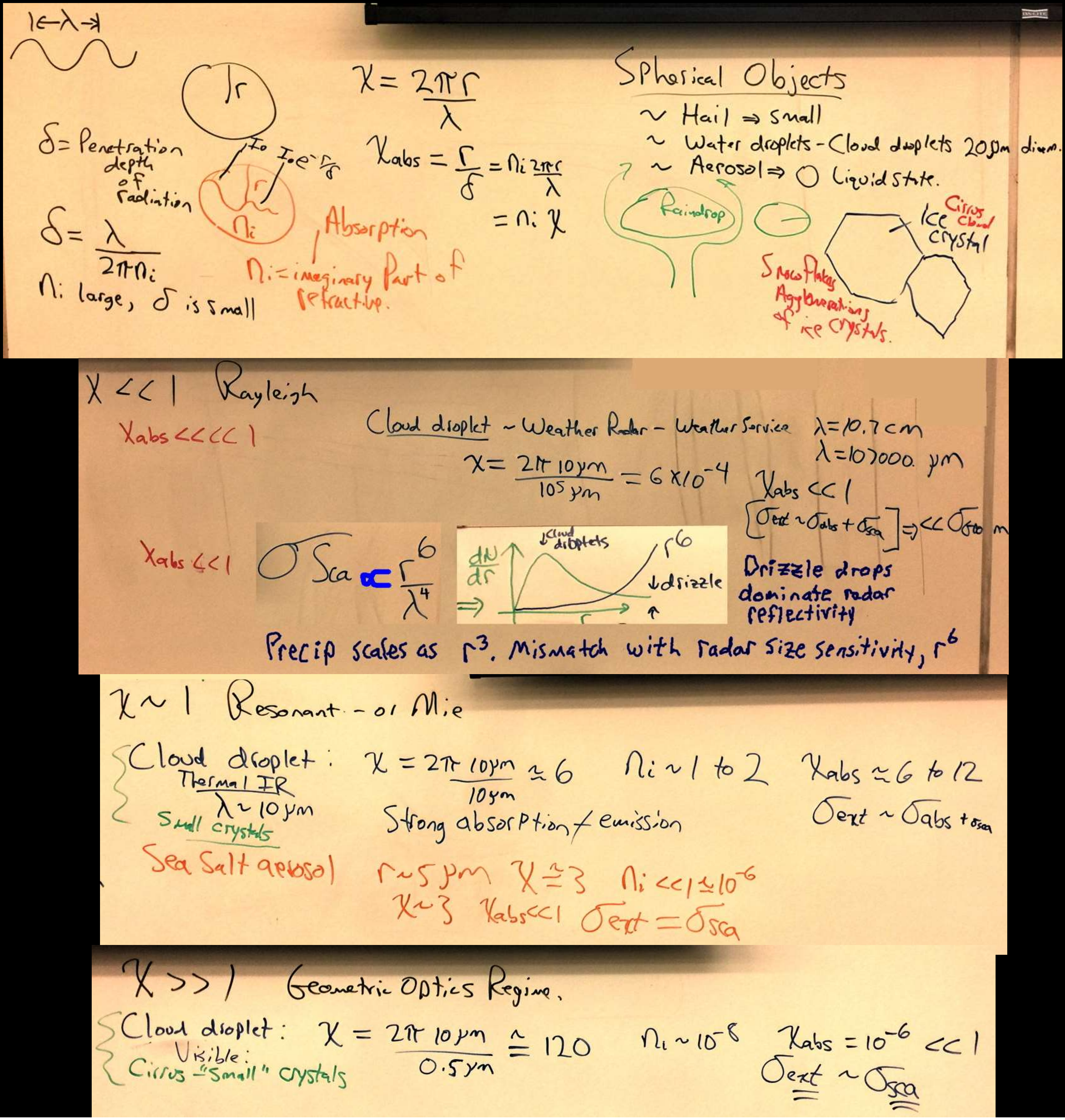
$\lambda_{550m} \Rightarrow$ green $\chi = \frac{\pi d}{\lambda} \cong 3/550 = 0.0054$

Scattering and Absorption by Gases, Aerosol, and Hydrometeors





Hydrometeor Size Parameter and Absorption Scale Factor



δ = Penetration depth of radiation

$$\delta = \frac{\lambda}{2\pi n_i}$$

n_i large, δ is small

Absorption

n_i = imaginary Part of refractive.

$$\chi = \frac{2\pi r}{\lambda}$$

$$\chi_{abs} = \frac{r}{\delta} = n_i \frac{2\pi r}{\lambda} = n_i \chi$$

Spherical Objects

~ Hail ⇒ small

~ Water droplets - Cloud droplets 20 μm diam.

~ Aerosol ⇒ ○ Liquid state.

Cirrus Cloud

Snowflakes Agglomerations of ice crystals.

$\chi << 1$ Rayleigh

$\chi_{abs} <<<< 1$

Cloud droplet ~ Weather Radar - Weather Service $\lambda = 10.7$ cm

$\lambda = 107000$ μm

$$\chi = \frac{2\pi\ 10\,\mu m}{10^5\,\mu m} = 6 \times 10^{-4}$$

$\chi_{abs} << 1$

$[\sigma_{ext} \sim \sigma_{abs} + \sigma_{sca}] \Rightarrow << \sigma_{geo}$ m

$\chi_{abs} << 1$ $\sigma_{sca} \propto \frac{r^6}{\lambda^4}$ ⇒

Drizzle drops dominate radar reflectivity

Precip scales as r^3. Mismatch with radar size sensitivity, r^6

$\chi \sim 1$ Resonant - or Mie

Cloud droplet: Thermal IR $\lambda \sim 10$ μm

Small crystals

$$\chi = \frac{2\pi\ 10\,\mu m}{10\,\mu m} \simeq 6$$

$n_i \sim 1$ to 2 $\chi_{abs} \simeq 6$ to 12

Strong absorption / emission

$\sigma_{ext} \sim \sigma_{abs} + \sigma_{sca}$

Sea Salt aerosol $r \sim 5$ μm $\chi \simeq 3$ $n_i << 1 \simeq 10^{-6}$

$\chi \sim 3$ $\chi_{abs} << 1$ $\sigma_{ext} = \sigma_{sca}$

$\chi >> 1$ Geometric Optics Regime.

Cloud droplet: Visible: Cirrus "Small" crystals

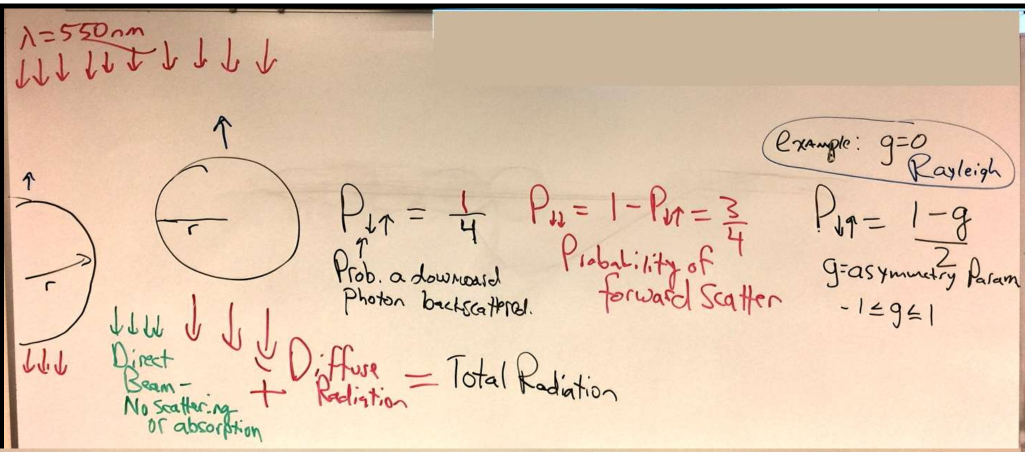
$$\chi = \frac{2\pi\ 10\,\mu m}{0.5\,\mu m} \hat{=} 120$$

$n_i \sim 10^{-8}$ $\chi_{abs} = 10^{-6} << 1$

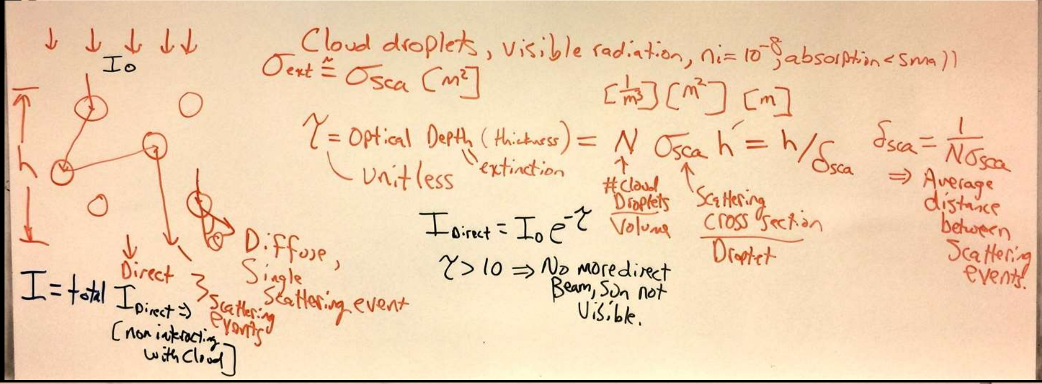
$\sigma_{ext} \sim \sigma_{sca}$

Multiple Scattering of Solar Radiation by Clouds

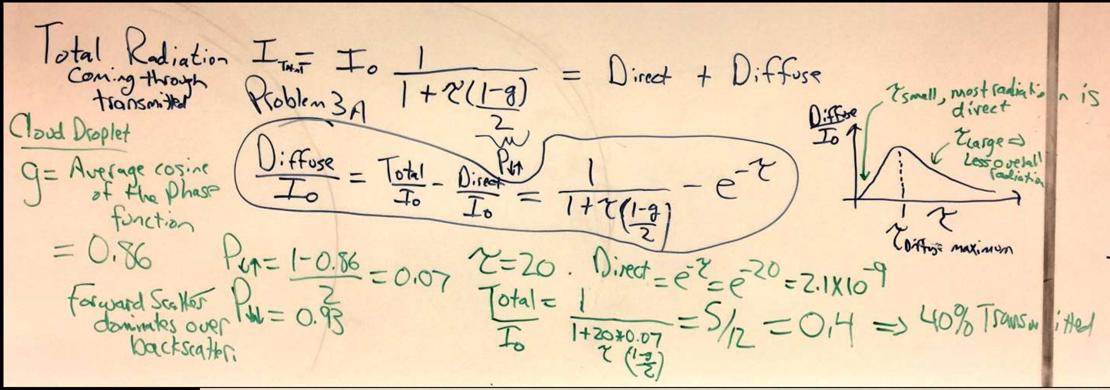
Asymmetry parameter
and direct and diffuse



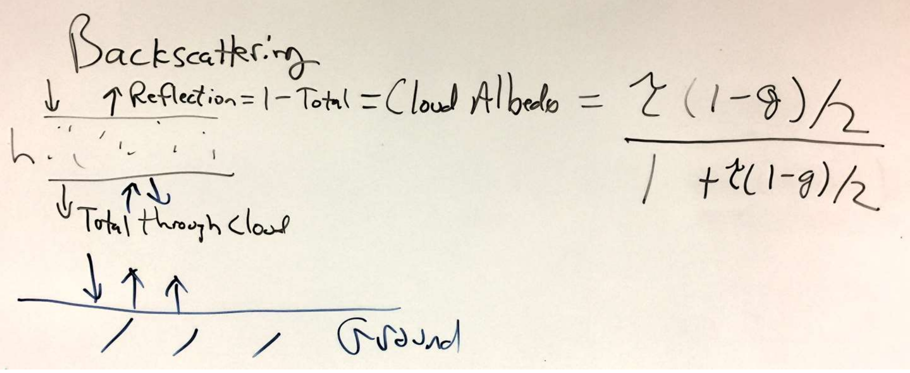
Optical depth calculation,
geometrical optics regime,
ext. cross section is twice
the geometric cross section.

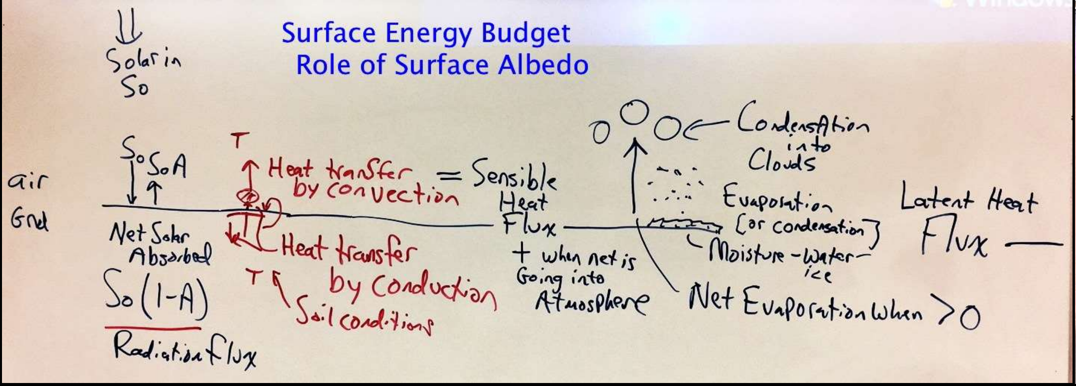


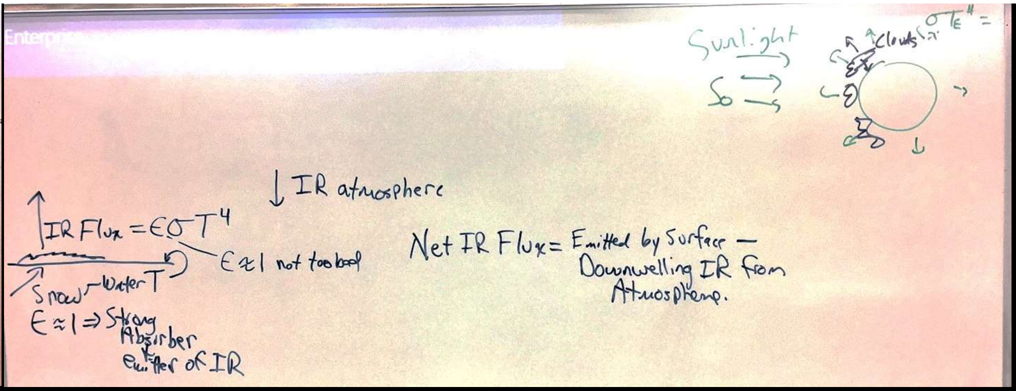
Total transmitted radiation



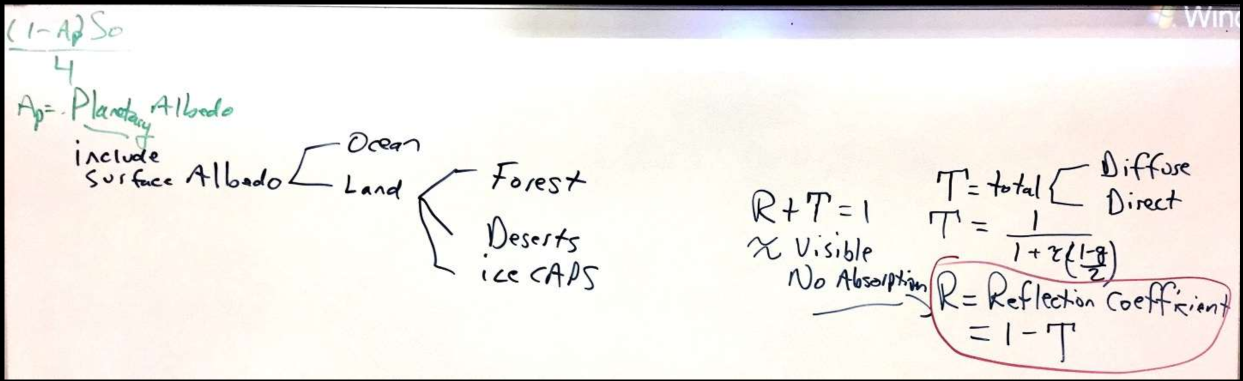
Radiation reflected by cloud,
not taking into account that reflected
by the ground. (see next slide).

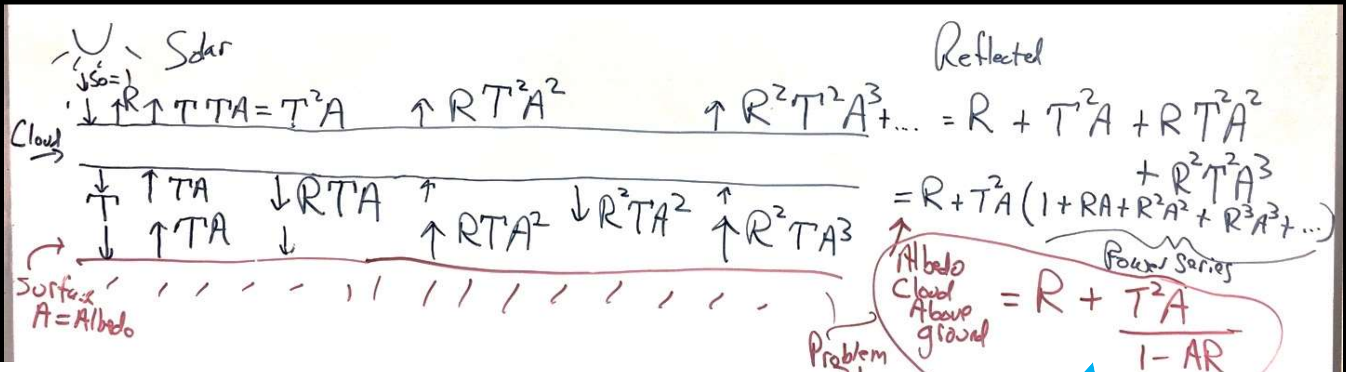


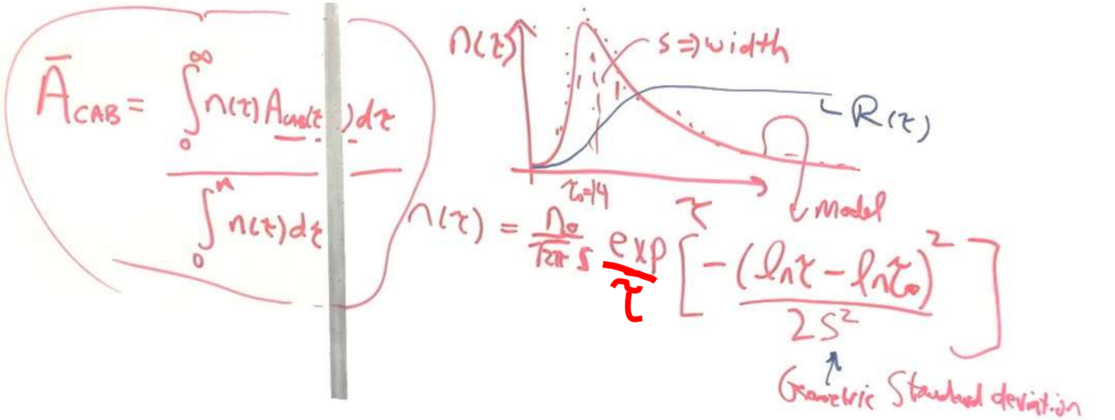




Surface Energy Budget and Solar radiation reflected by cloud above ground

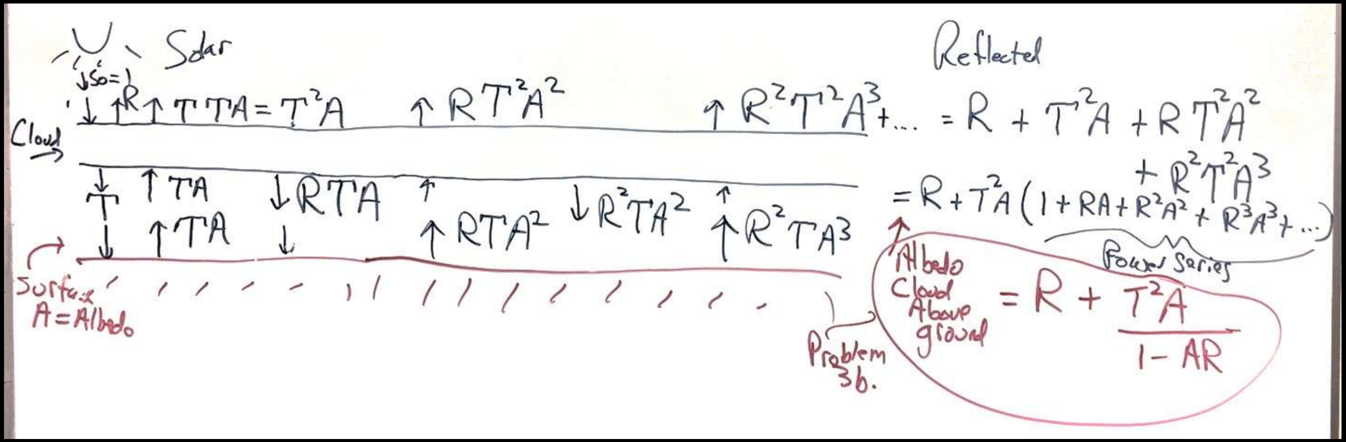


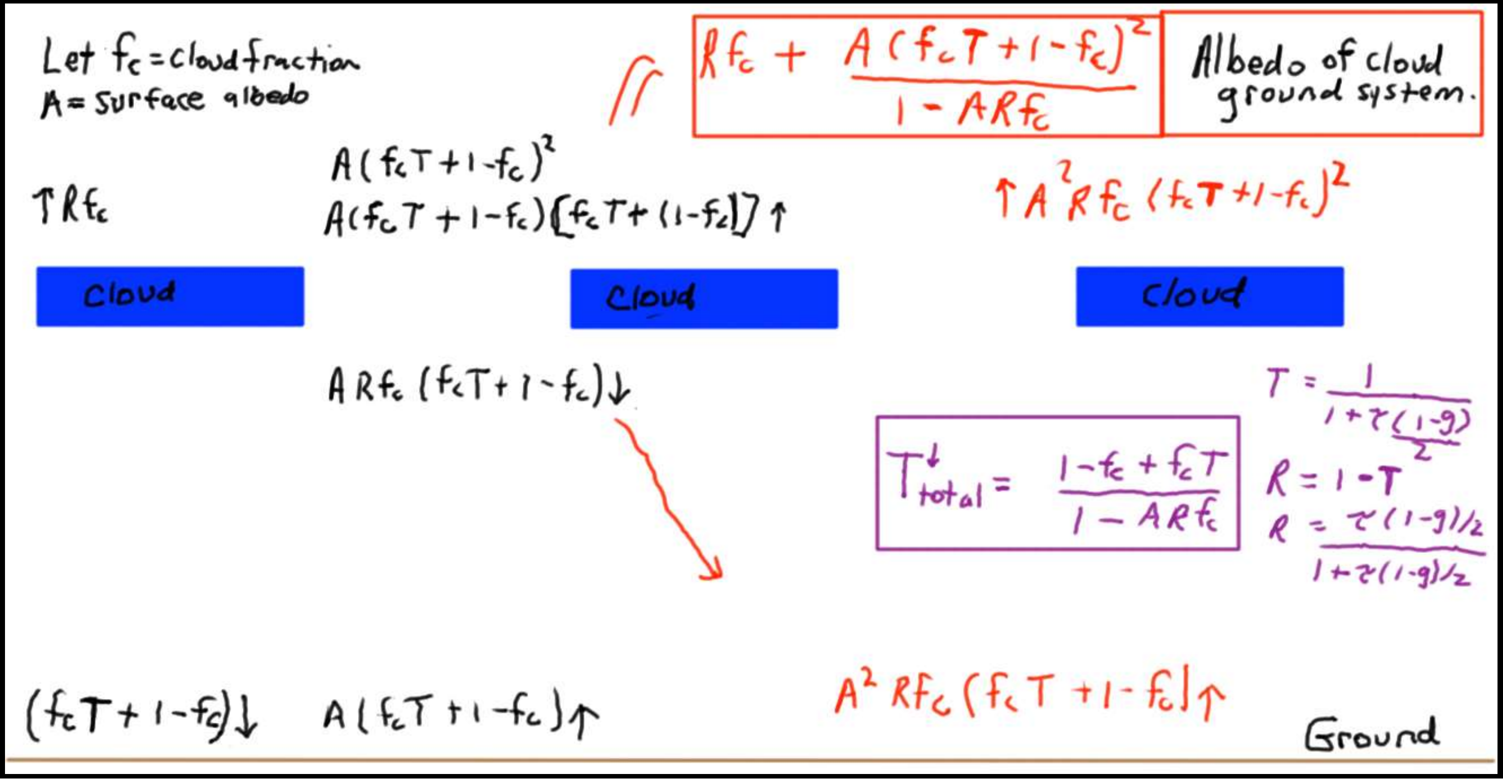




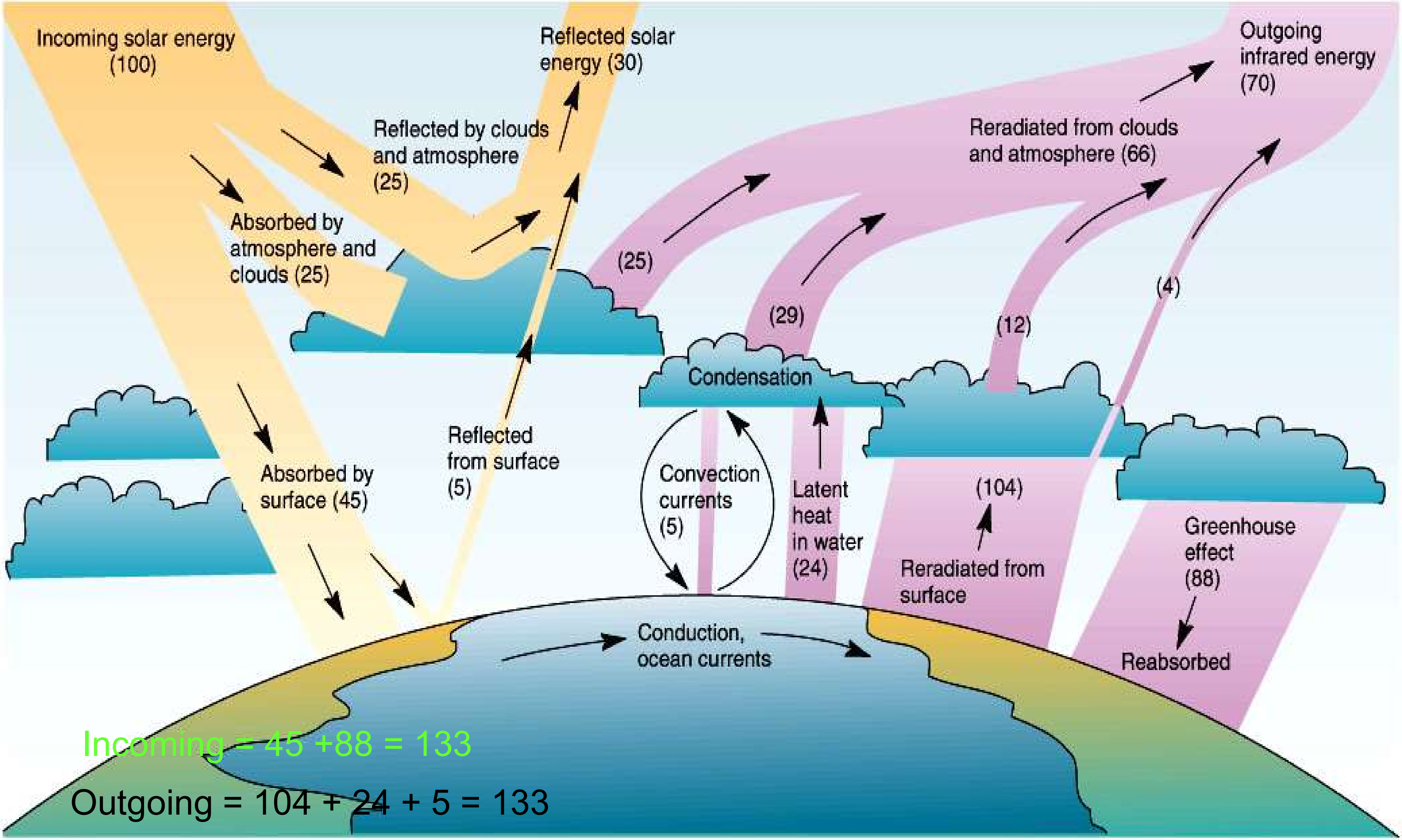
Cloud albedo above ground

Surface Energy Budget and Solar radiation reflected by cloud above ground with partly cloudy sky conditions





Global Energy Balance

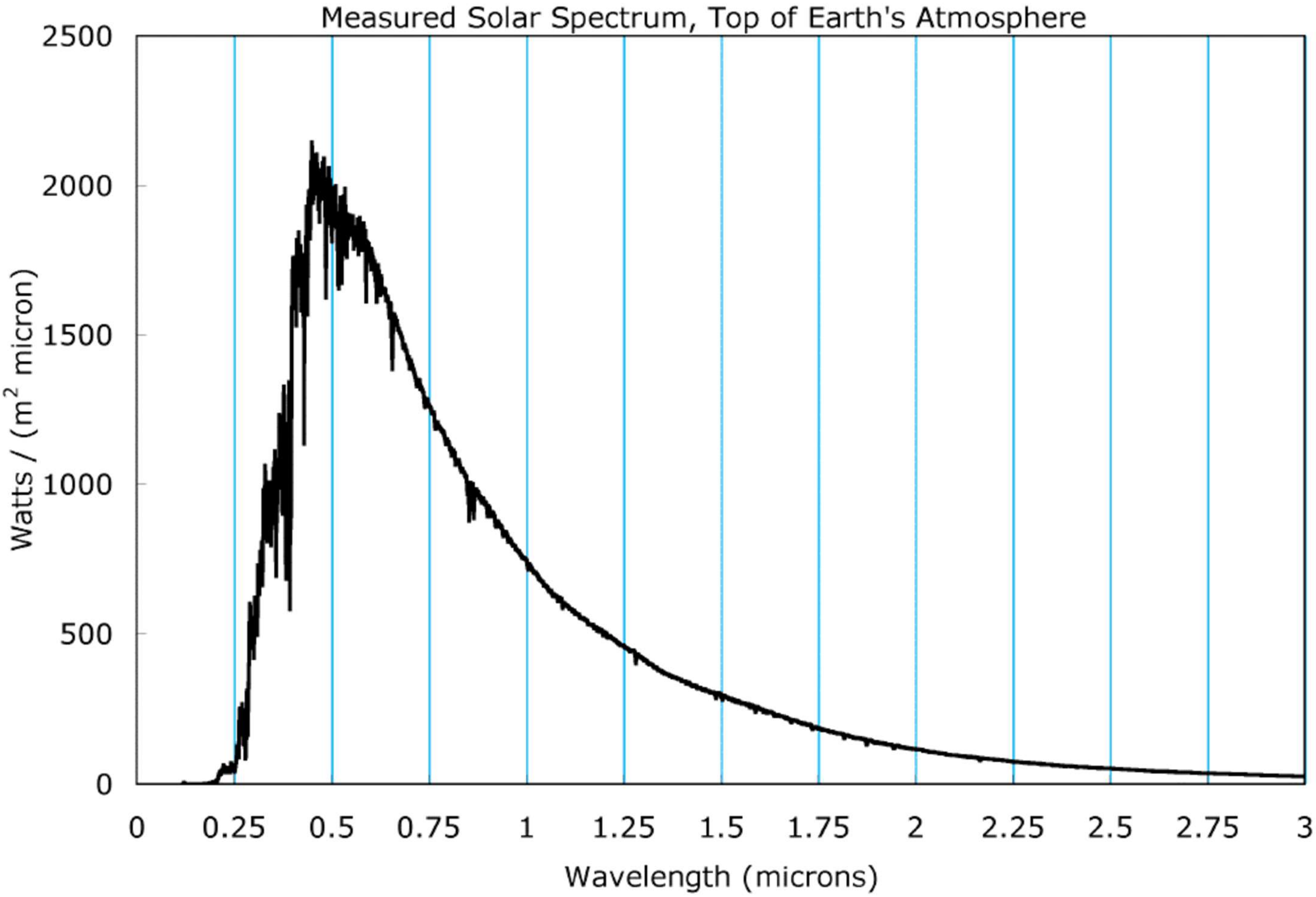


Incoming = 45 +88 = 133

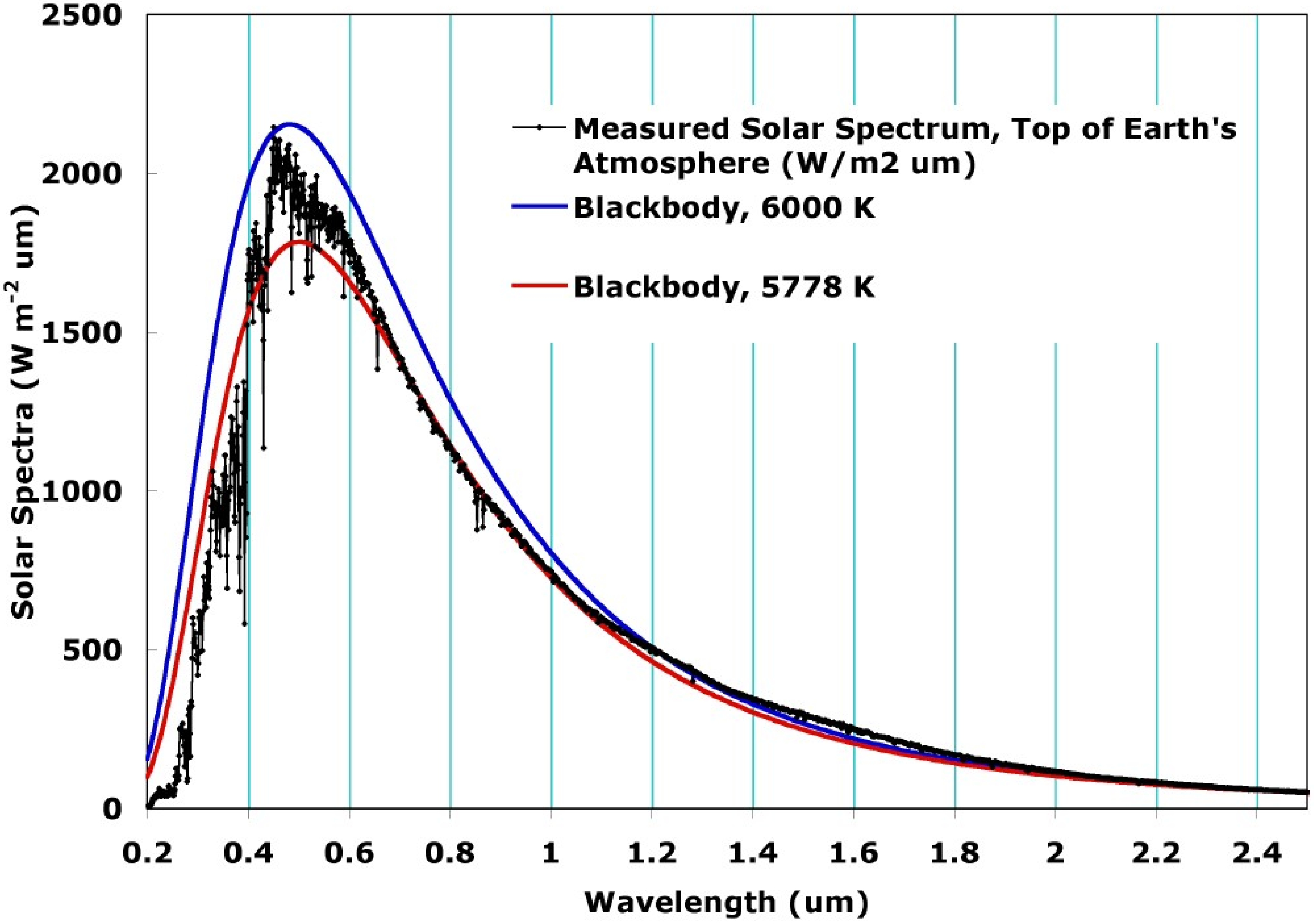
Outgoing = 104 + 24 + 5 = 133

From Cunningham & Cunningham, 2004, Fig. 9.2

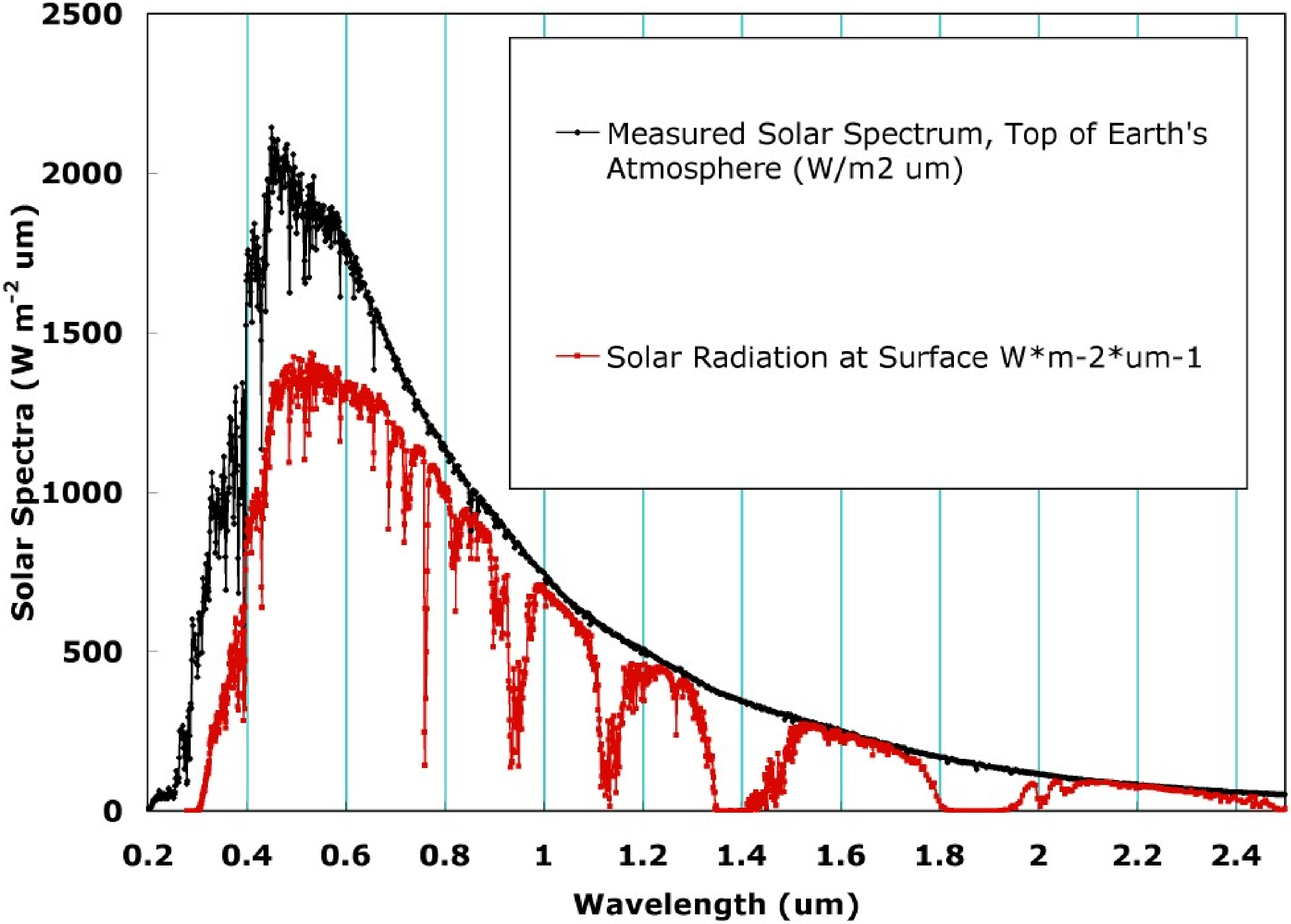
Top of the Atmosphere Solar Radiation:



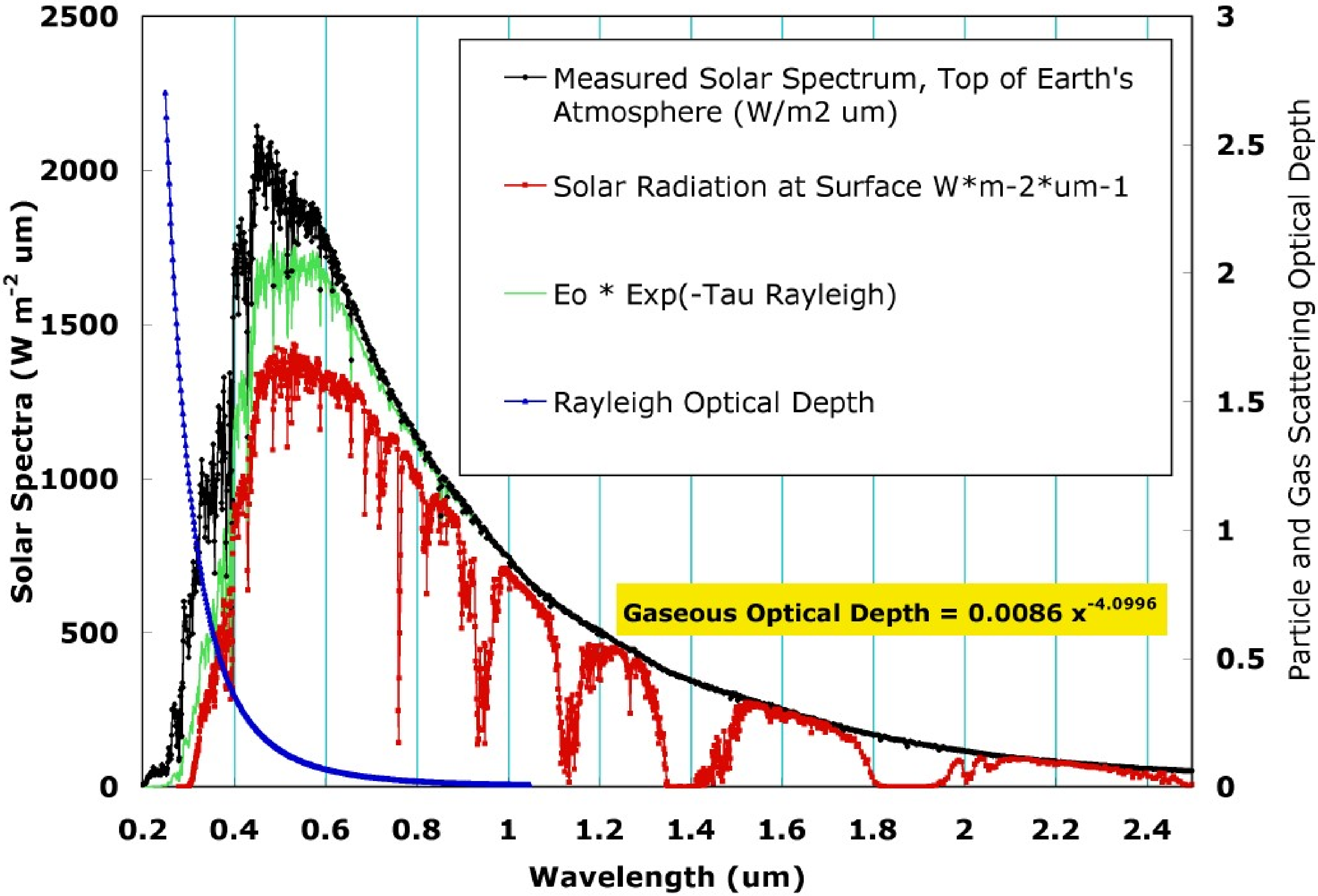
SOLAR SPECTRUM:
TOP OF THE ATMOSPHERE



SOLAR SPECTRUM:
TOP OF THE ATMOSPHERE AND AT THE SURFACE

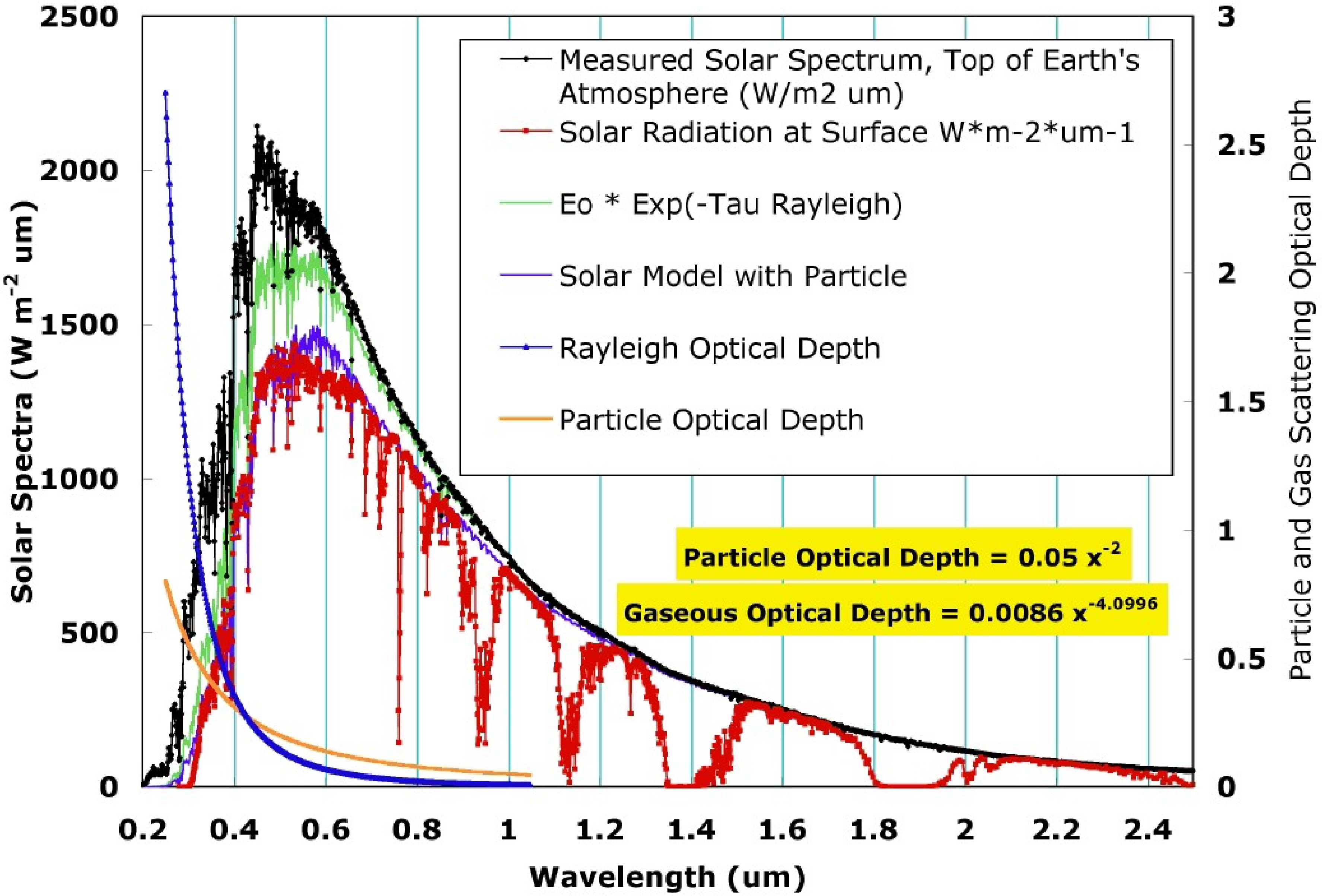


SOLAR SPECTRUM:
Effects of Rayleigh (gas) scattering, O_2 and N_2.

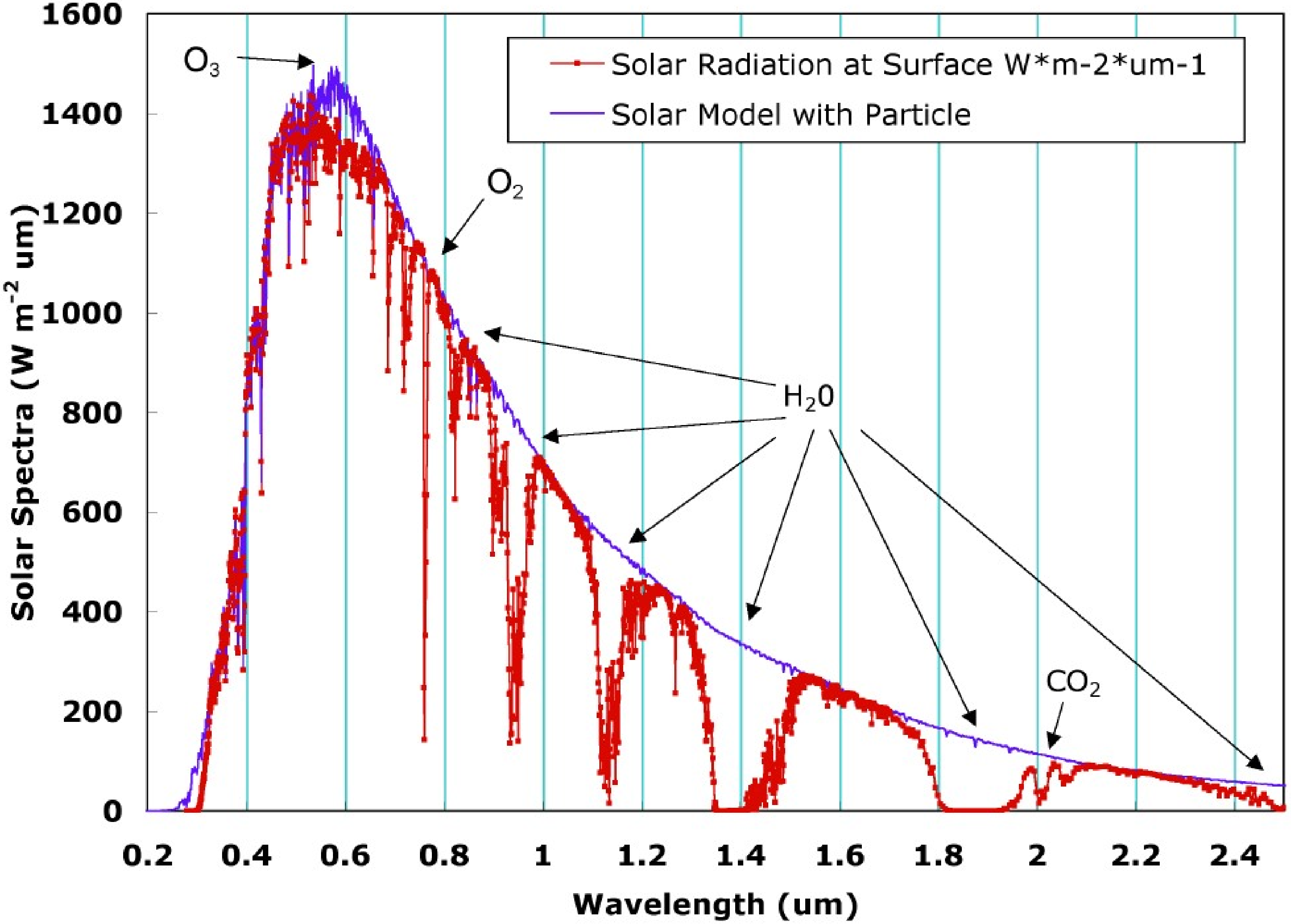


SOLAR SPECTRUM:

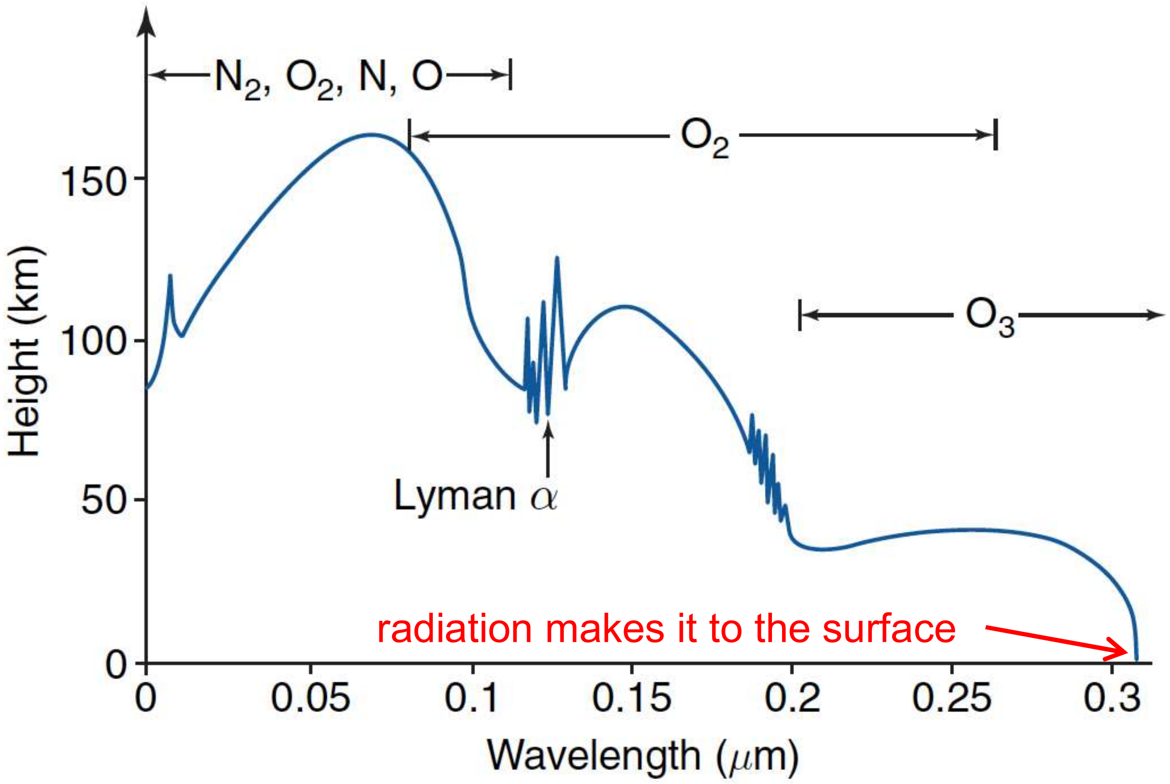
Effects of Rayleigh (gas) scattering, O_2 and N_2, And effects of extinction by aerosol particles.



SOLAR SPECTRUM:
Effects of gaseous absorption.



Absorption by Gases: Solar Radiation Penetration Depth



radiation makes it to the surface

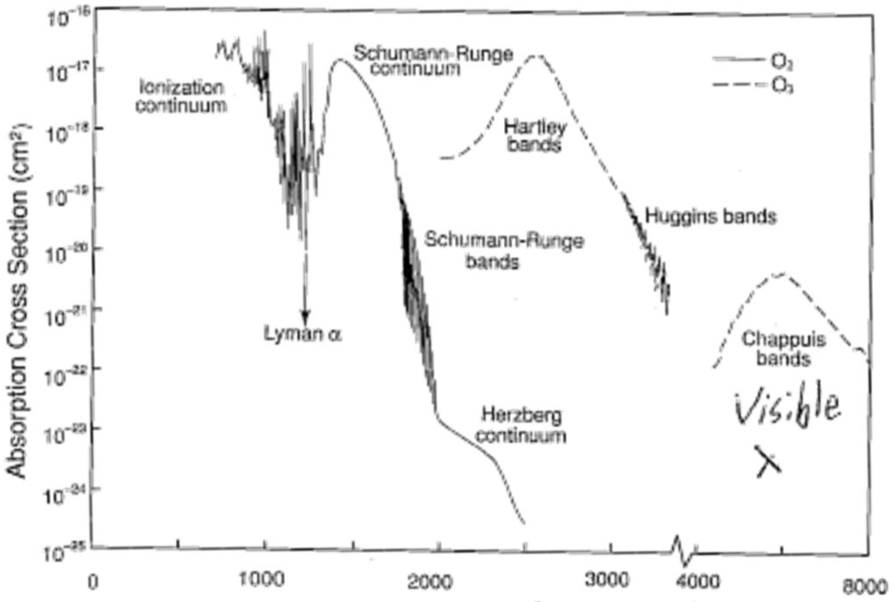


Figure 3.5 Absorption cross section of ozone and molecular oxygen in the ultraviolet spectral region. Data taken from Brasseur and Solomon (1986), Vigroux (1953), and Griggs (1968).

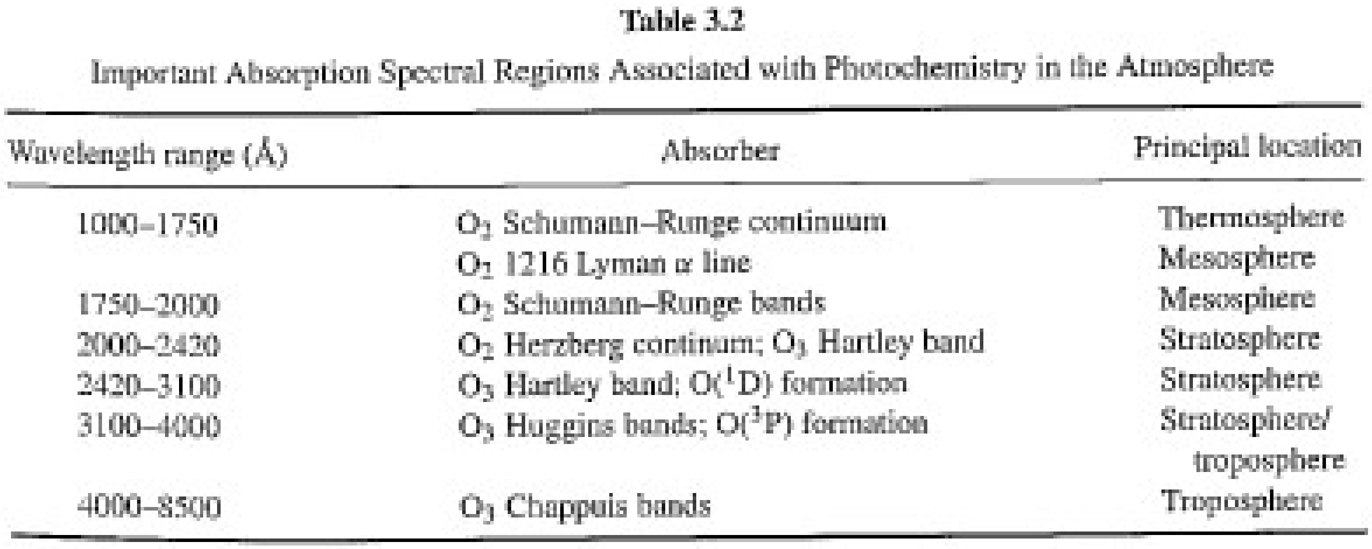
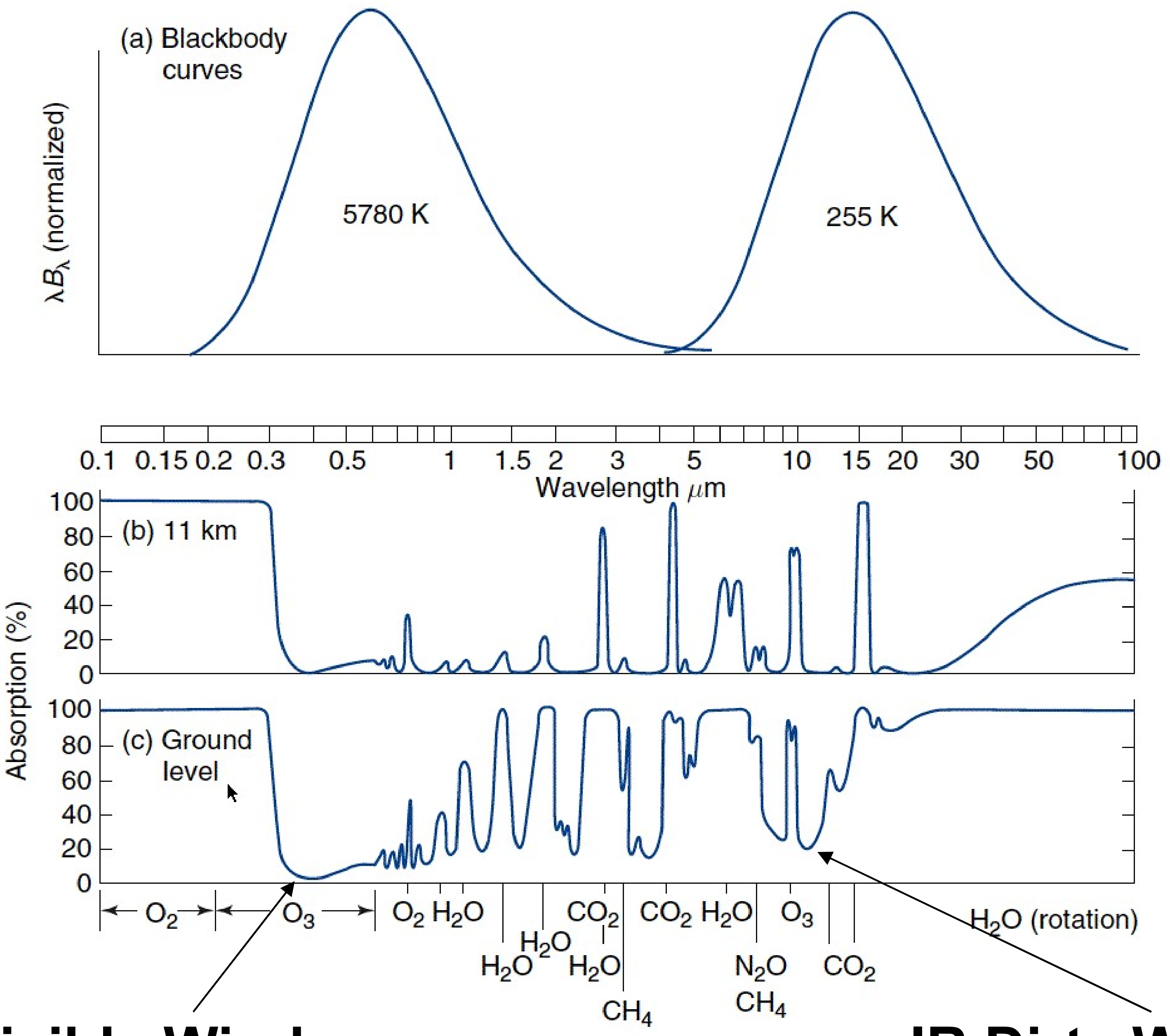


Table 3.2

Important Absorption Spectral Regions Associated with Photochemistry in the Atmosphere

Wavelength range (Å)	Absorber	Principal location
1000–1750	O_2 Schumann–Runge continuum	Thermosphere
	O_2 1216 Lyman α line	Mesosphere
1750–2000	O_2 Schumann–Runge bands	Mesosphere
2000–2420	O_2 Herzberg continum; O_3 Hartley band	Stratosphere
2420–3100	O_3 Hartley band; O(^{1}D) formation	Stratosphere
3100–4000	O_3 Huggins bands; O(^{3}P) formation	Stratosphere/ troposphere
4000–8500	O_3 Chappuis bands	Troposphere

Sun and Earth Spectra and Atmospheric Windows



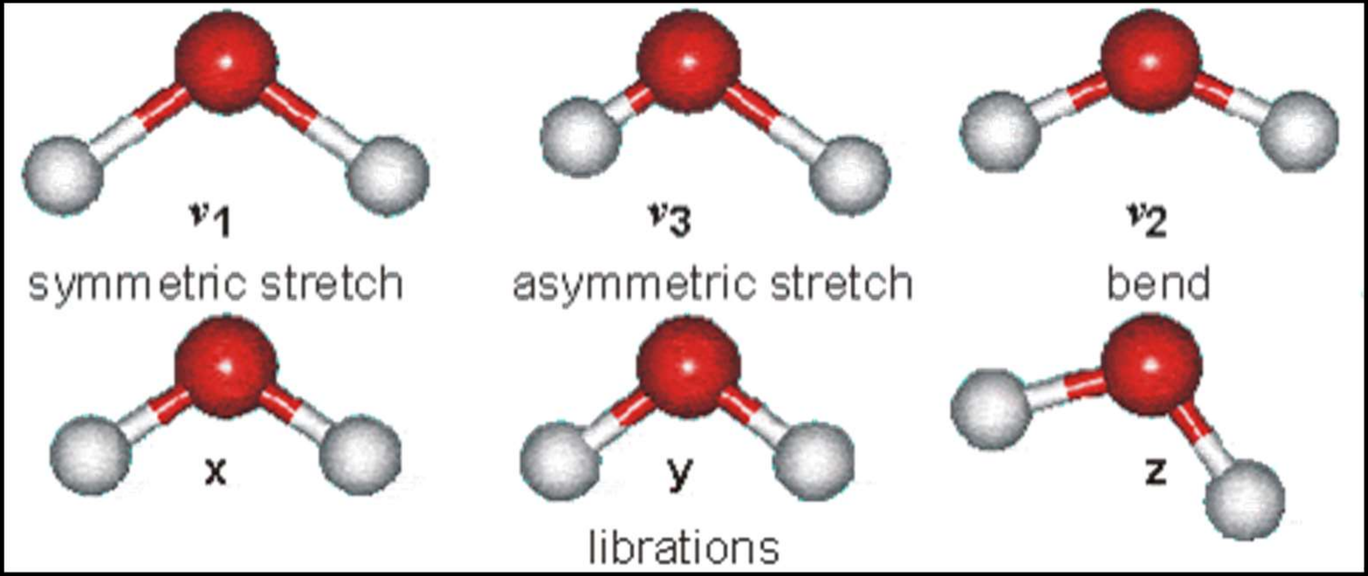
Visible Window

IR Dirty Window

Absorption only, not effects of Rayleigh Scattering by gases.

Some Energy States of Water Molecules

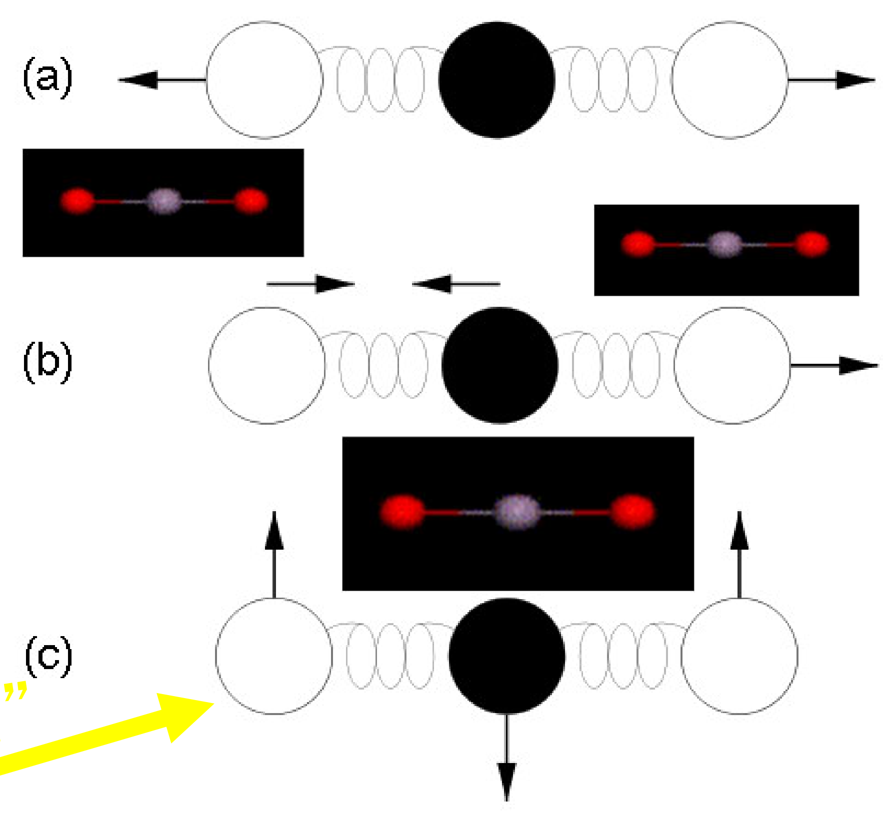
Atmospheric molecules lead very active ‘lives’ due to thermal and electromagnetic energy exchange.



http://www.lsbu.ac.uk/water/vibrat.html

... of Carbon Dioxide Molecules

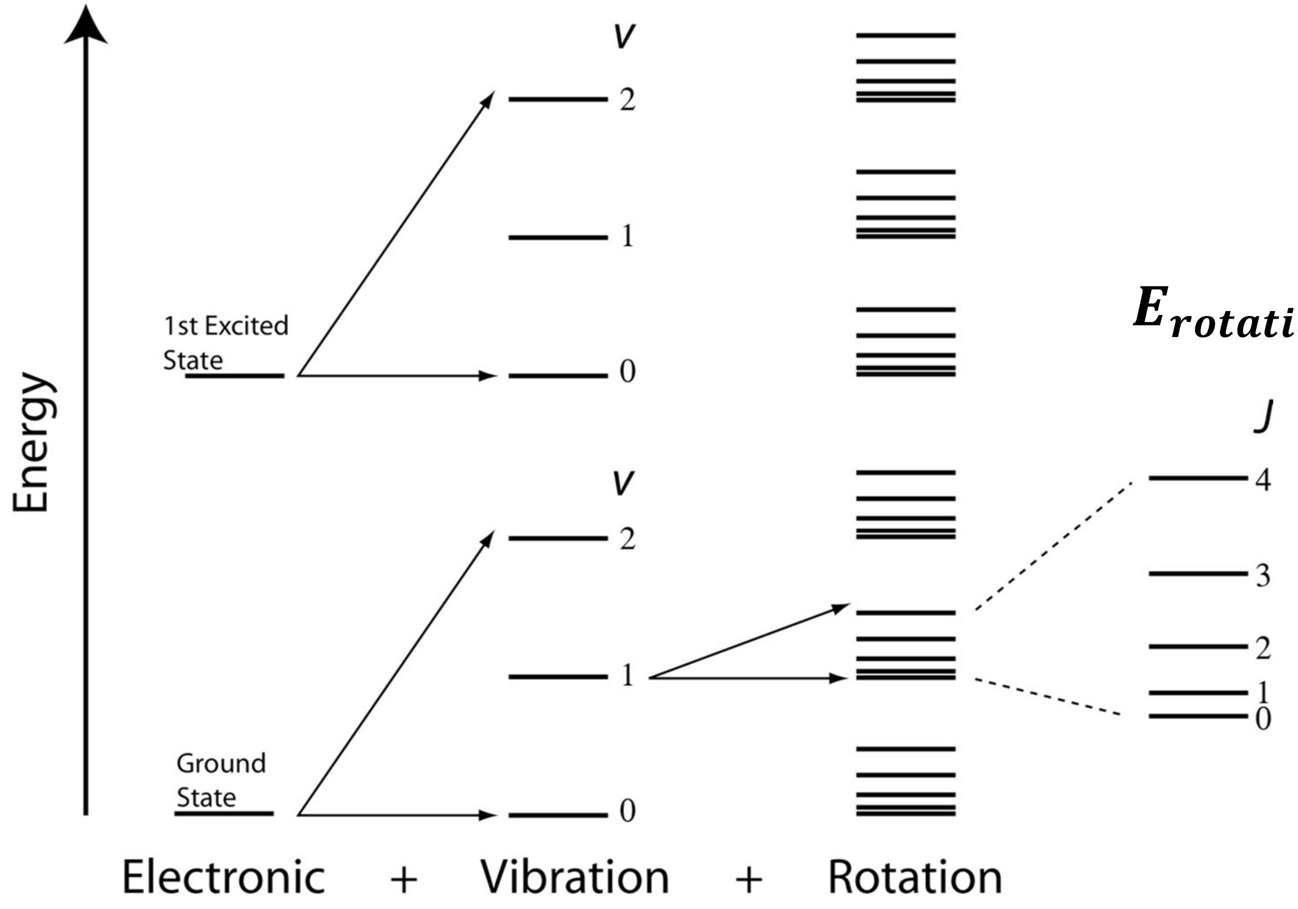
Vibration modes of carbon dioxide. Mode (a) is symmetric and results in no net displacement of the molecule's "center of charge", and is therefore not associated with the absorption of IR radiation. Modes (b) and (c) do displace the "center of charge", creating a "dipole moment", and therefore are modes that result from EM radiation absorption, and are thus responsible for making CO2 a greenhouse gas.



“15 um motion”

Pat Arnott

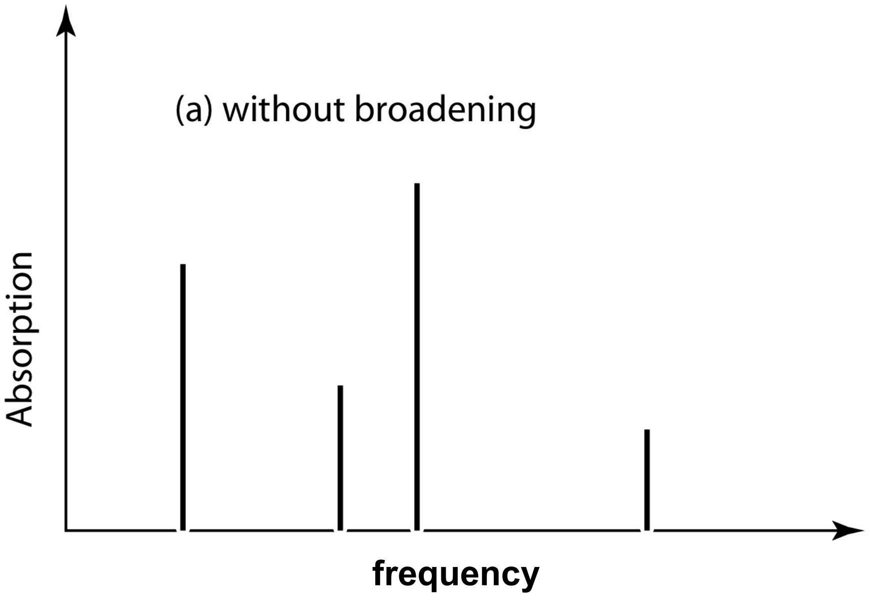
Energy Transitions in Molecules

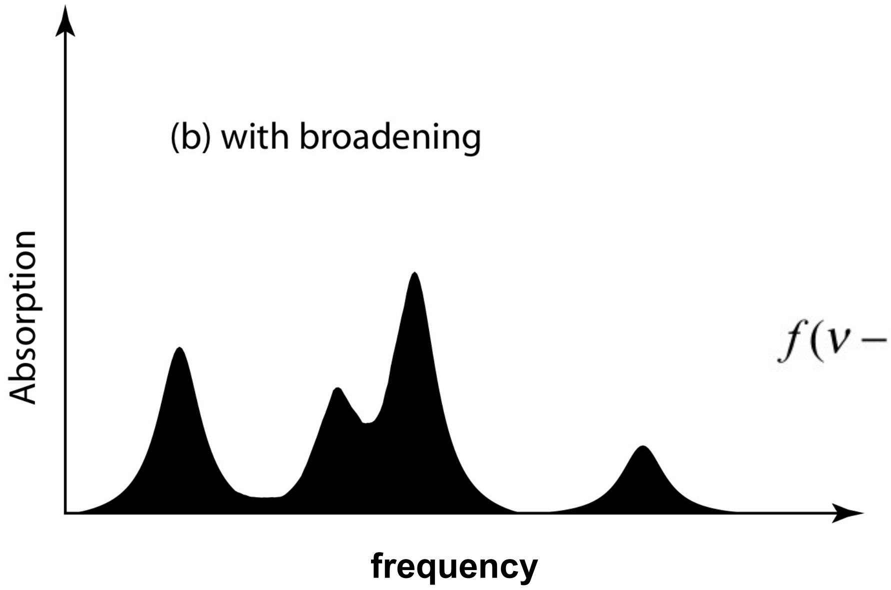


$$E_{rotation} = \frac{J(J+1)h^2}{8\pi^2 \mu r_0^2}$$

$$E_{vibration} = \left(\nu + \frac{1}{2}\right)\mathrm{hf} \quad \nu = 0, 1, 2, \ldots$$

Line Broadening





Natural Broadening:
Finite time, finite widths (Heisenberg is uncertain about widths, certain they are not infinitely narrow!)

Doppler Broadening:

$$f_D(\nu-\nu_0)=\frac{1}{\alpha_D\sqrt{\pi}}\exp\left[-\frac{(\nu-\nu_0)^2}{\alpha_D^2}\right],\quad \alpha_D=\nu_0\sqrt{\frac{2k_BT}{mc^2}}$$

Molecules with relative motions due to thermal energy ‘see’ doppler shifts of the light. Important in the mesosphere.

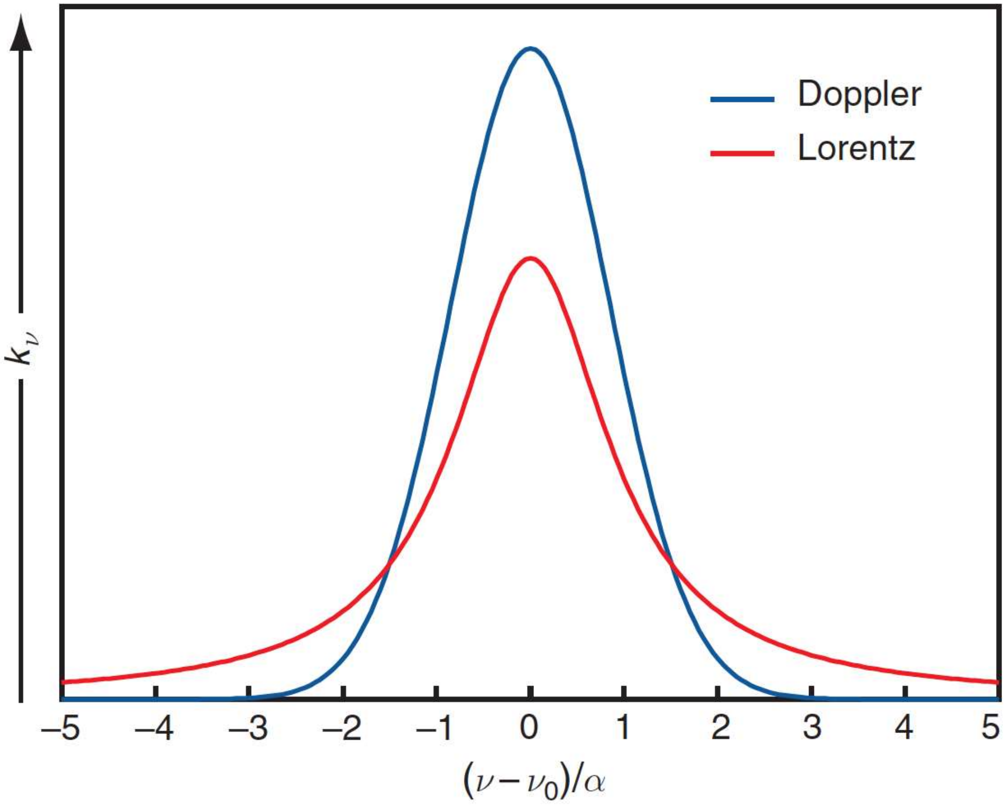
Pressure Broadening: Lorentz line shape

$$f(\nu-\nu_0)=\frac{\alpha_L/\pi}{(\nu-\nu_0)^2+\alpha_L^2},\quad \alpha_L\propto pT^{-1/2},\quad \alpha_L=\alpha_\infty\left(\frac{p}{p_0}\right)\left(\frac{T_0}{T}\right)^n$$

Molecular collisions distort energy levels for absorption and emission. Emperically determined (by measurement). Very important for the troposphere and lower stratosphere.

Individual Absorption Lines Are Pressure and Doppler Broadened (absorb more radiation as a result).

gas absorption cross section



$$k_\nu = Sf(\nu - \nu_0)$$

$$S = \int_0^\infty k_\nu d\nu$$

S = absorption strength

ν_0=wavenumber of absorption peak

- *Doppler broadening*: the Doppler shifting of frequencies at which the gas molecules experience the incident radiation by virtue of their random motions toward or away from the source of the radiation, and

$$f = \frac{1}{\alpha_D\sqrt{\pi}}\exp\left[-\left(\frac{\nu-\nu_0}{\alpha_D}\right)^2\right]$$

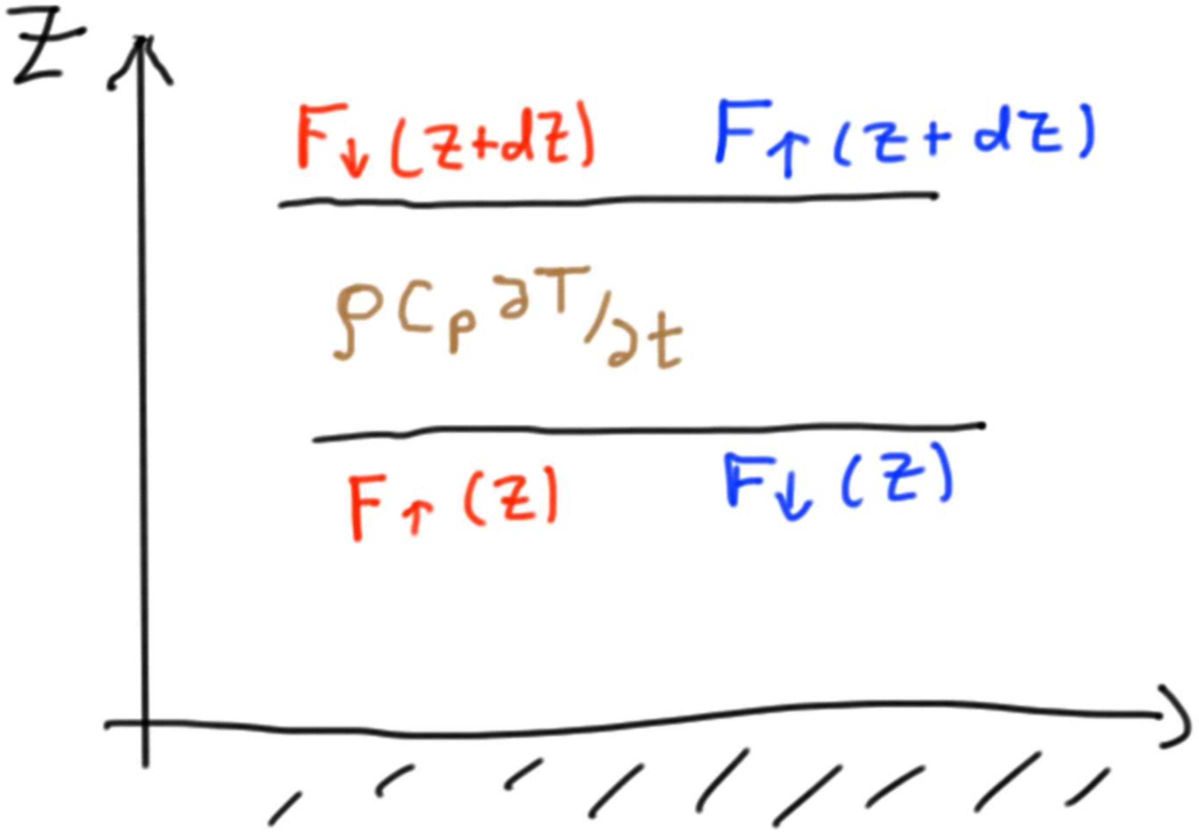
$$\alpha_D = \frac{\nu_0}{c^*}\left(\frac{2kT}{m}\right)^{1/2}$$

- *pressure broadening*: (also referred to as *collision broadening*) associated with molecular collisions.

$$f = \frac{\alpha}{\pi[(\nu-\nu_0)^2+\alpha^2]} \qquad \alpha \propto \frac{p}{T^N}$$

$0.5 \le N \le 1$

Heating Rate = Divergence of the Net Flux



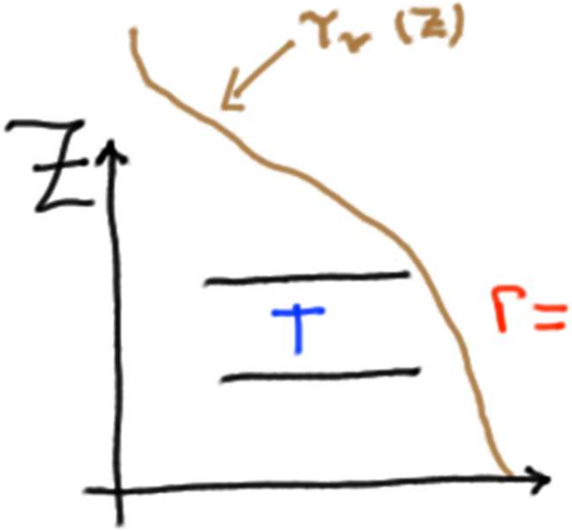
$$F_{net}(z) = F_\uparrow(z) - F_\downarrow(z)$$

$$\boxed{\rho c_p \frac{\partial T}{\partial t} = -\frac{\partial F_{net}(z)}{\partial z}}$$

$$dz\,\rho c_p \frac{\partial T}{\partial t} = \text{In} - \text{out} = F_\downarrow(z+dz) - F_\uparrow(z+dz) + F_\uparrow(z) - F_\downarrow(z)$$

$$= -F_{net}(z+dz) + F_{net}(z) = -\frac{\partial F_{net}}{\partial z}\,dz$$

Heating Rate = Divergence of the Net Flux



r = mixing ratio of absorber = w for water vapor

$$\left(\frac{dT}{dt}\right)_\nu = -\frac{\pi}{c_p}\overbrace{\underline{k_\nu} r B_{\underline{\nu}}(z)}^{\text{emission}}\frac{\overbrace{e^{-\tau_\nu/\bar{\mu}}}^{\text{Transmission}}}{\bar{\mu}}$$

$\bar{\mu} = 1.66 \equiv$ diffusivity factor

$k_\nu r = \dfrac{\text{Absorption Cross section}}{\text{mass}}$ at ν for molecule i.

Clear Sky Heating Rate Total: Solar and IR

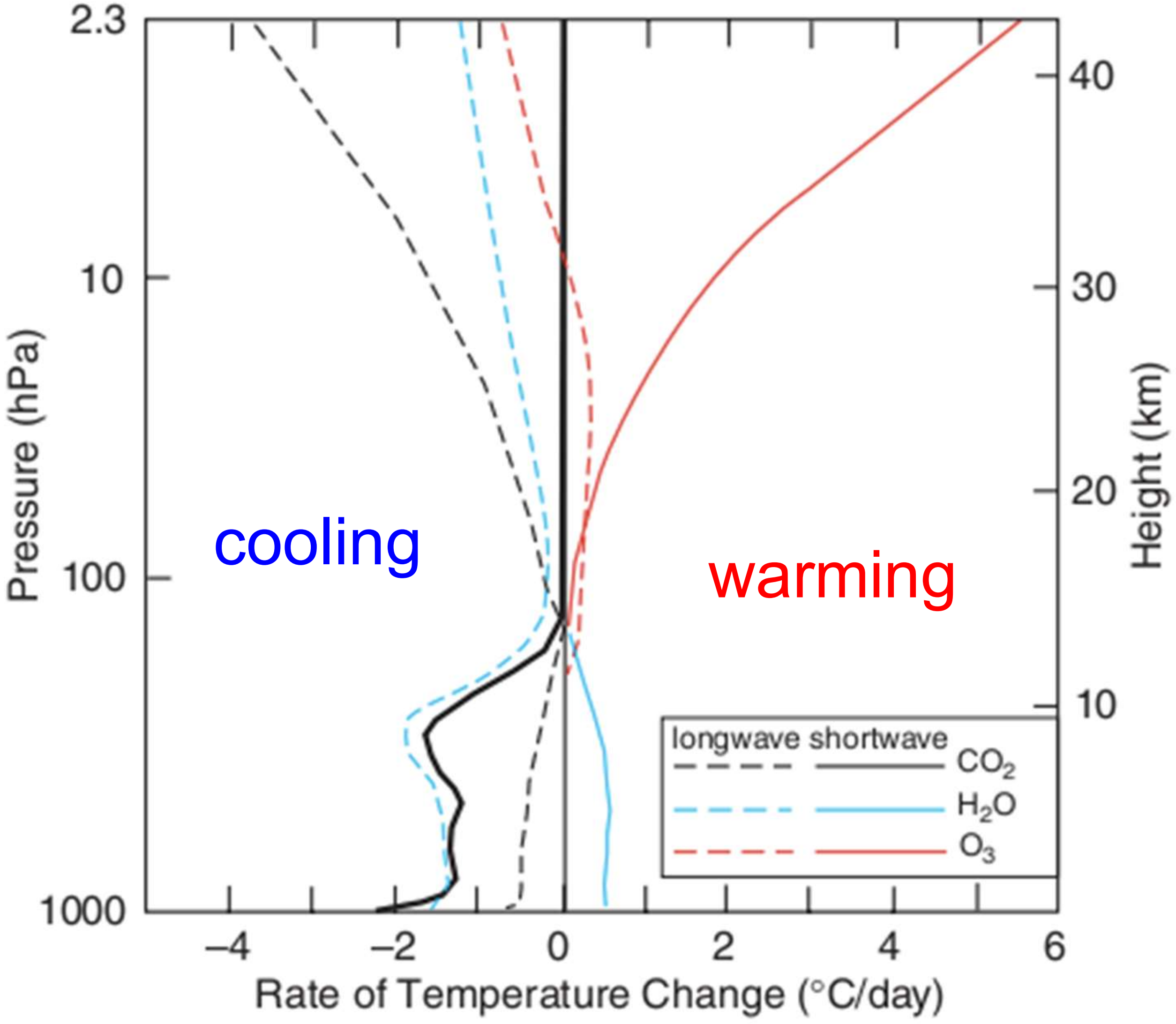


Fig. 4.29 Vertical profiles of the time rate of change of temperature due to the absorption of solar radiation (solid curves) and the transfer of infrared radiation (dashed curves) by water vapor (blue), carbon dioxide (black), and ozone (red). The heavy black solid curve represents the combined effects of the three gases. [Adapted from S. Manabe and R. F. Strickler, *J. Atmos. Sci.*, **21**, p. 373 (1964).]

Role of Clouds in Heating Rate

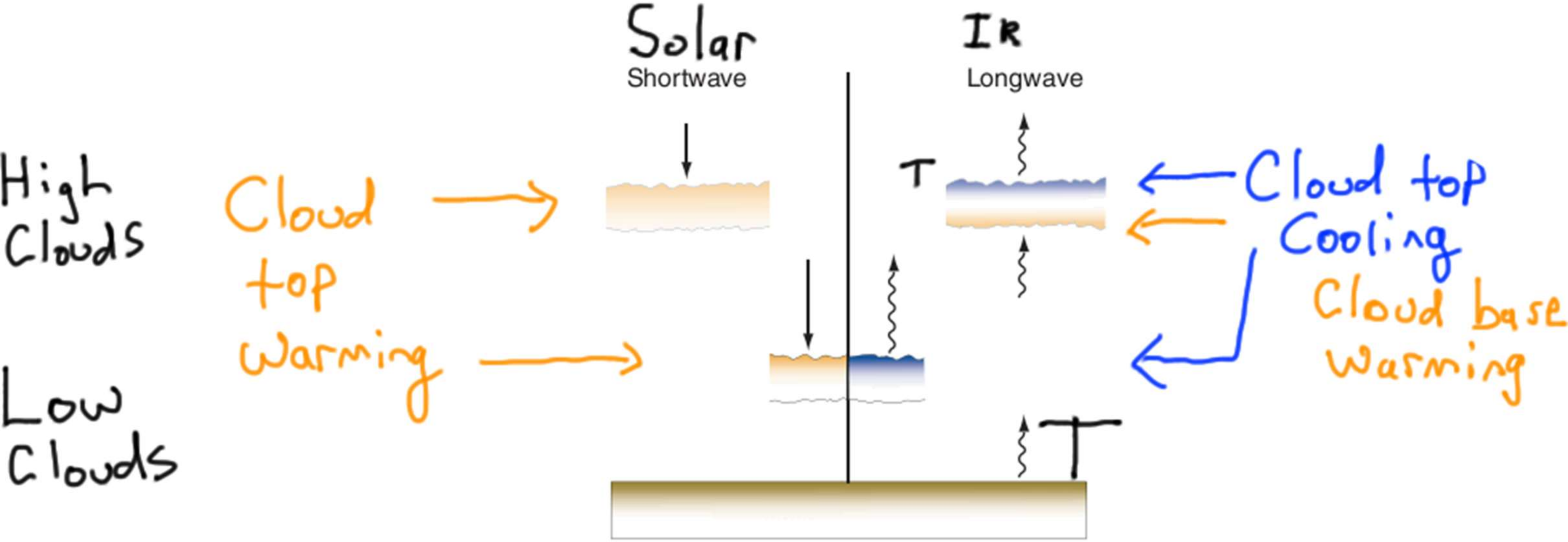
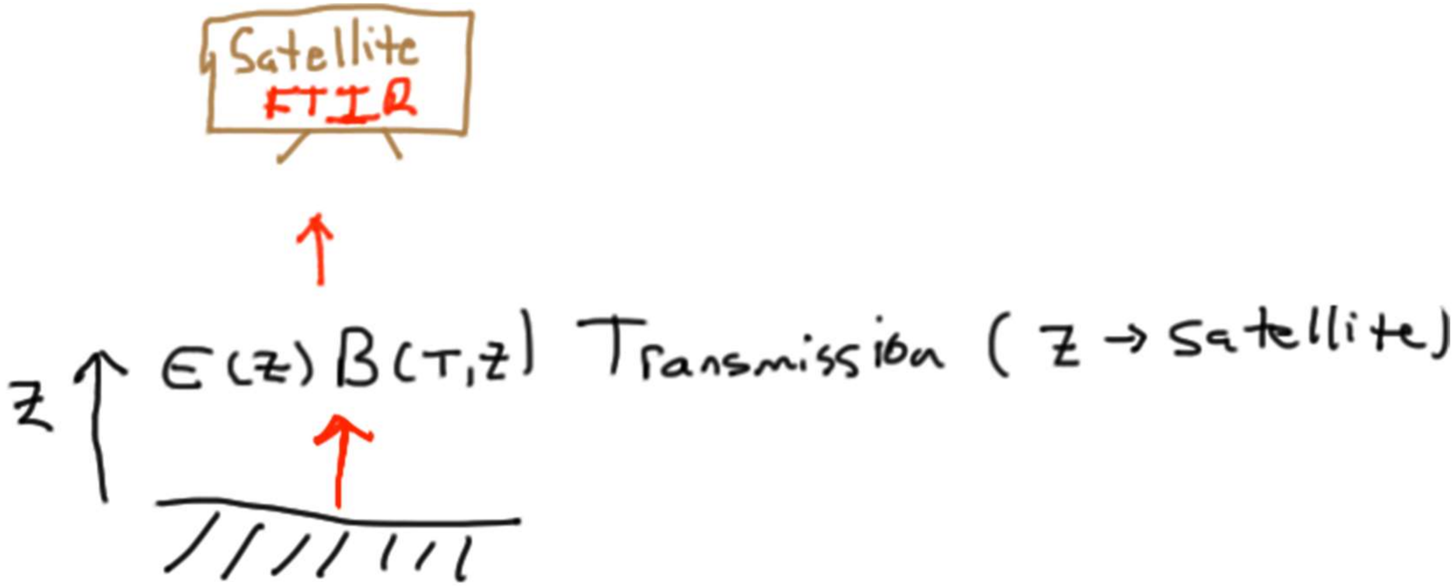
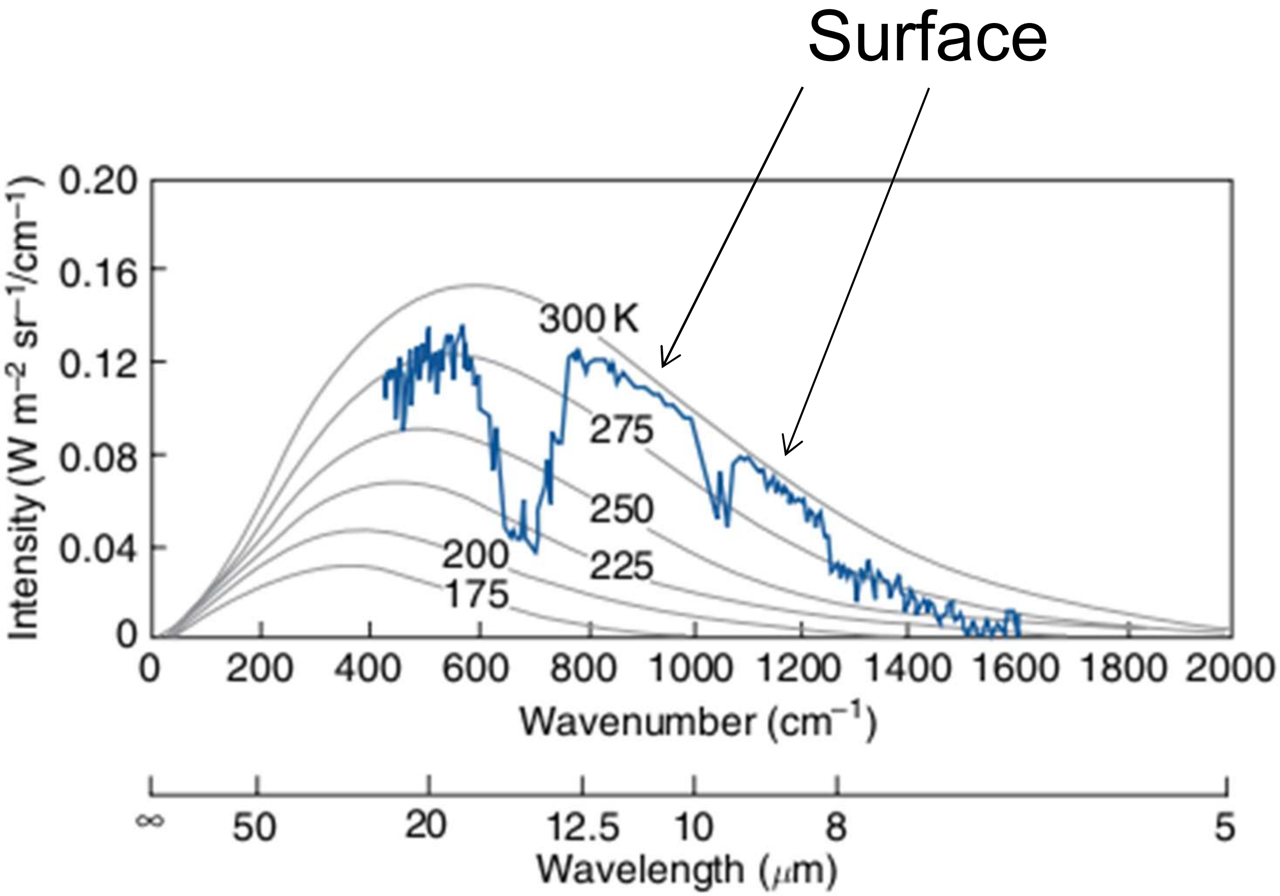


Fig. 4.30 Schematic of vertical profiles of heating in cloud layers at various heights in the atmosphere as indicated. Orange shading indicates warming and blue shading indicates cooling. Effects of shortwave radiation are represented on the left, and effects of longwave radiation on the right.

Remote Sensing With Down Looking IR Spectrometer

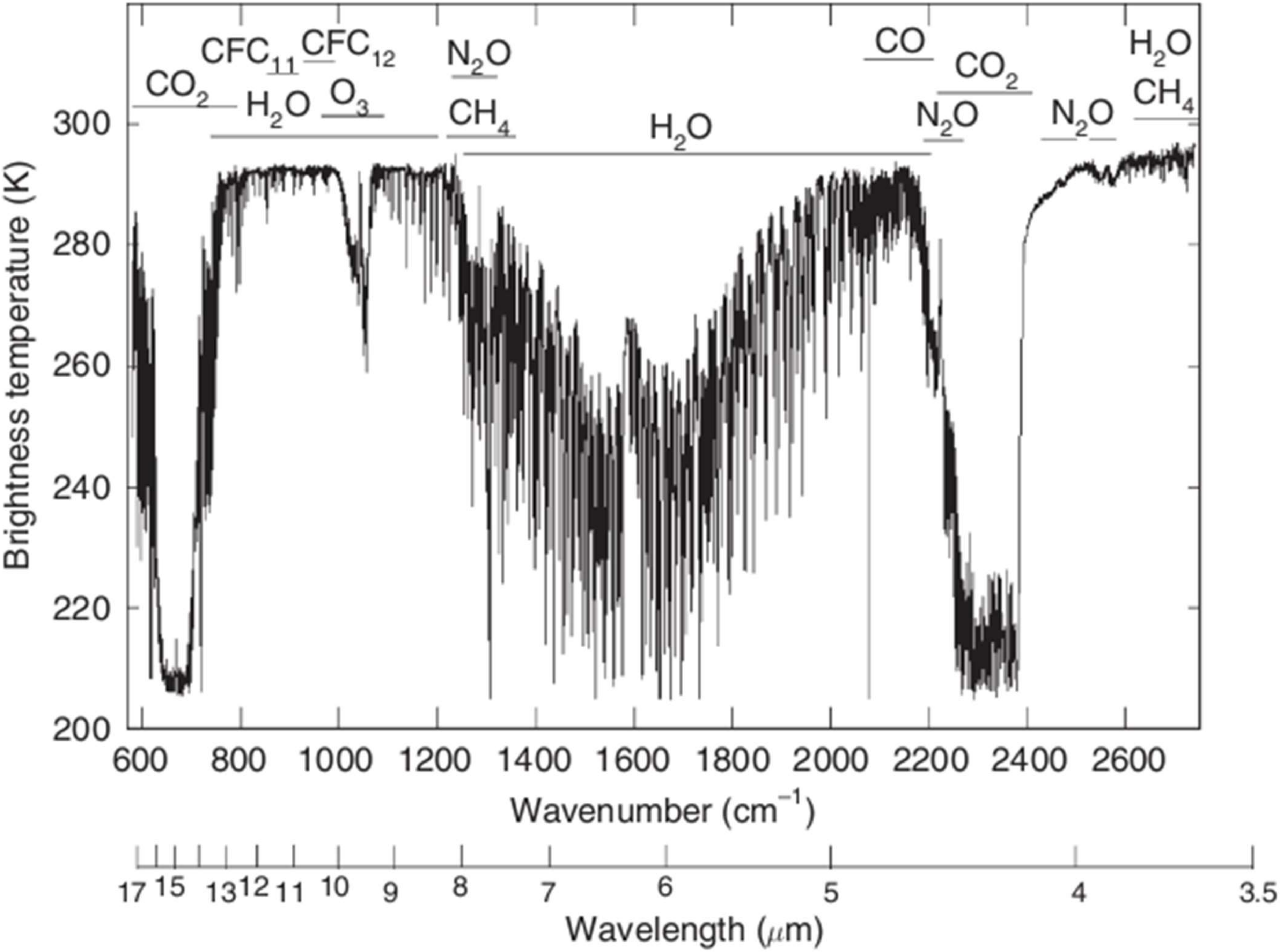


$\epsilon(z) B(T,z)$ Transmission ($z \rightarrow$ satellite)

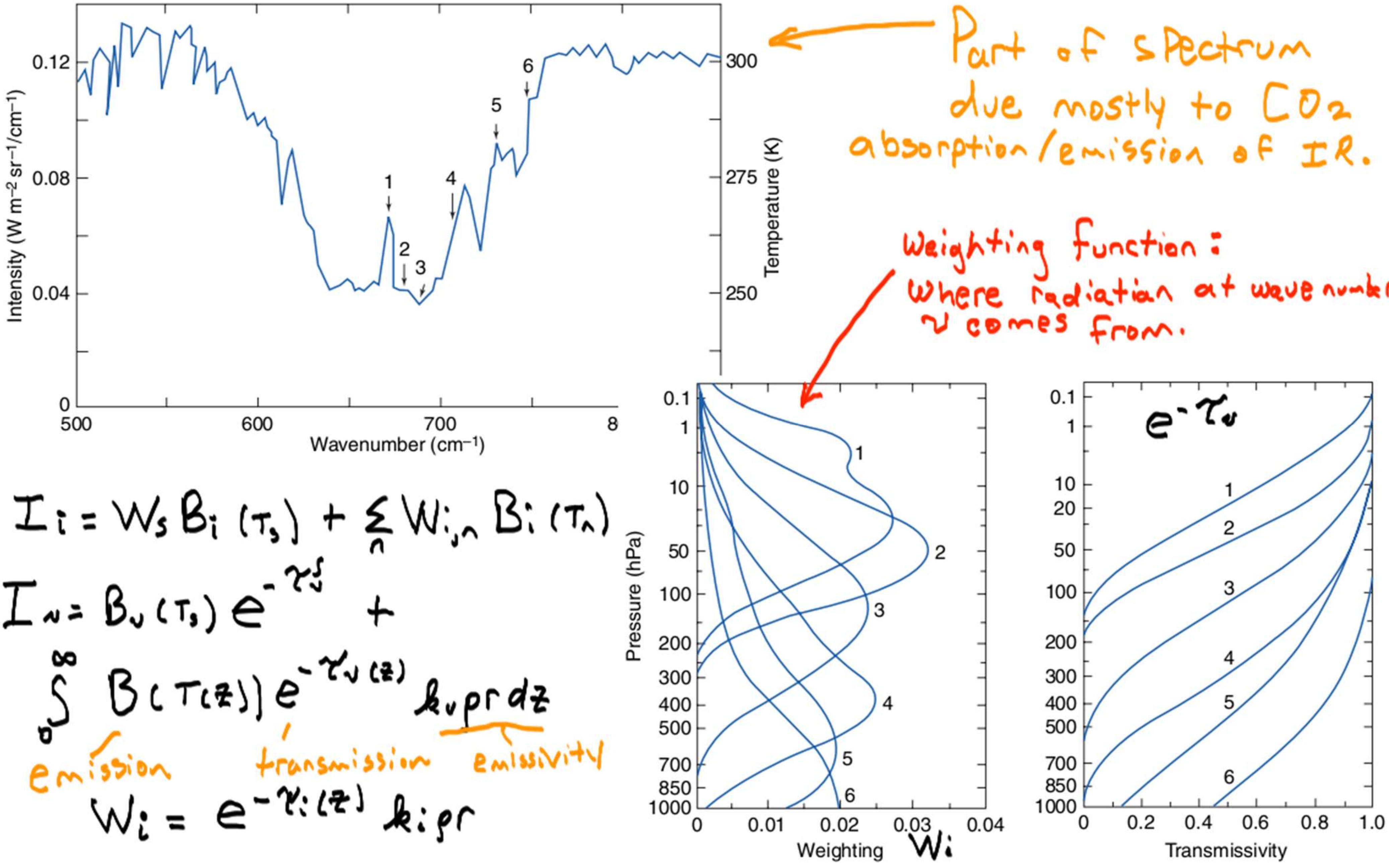


Roughly, the temperature where the emission comes from.

Top of Atmosphere Outgoing IR As Brightness Temperature



Use of CO2 (well-mixed) Spectra For Temperature Retrieval



Part of spectrum due mostly to CO2 absorption/emission of IR.

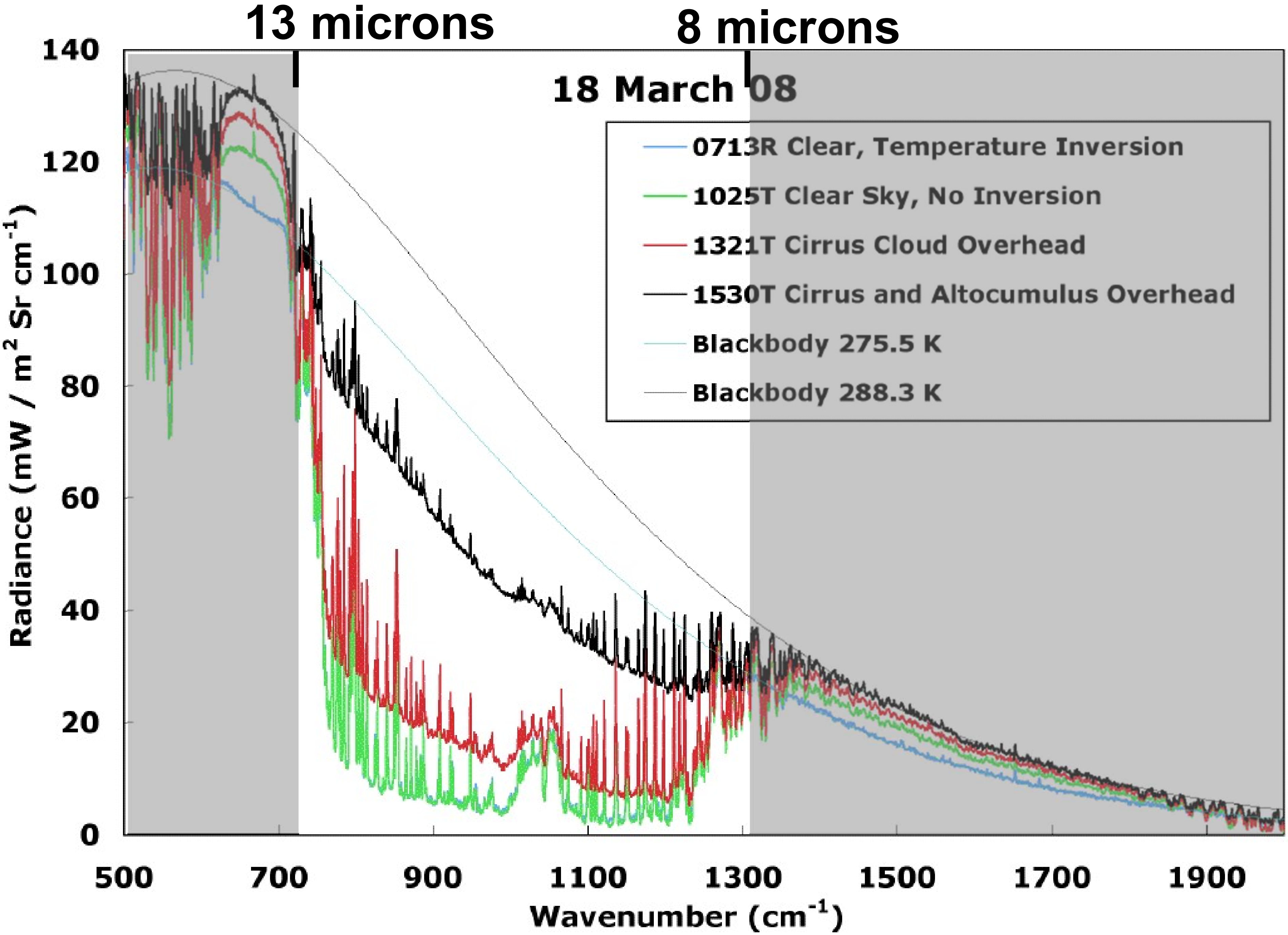
Weighting function: where radiation at wave number ν comes from.

$$I_i = W_s B_i(T_s) + \sum_n W_{i,n} B_i(T_n)$$

$$I_\nu = B_\nu(T_s) e^{-\tau_\nu^s} + \int_0^\infty \underbrace{B(T(z))}_{\text{emission}} \underbrace{e^{-\tau_\nu(z)}}_{\text{transmission}} \underbrace{k_\nu \rho r\, dz}_{\text{emissivity}}$$

$$W_i = e^{-\tau_i(z)} k_i \rho r$$

DOWNWELLING FTIR Radiance at UNR: Atmospheric IR Window



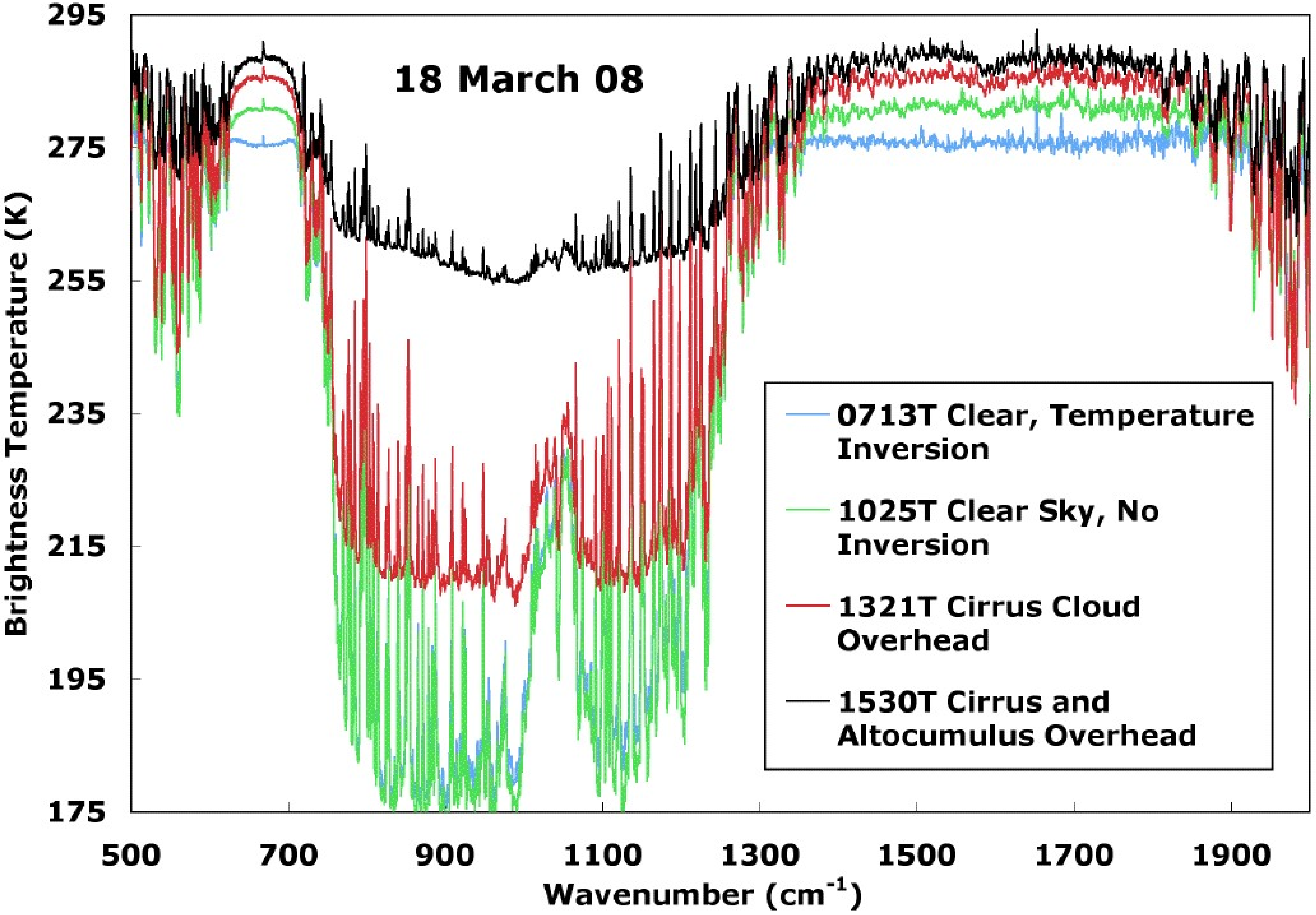
DEFINITION OF THE BRIGHTNESS TEMPERATURE
T_B

$$Radiance(\nu) = B(T_B, \nu)$$

Measured Radiance at wavenumber v
=
Theoretical Radiance of a Black Body at temperature T_B

$$B(T_B, \nu) = 2x10^{11}\, hc^2\nu^3 \frac{1}{e^{100\, hc\nu / kT} - 1} \quad \frac{mW}{m^2\ Sr\ cm^{-1}}$$

FTIR Brightness Temperatures



Atmosphere Emission Measurements, Downwelling Radiance

Notes:

1. Wavelength range for CO_2, H_20, O_3, CH_4.
2. Envelope blackbody curves.
3. Monster inversion in Barrow.
4. Water vapor makes the tropical window dirty.

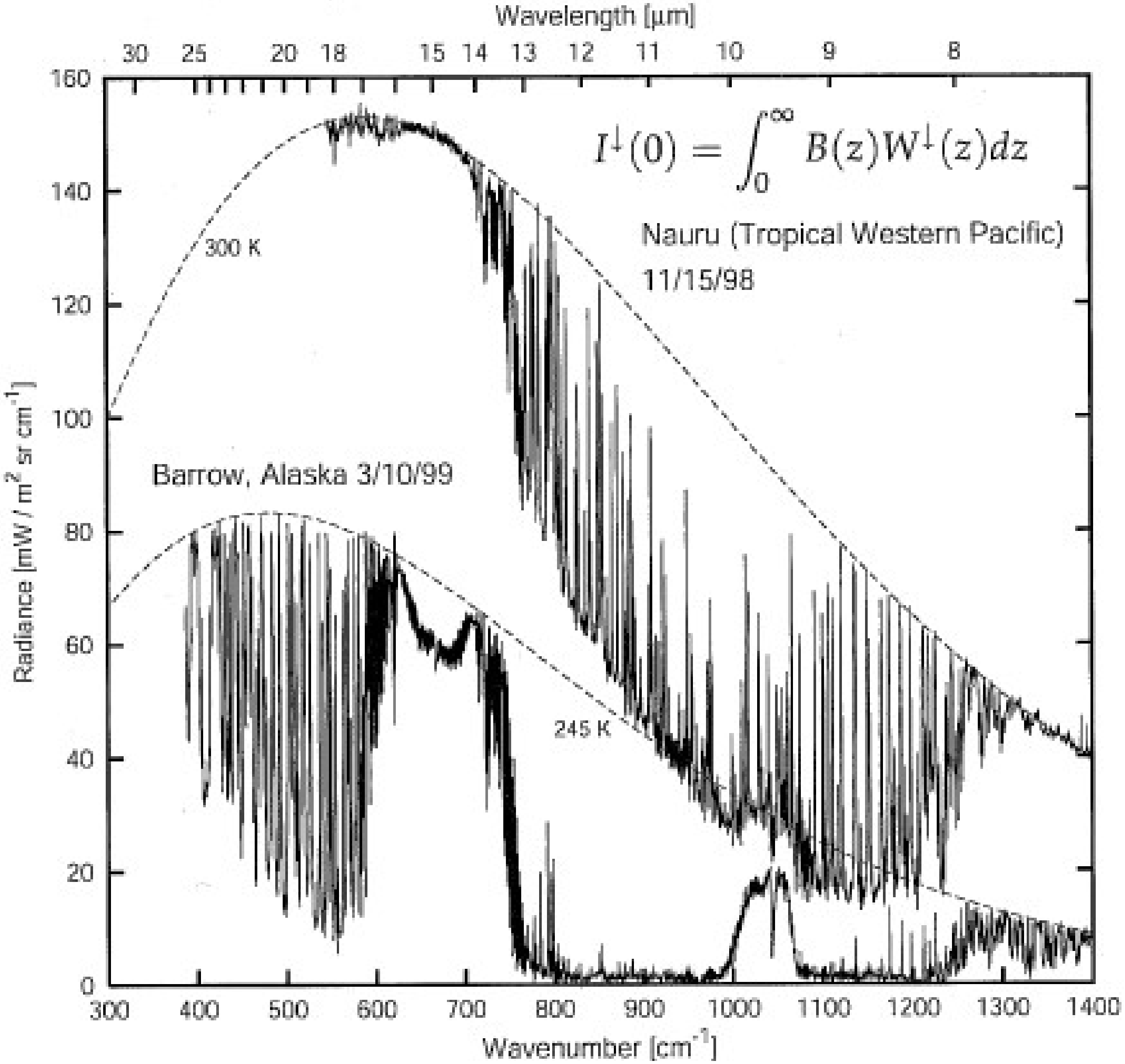
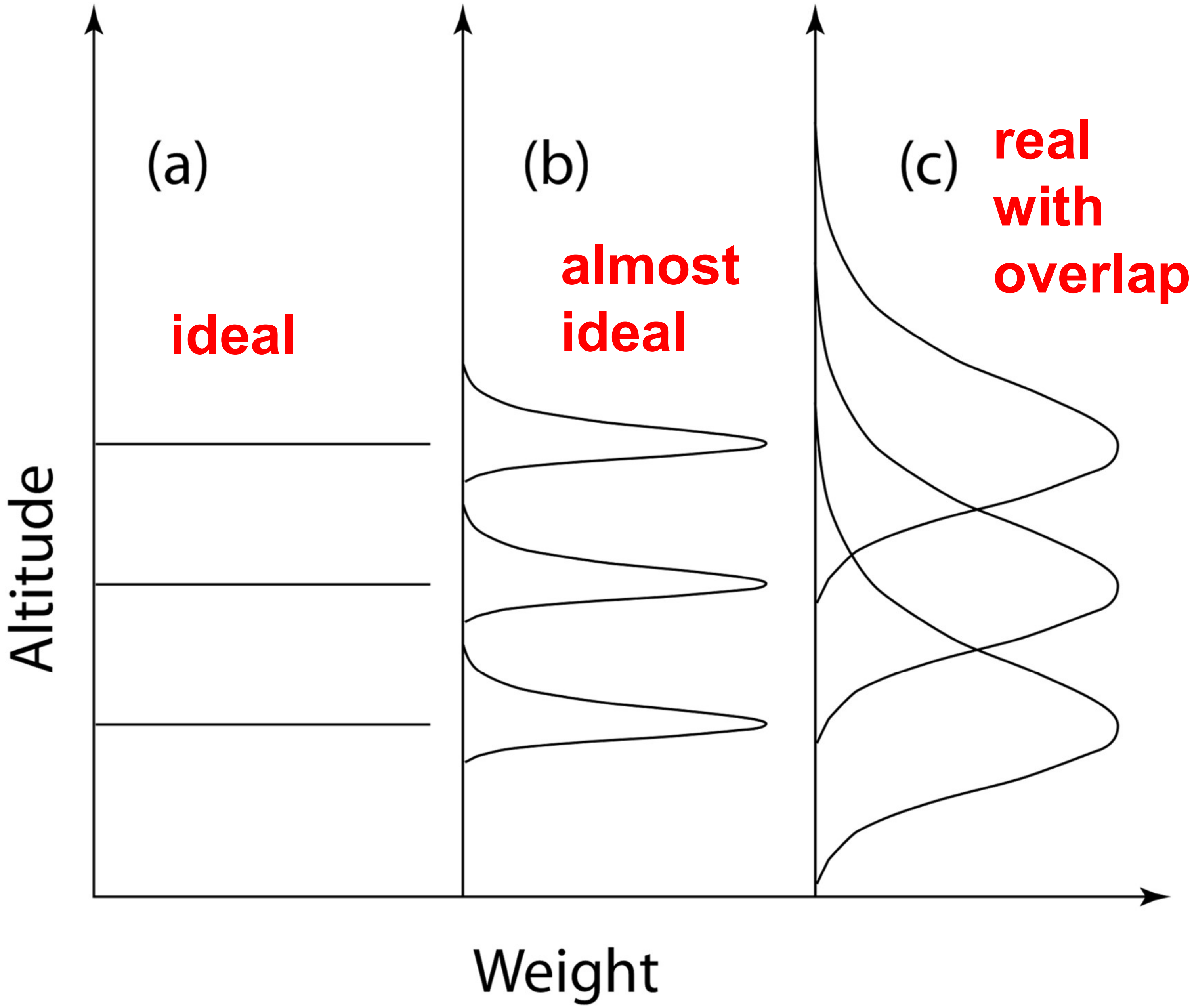
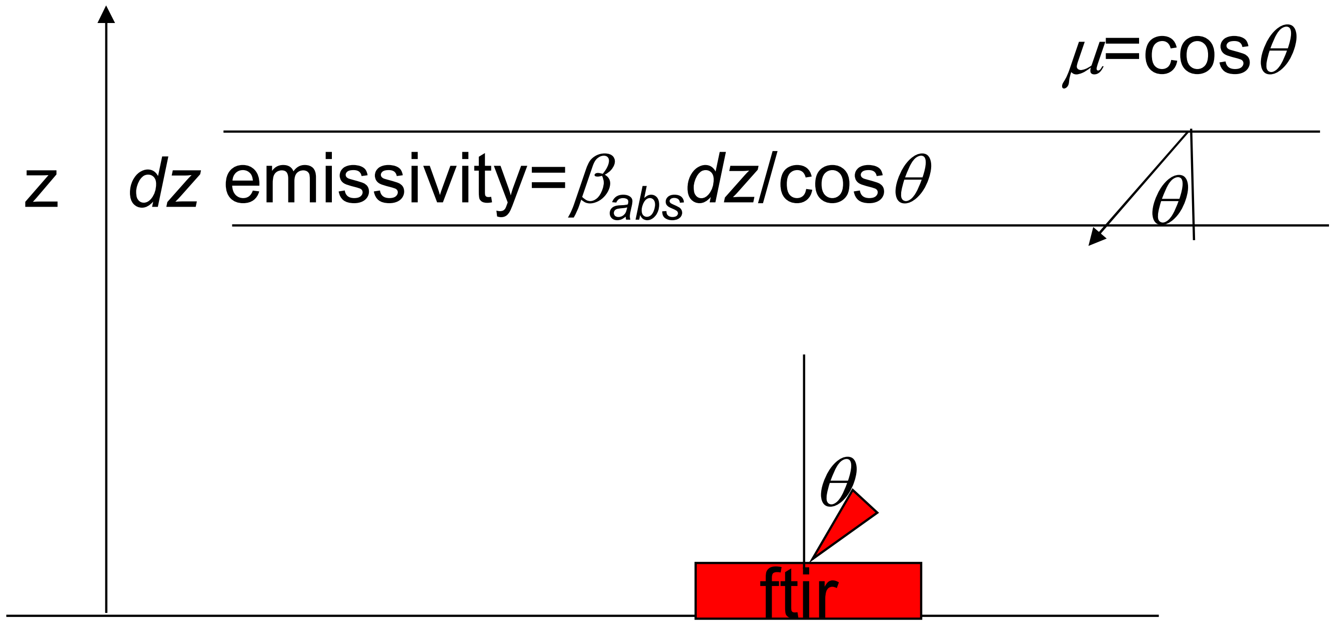


Fig. 8.1: Two examples of measured atmospheric emission spectra as seen from ground level looking up. Planck function curves corresponding to the approximate surface temperature in each case are superimposed (dashed lines). *(Data courtesy of Robert Knuteson, Space Science and Engineering Center, University of Wisconsin-Madison.)*

Ideal Weighting Function W_i: Where in the atmosphere the main contribution to the radiation at wavenumber ν_i comes from.



Downwelling Intensity Emitted by the Atmosphere to the Detector (Radiance)



emissivity=$\beta_{abs}dz/\cos\theta$

$B[T(z)]$

blackbody radiance,
T = temperature.

$$dI_\downarrow(z) = \overset{\textit{emission}}{\boxed{\frac{\beta_{abs}(z)\;dz}{\mu}\;B[T(z)]}}\;\overset{\textit{transmission}}{\boxed{\exp\left(-\int_0^z \frac{\beta_{abs}(z')\;dz'}{\mu}\right)}}\;\equiv W(z)\;B[T(z)]\;dz$$

$$I_\downarrow(0) = \int_0^\infty dI_\downarrow(z) = \int_0^\infty \frac{\beta_{abs}(z)\;dz}{\mu}\;B[T(z)]\exp\left(-\int_0^z \frac{\beta_{abs}(z')\;dz'}{\mu}\right) = \int_0^\infty W(z)\;B[T(z)]\;dz$$

$$\boxed{\begin{aligned}&W(z) \equiv \frac{\beta_{abs}(z)}{\mu}\;\exp\left(-\int_0^z \frac{\beta_{abs}(z')\;dz'}{\mu}\right) = \frac{\beta_{abs}(z)}{\mu}\;\exp\left(-\tau_{abs}(z)/\mu\right)\\ &\tau_{abs}(z) \equiv \int_0^z \beta_{abs}(z')\;dz' \qquad \boxed{\beta_{abs} = \sigma_{abs}(T,P)\left(\frac{m^2}{molecule}\right)N\left(\frac{molecules}{m^3}\right)}\end{aligned}}$$

weighting function

Weighting Functions for Satellite Remote Sensing using the strong CO_2 absorption near 15.4 um. (from Wallace and Hobbs, 2nd edition)

$$I_i = B(T_s)\exp\left(-\tau_{abs}^{All\ Atmos}\right) \quad (surface)$$

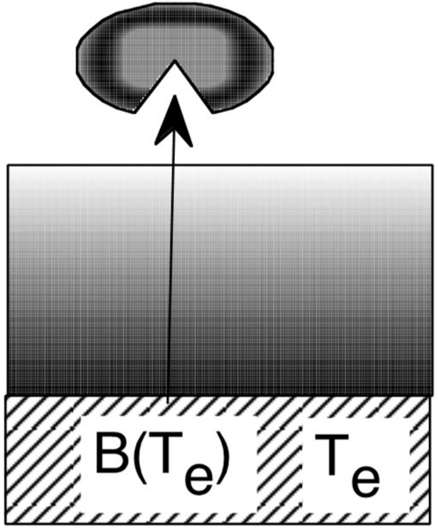
$$+\int_0^\infty B[T(z)]\exp(-\tau_{abs}(z))\,\beta_{abs}(z)\,dz \quad (atmos)$$

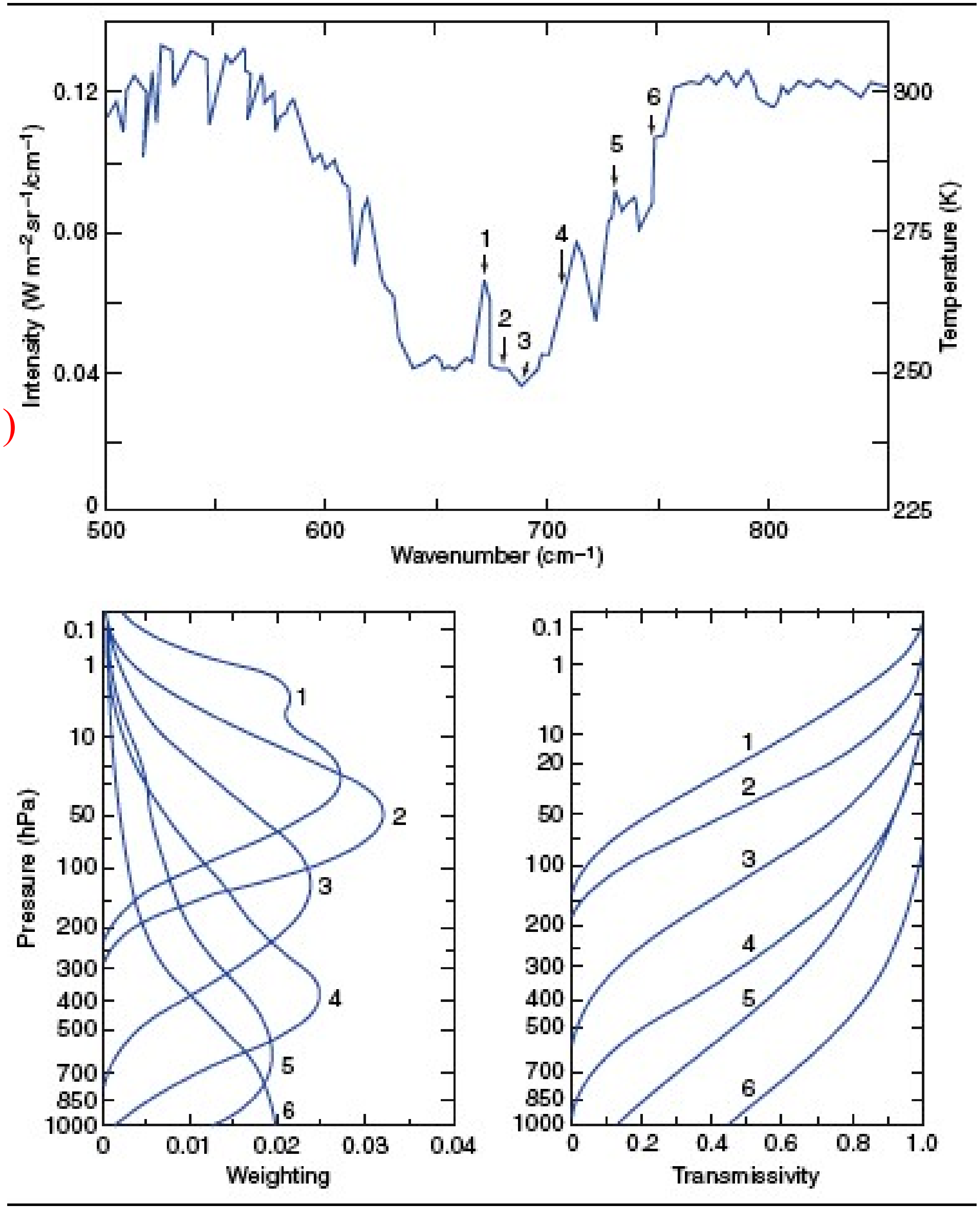
or

$$I_i = B(T_s)\exp\left(-\tau_{abs}^{All\ Atmos}\right) \quad (surface)$$

$$+\int_0^\infty B[T(z)]\,W_i(z)\,dz \quad (atmos)$$

Satellite with FTIR Looking Down

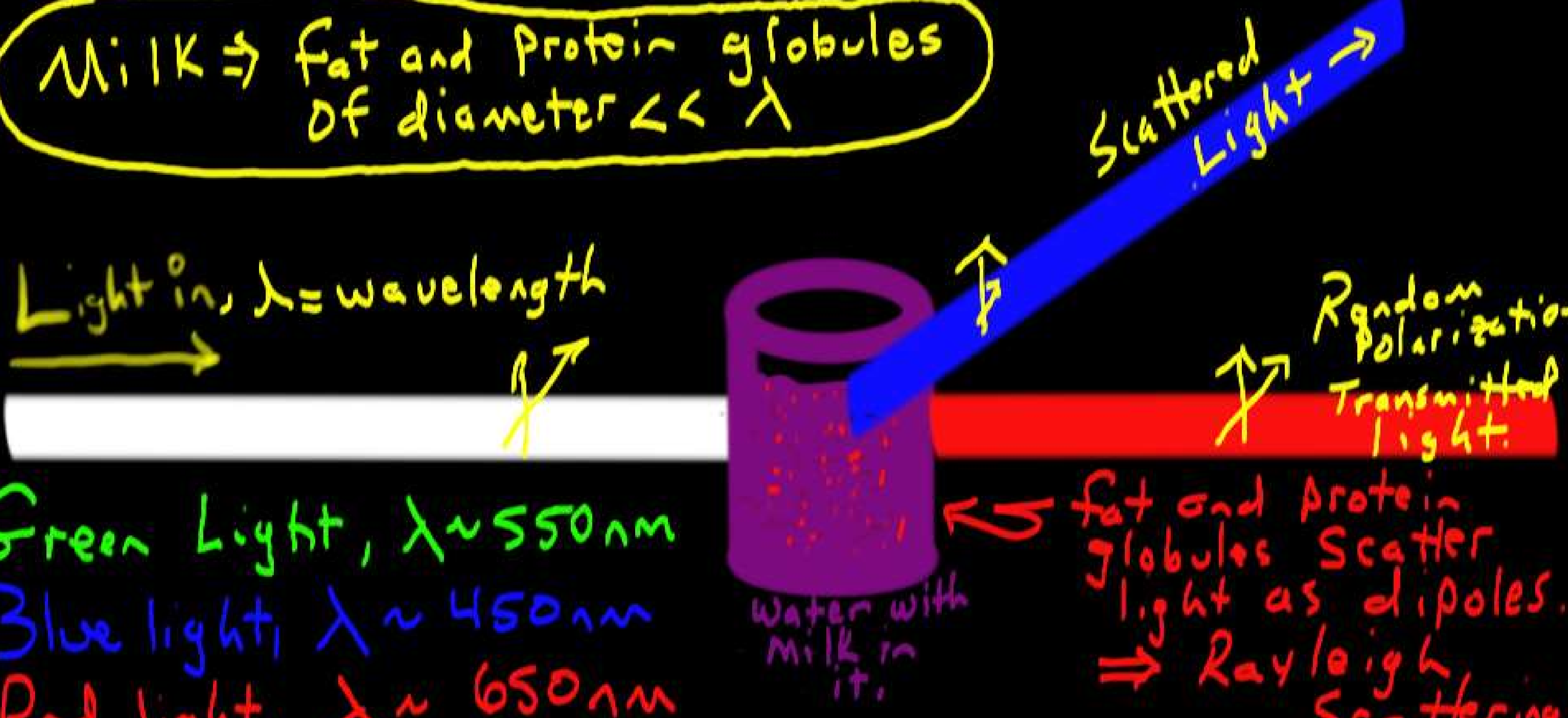




Milk!

Milk ⇒ fat and protein globules of diameter $<< \lambda$

Light in, λ = wavelength →



Green Light, $\lambda \sim 550 nm$

Blue light, $\lambda \sim 450 nm$

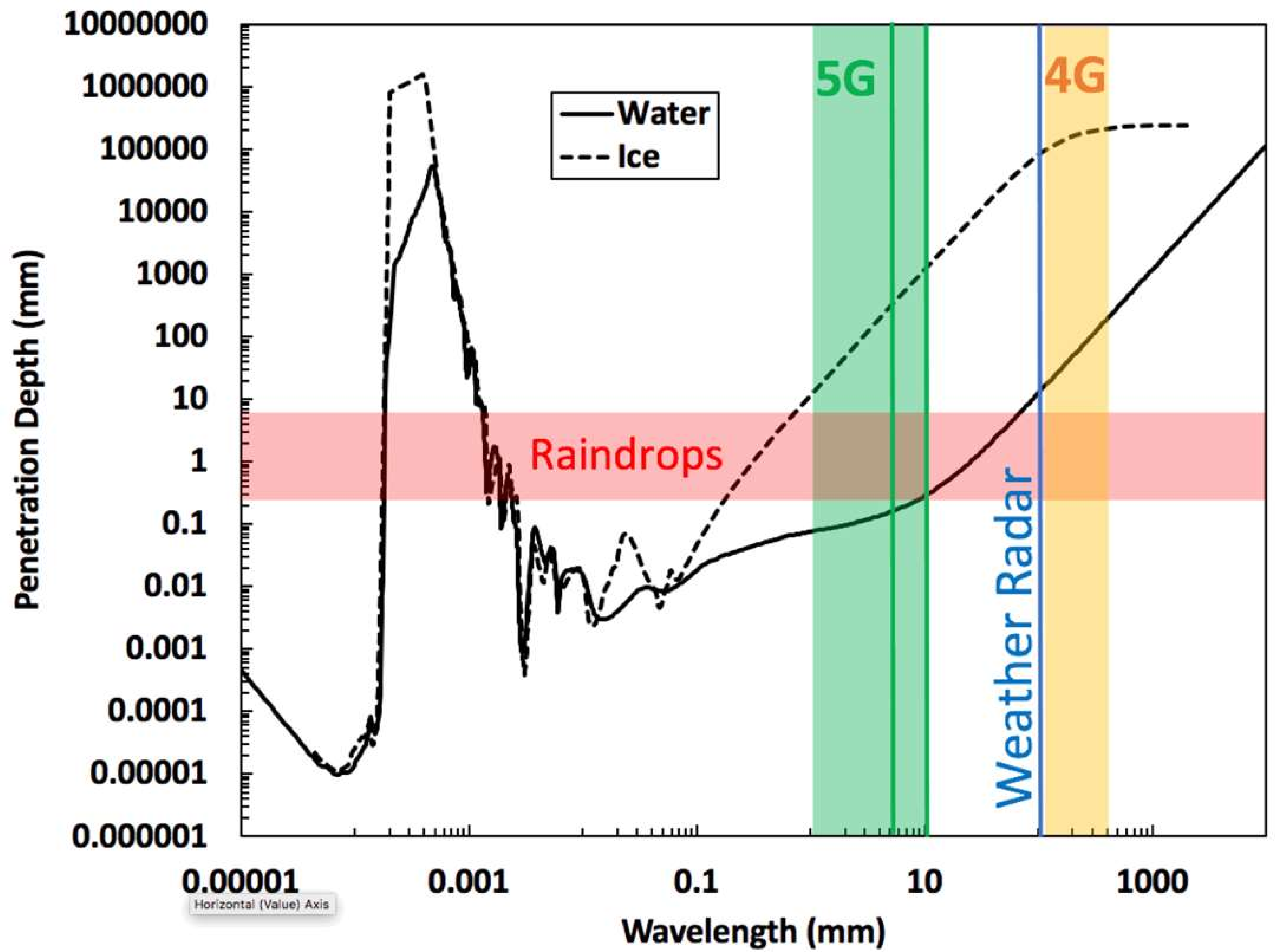
Red light, $\lambda \sim 650 nm$

← fat and protein globules Scatter light as dipoles. ⇒ Rayleigh Scattering!

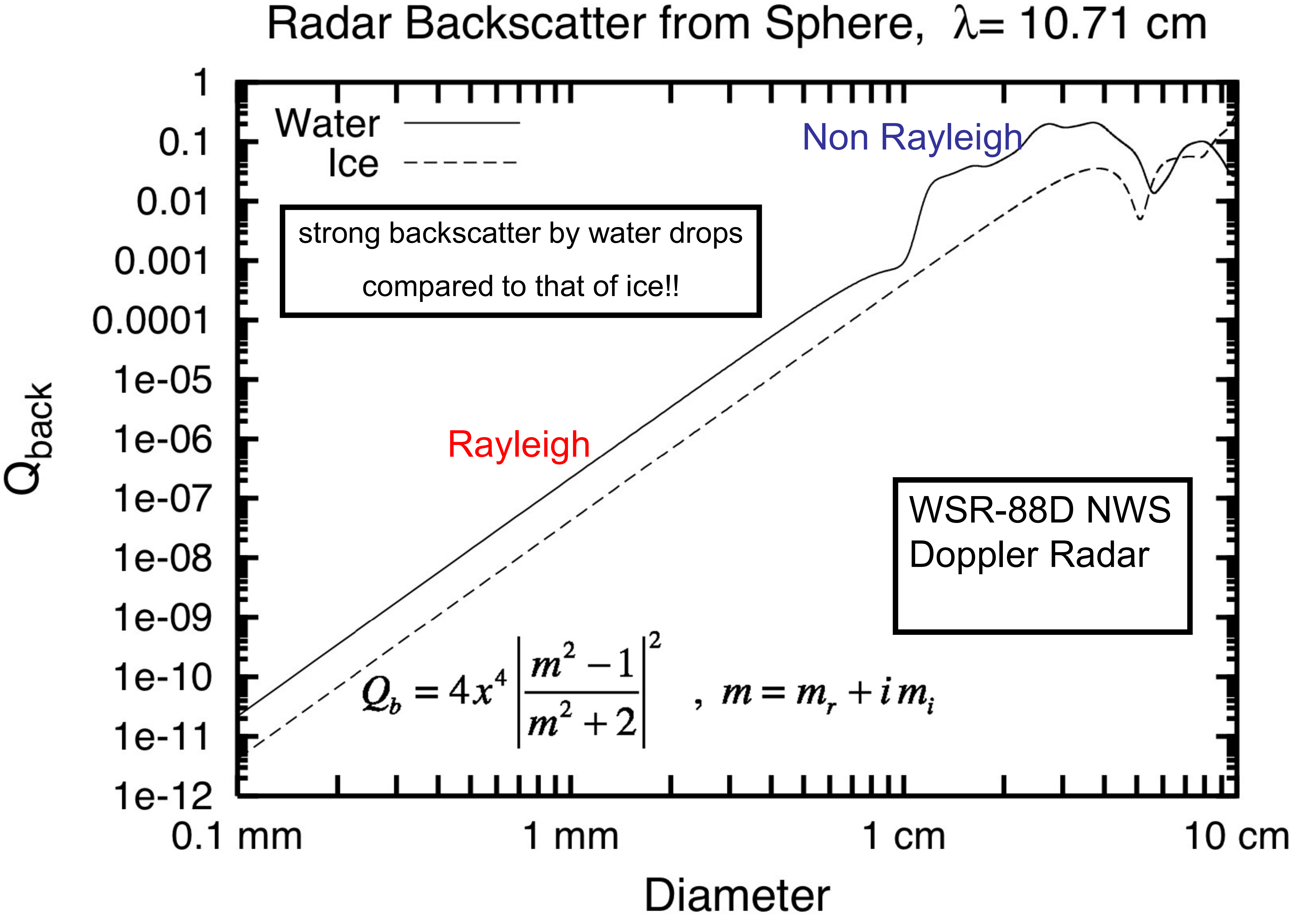
βsca = Scattering amount by milk.

$\beta sca \propto D^6/\lambda^4$ ⇒ Much Stronger for blue light than red light!

Electromagnetic Penetration Depth for Water and Ice



Mie Radar Backscatter Efficiency for Water and Ice Spheres



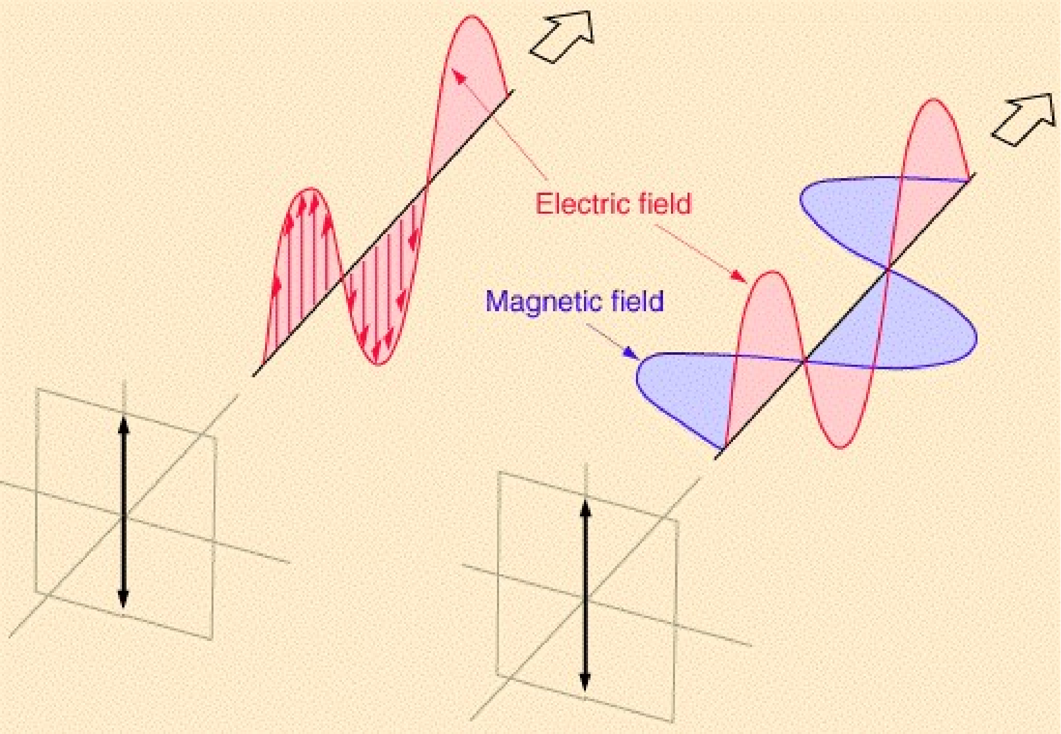
Radar Backscatter from Sphere, λ= 10.71 cm

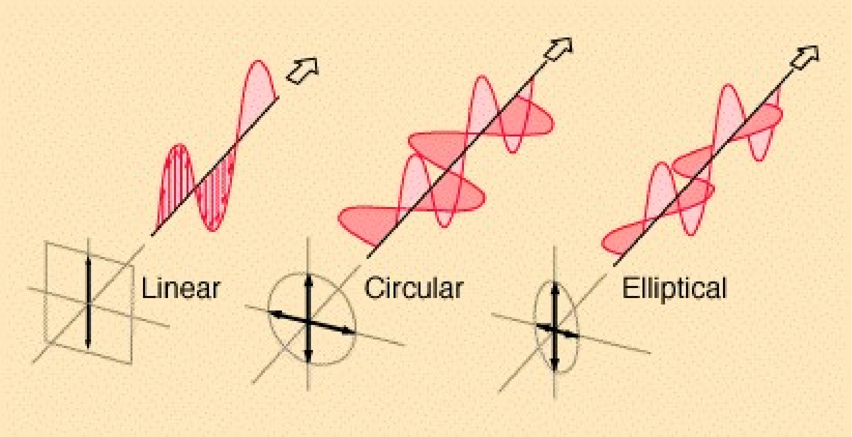
strong backscatter by water drops compared to that of ice!!

WSR-88D NWS Doppler Radar

$$Q_b = 4x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \quad , \; m = m_r + i\, m_i$$

Polarization of Light





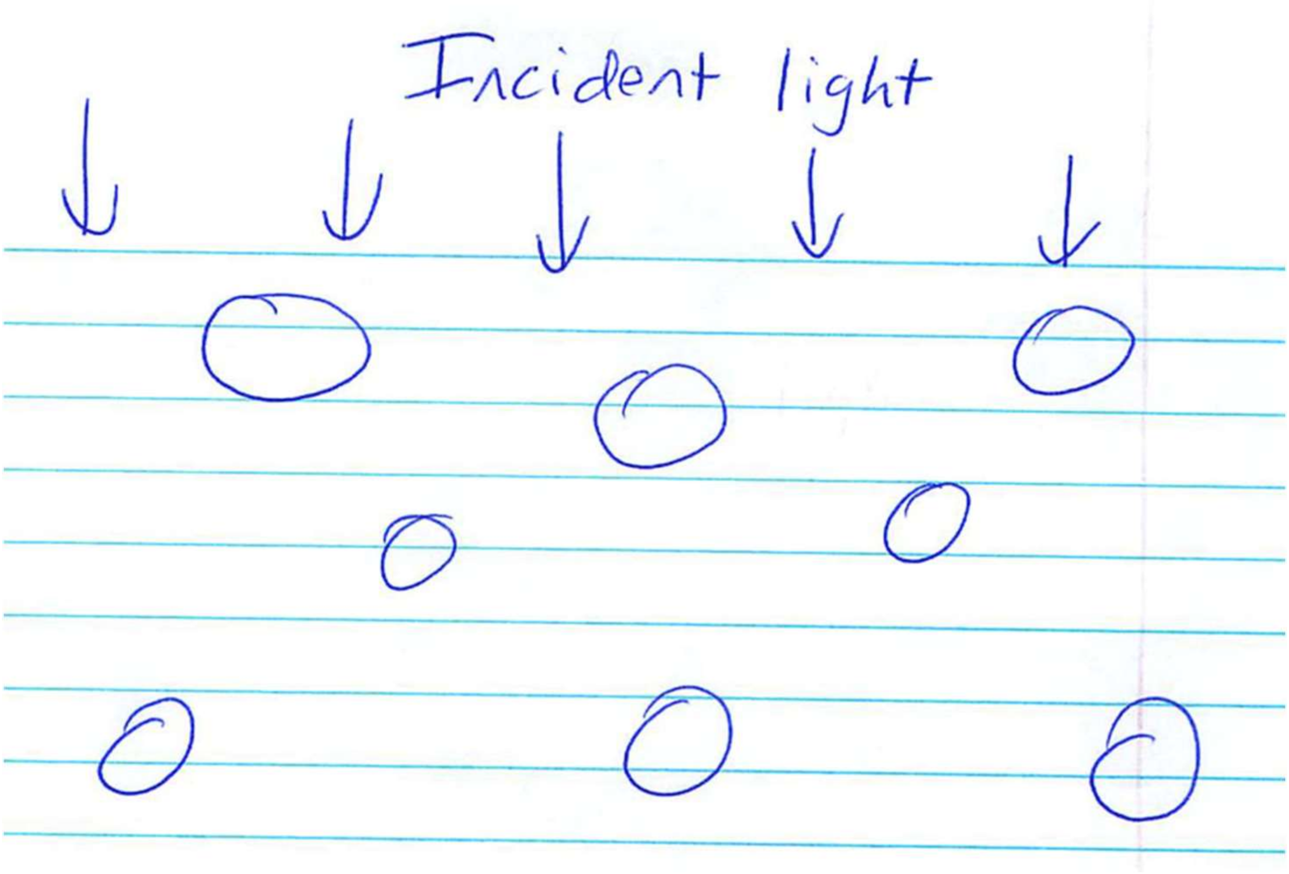
Wave/Photon boson: Polarization.

Linear Polarization: E-field in one direction.

Circular, elliptical polarization: E-Efield rotates due to phase difference between horizontal and vertical components.

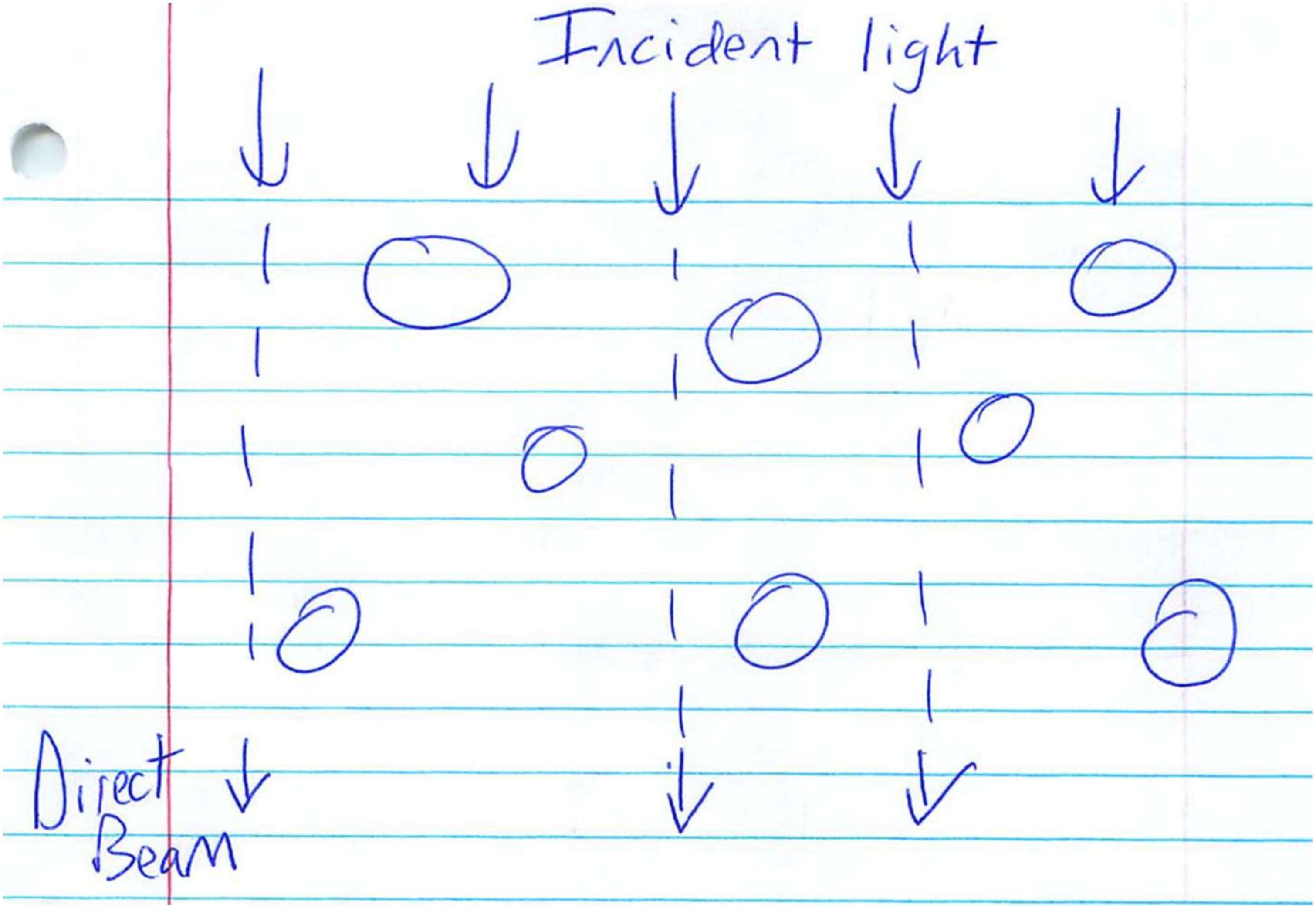
From: http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polclas.html

Scattering and Multiple Scattering

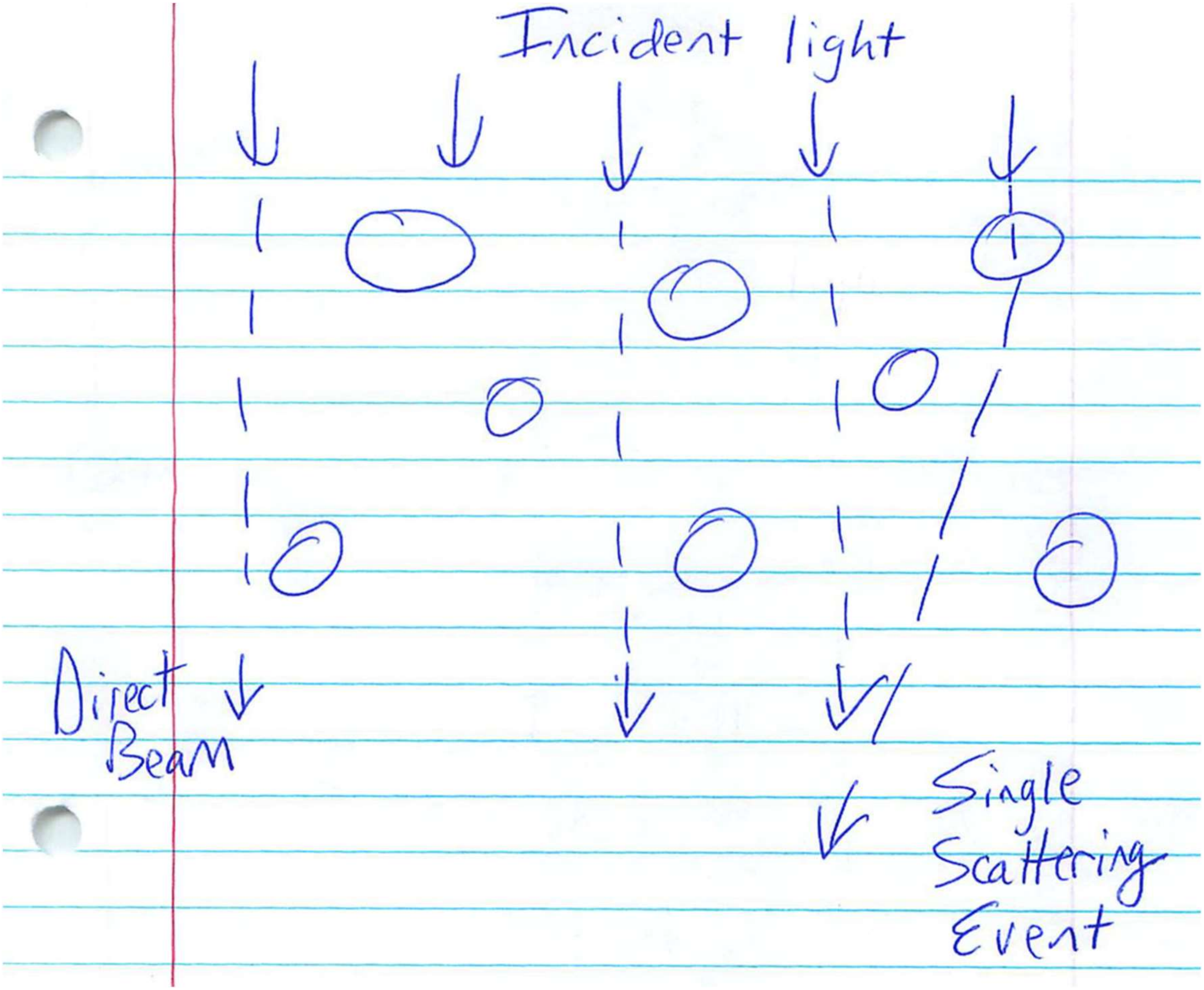


Particles or gases (circles actually represent extinction cross section for the object; it could be smaller or larger than the physical cross section)

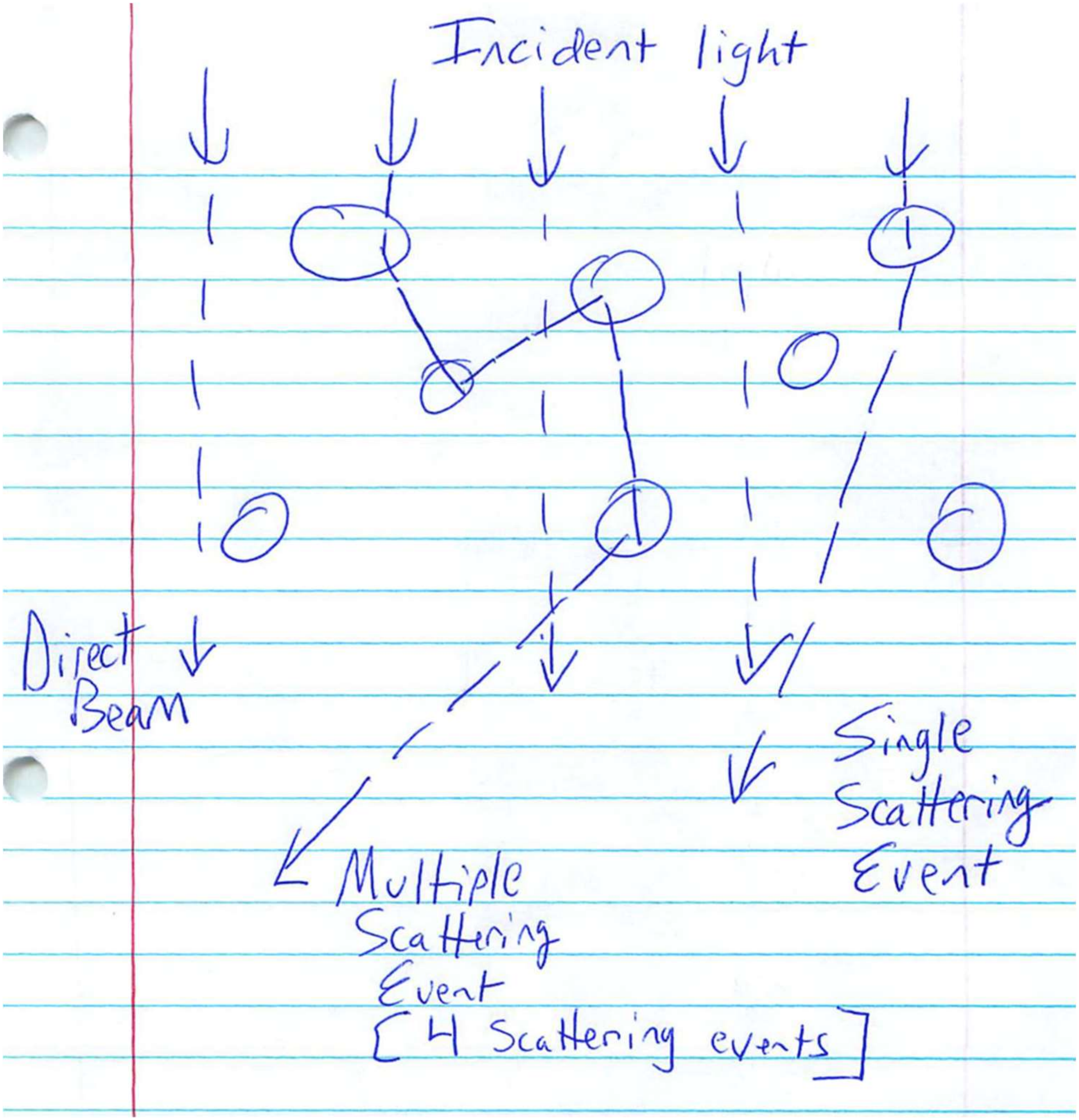
Scattering and Multiple Scattering: Direct Beam, no scattering or absorption at all. 'Sneaky light'



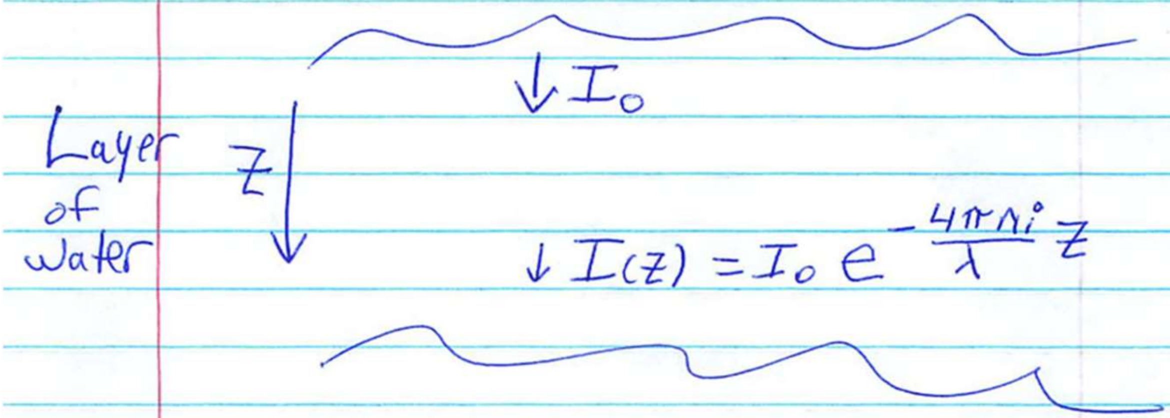
Scattering and Multiple Scattering



Scattering and Multiple Scattering: Diffuse Radiation



How to calculate the direct beam transmission through various media.



$$\downarrow I(z) = I_o e^{-\frac{4\pi n_i}{\lambda} z}$$

Light transmission
in the ocean or a lake
with no suspended particles

Let $\beta_{abs} \equiv \frac{4\pi n_i}{\lambda}$ λ = wavelength

Absorption Coefficient

n_i = imaginary part of refractive index

Let $\delta \equiv 1/\beta_{abs} = \frac{\lambda}{4\pi n_i}$ = Penetration depth

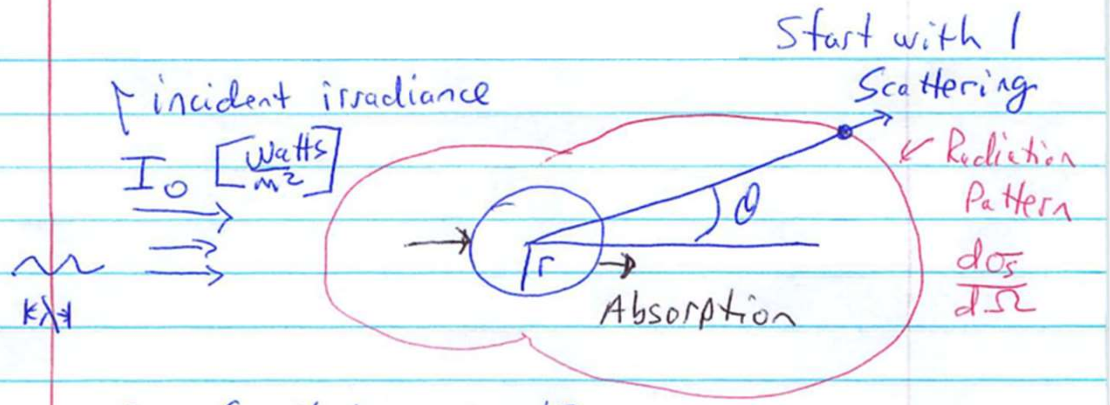
Penetration Depth

$$I(z) = I_o e^{-z/\delta} = I_o e^{-\beta_{abs} z}$$

Explains blue color of water since $n_{i\,blue} < n_{i\,red}$,

$$\delta_{Blue} > \delta_{Red}$$

Like in a swimming pool



Scattering by particles (aerosol, hydrometeors)

ideas relevant to scattering by air as well.

θ = Scattering angle
r = Particle radius $2r$ = diameter = D

Power removed from incident beam due to scattering $\equiv P_{sca} = I_o Q_s \sigma$
$\sigma = \pi r^2$
Scattering efficiency

Book uses $K_\lambda = Q$

$\sigma_s \equiv$ Scattering Cross Section (total in all directions)

$\sigma_s = Q_s \sigma$

$$\sigma_s = \int \left(\frac{d\sigma_s}{d\Omega}\right) d\Omega$$

solid angle

Differential Scattering Cross Section

Scaling

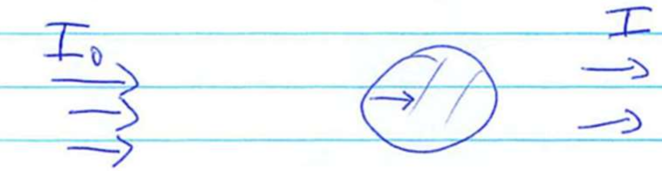
$x \equiv$ Size Parameter $= 2\pi r / \lambda$

$x << 1,\ Q_s << 1$ Rayleigh Regime

$x >> 1,\ Q_s \approx 1$ or 2 Geometrical optics
absorption $\neq 0$

dimensionless size parameter x

Absorption of light



P_a = Power Removed from beam due to absorption.

$$P_{abs} = I_0 \underbrace{Q_{abs}\,\sigma}_{}$$

σ_{abs} = Absorption cross section

extinction

$P_{ext} = P_{sca} + P_{abs}$ = Total Power removed by both Processes

Examples

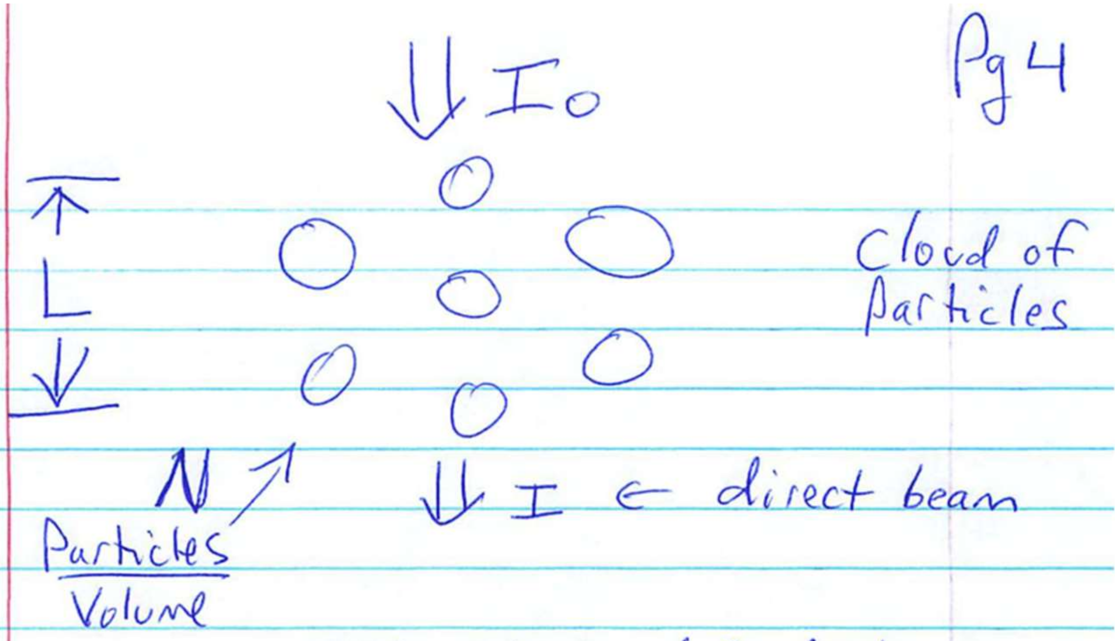
$x << 1$ $\quad \sigma_{sca} \propto \frac{D^6}{\lambda^4}$ Rayleigh Regime

[NOT D^3, radar ↗ so Precip amount not directly given by radar]

$x >> 1$ $\quad \sigma_{sca} \cong 2\sigma = 2\pi r^2$

$[n_i \approx 0]$

Good for cloud droplets and Visible sunlight range.



If identical particles,

$$\beta_{\substack{ext\\sca\\abs}} = N \frac{\text{Particles}}{\text{Volume}} \cdot \sigma_{\substack{ext\\sca\\abs}} \text{ [Area]}$$

Units: $\frac{1}{\text{distance}}$

Direct beam [ONLY]!

Beer Lambert relation

$$I(L) = I_o e^{-\beta_{ext} L}$$

$$\tau_{ext} \equiv \beta_{ext} L$$

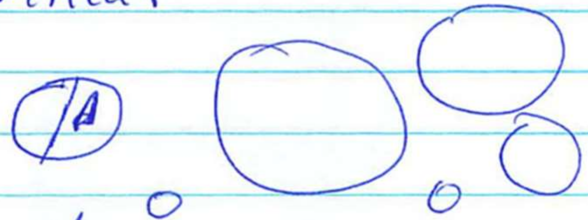
Dimensionless optical depth

Must add diffuse beam when

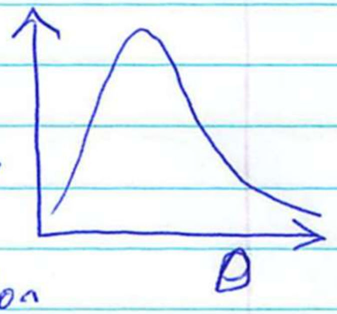
$$\tau_{sca} \equiv \beta_{sca} L \text{ is } \gtrsim 1$$

Scattering optical depth

When particles are not identical



$$N = \int_0^\infty \left(\frac{dN}{dD}\right) dD$$



Size distribution

$$\beta_{\substack{ext\\sca\\abs}} = \int_0^\infty \left(\frac{dN}{dD}\right) \sigma_{\substack{ext\\sca\\abs}}(\lambda, D)\, dD$$

Single Scattering Albedo

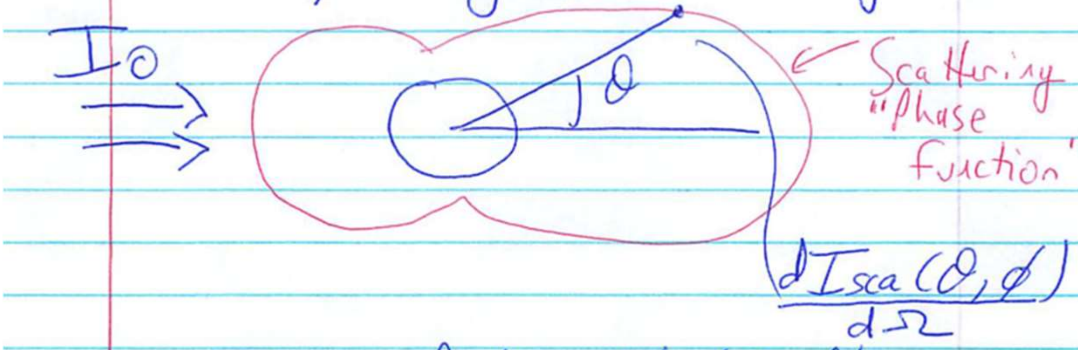
single scattering albedo

$$\tilde{\omega}_0 = \frac{\beta_{sca}}{\beta_{ext}} = \frac{\beta_{ext} - \beta_{abs}}{\beta_{abs} + \beta_{sca}}$$

= fraction of extinction due to scattering

Pg 6

Asymmetry Parameter, g



Idea: Replace detailed Phase function with forward or backward scattering Probability

$$g \equiv \frac{\int \cos\theta \left(\frac{d\sigma_{sca}}{d\Omega}\right) d\Omega}{\int \left(\frac{d\sigma_{sca}}{d\Omega}\right) d\Omega}$$

$\sigma_{sca} \Longrightarrow$

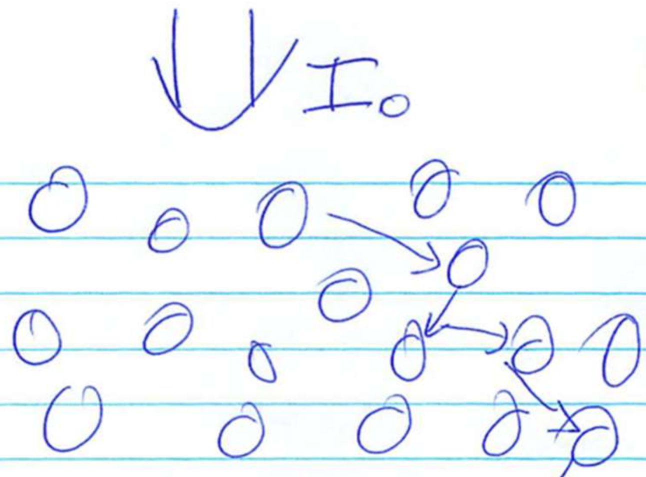
$$g = \frac{\int_0^\infty g(D,\lambda)\,\sigma_{sca}\left(\frac{dN}{dD}\right) dD}{\beta_{sca}}$$

for a cloud of Particles

---

g=1 All forward scattered
 =-1 All backscattered

Limits. What if g=0?
g=0 for Rayleigh Sca



$$I = \downarrow I^{\text{direct}}_{\text{Beam}} + \swarrow I^{\text{diffuse}}_{\text{Beam}}$$

Simple Case, $\tilde{\omega}_0 = 1$, $n_i = 0$
[no absorption]

$$\frac{I}{I_0} = \frac{2}{2 + \tau_{ext}(1-g)}$$

$$\frac{I^{\text{direct}}_{\text{Beam}}}{I_0} = e^{-\tau_{ext}}$$

$$\frac{I^{\text{diffuse}}_{\text{Beam}}}{I_0} = \frac{I}{I_0} - \frac{I^{\text{direct}}_{\text{Beam}}}{I_0}$$

$P_{\downarrow\uparrow} \equiv$ Probability down going photon is backscattered

$P_{\downarrow\uparrow} = (1-g)/2$

Similarly,

$$P_{\downarrow\downarrow} = \frac{1+g}{2}$$ = Probability a downward photon is scattered downward.

$$\text{Solar Visible } \lambda \begin{cases} g \approx 0.85 & \text{for cloud droplets} \\ g \approx 0.7 & \text{for aerosol} \\ g \approx 0.8 & \text{for ice crystals} \end{cases}$$

Note: $P_{\downarrow\uparrow} + P_{\downarrow\downarrow} = 1$

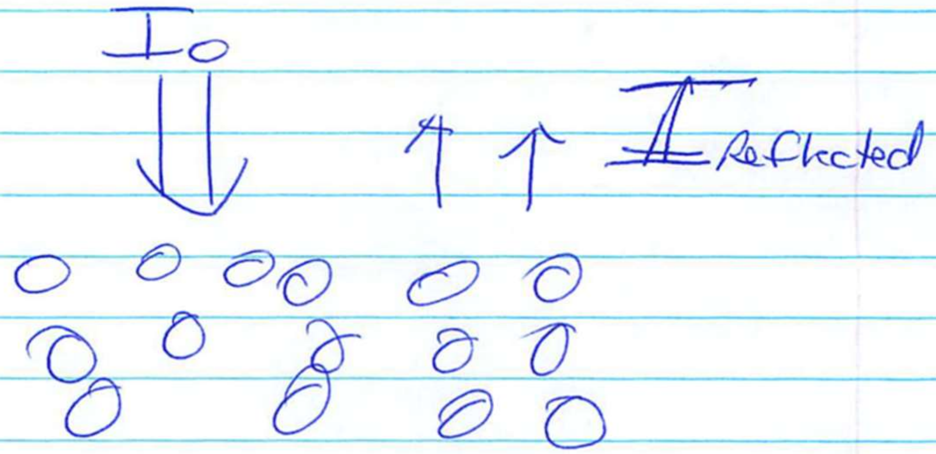
$$\frac{I}{I_0} = \frac{1}{1 + \tau_{ext} P_{\downarrow\uparrow}}$$

[backscattering prevents perfect transmission, $I/I_0 \cong 1$]

Can NOT get τ_{ext} and g uniquely from only I/I_0 measurements

Let $T = \frac{I}{I_0} = \frac{2}{2 + \tau_{ext}(1-g)}$

Transmission Coefficient



$$R \equiv \frac{I_{reflected}}{I_0} = 1 - T$$

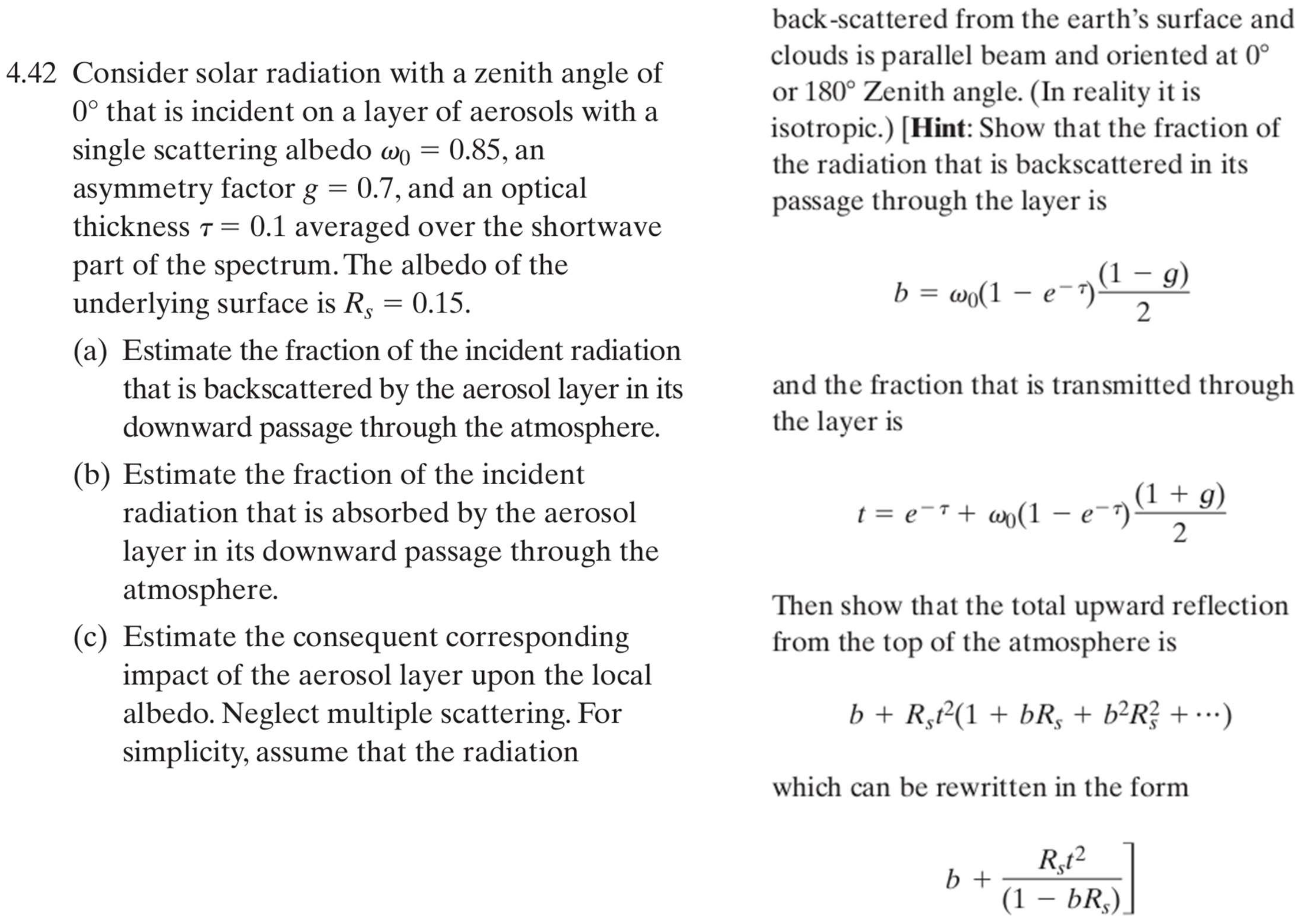
$$R = \frac{\tau_{ext}(1-g)}{2 + \tau_{ext}(1-g)}$$

Reflection Coefficient

When $\tilde{\omega}_0 = 1$, $n_i = 0$

Diffuse reflection coefficient when multiple scattering is strong

Problem 4.42 A good one for basics of aerosol optical depth determination from satellite and for direct forcing of aerosol on climate



4.42 Consider solar radiation with a zenith angle of 0° that is incident on a layer of aerosols with a single scattering albedo $\omega_0 = 0.85$, an asymmetry factor $g = 0.7$, and an optical thickness $\tau = 0.1$ averaged over the shortwave part of the spectrum. The albedo of the underlying surface is $R_s = 0.15$.

(a) Estimate the fraction of the incident radiation that is backscattered by the aerosol layer in its downward passage through the atmosphere.

(b) Estimate the fraction of the incident radiation that is absorbed by the aerosol layer in its downward passage through the atmosphere.

(c) Estimate the consequent corresponding impact of the aerosol layer upon the local albedo. Neglect multiple scattering. For simplicity, assume that the radiation back-scattered from the earth's surface and clouds is parallel beam and oriented at 0° or 180° Zenith angle. (In reality it is isotropic.) [**Hint**: Show that the fraction of the radiation that is backscattered in its passage through the layer is

$$b = \omega_0(1 - e^{-\tau})\frac{(1 - g)}{2}$$

and the fraction that is transmitted through the layer is

$$t = e^{-\tau} + \omega_0(1 - e^{-\tau})\frac{(1 + g)}{2}$$

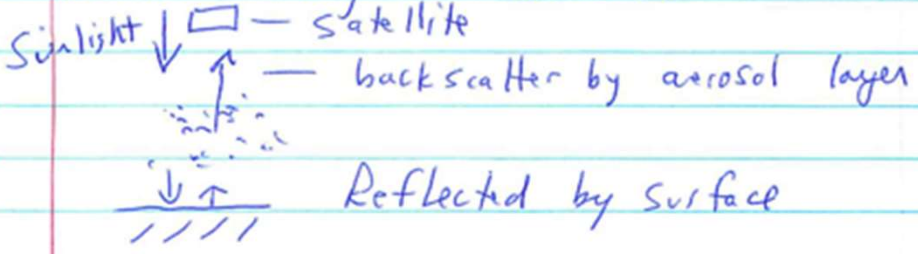
Then show that the total upward reflection from the top of the atmosphere is

$$b + R_s t^2(1 + bR_s + b^2 R_s^2 + \cdots)$$

which can be rewritten in the form

$$b + \frac{R_s t^2}{(1 - bR_s)}]$$

Problem 4.42. Single model of satellite remote sensing of aerosol.

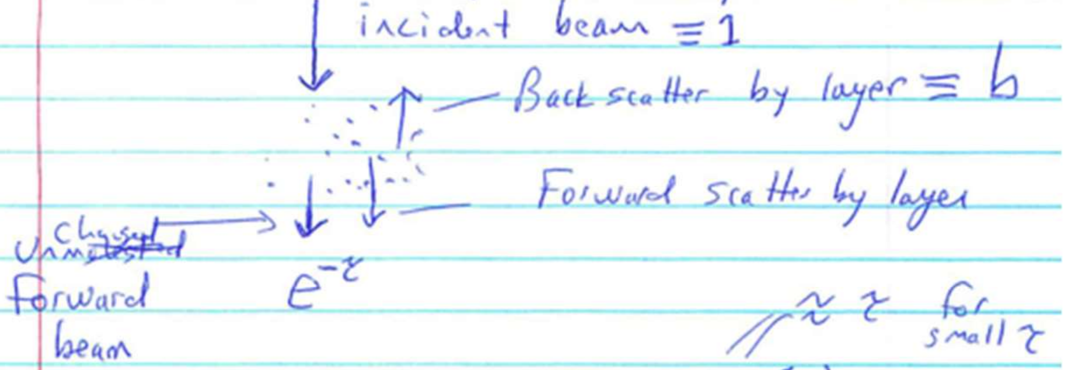


Let: $\omega_0 = \tau_s / \tau_{ext}$ = Single Scattering albedo.

g = asymmetry parameter

$\tau \equiv \tau_{ext}$ = aerosol optical depth.

First: Backscatter by aerosol layer.



$$b = \underbrace{\omega_0 \frac{(1-g)}{2}}_{\text{Probability scattering event results in backscatter}} \underbrace{\overset{\approx \tau \text{ for small } \tau}{(1 - e^{-\tau})}}_{\text{amount of radiation taken out of direct beam, that's available for scattering, forward or backward}}$$

The total radiation transferred through the layer is

$$t = \underset{\text{direct beam}}{e^{-\tau}} + \underbrace{\frac{\omega_0(1+g)}{2}}_{\text{Probability of forward scatter}} \underset{\text{Same as in b.}}{\left(1-e^{-\tau}\right)}$$

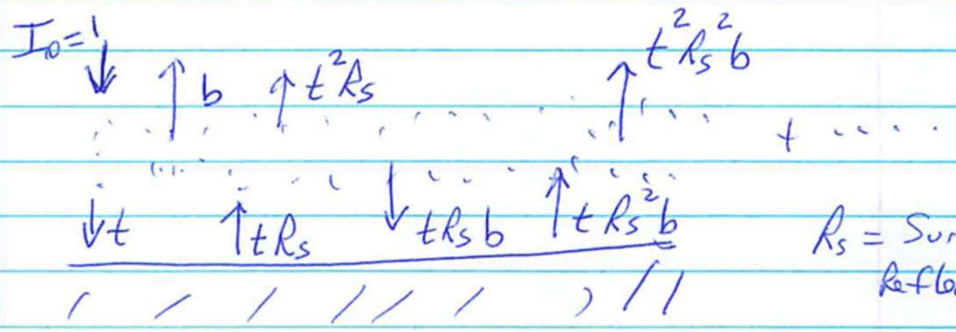
Note: $\bar{\omega}_0 = \tau_s/\tau$ is the probability an extinction event is associated with scattering and not absorption.

$P_{\downarrow\uparrow} \equiv (1-g)/2$ is Probability backscattering occurs

$P_{\downarrow\downarrow} \equiv (1+g)/2$ is Probability forwardscattering occurs

$$P_{\downarrow\uparrow} + P_{\downarrow\downarrow} = 1$$

Multiple scattering between layer and Surface :



R_s = Surface Reflectivity

The total backscatter is

$$\text{Albedo} = b + t^2 R_s \underbrace{\left[1 + (R_s b) + (R_s b)^2 + \dots \right]}_{\substack{\text{Power Series} \\ \frac{1}{1 - R_s b}}}$$

$$\boxed{\text{Albedo} = b + \frac{t^2 R_s}{1 - R_s b}}$$

$$b = \omega_0 \frac{(1-g)}{2} (1 - e^{-\tau})$$

$$t = e^{-\tau} + \frac{\omega_0 (1+g)}{2} (1 - e^{-\tau})$$

With no aerosol, $\tau = 0$,

$$\text{Albedo} = R_s$$

Example: $\bar{\omega}_0$ = Variable

$\tau = 0.1$

$g = 0.7$

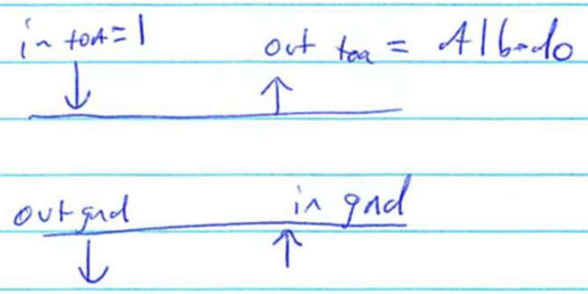
R_s = variable

Note how b an t involve g and τ in such a way that they are not separately determined by just one Albedo measurement.

Note: when $\bar{\omega}_0 \cong 0.73$, $R_s = 0.15$
$\Rightarrow$ Albedo $\cong 0.15$ as well
and Albedo < 0.15 for $\omega_0 < 0.73$

Note: For large $R_s \sim 0.8$, Albedo $\leq R_s$ for all $\bar{\omega}_0$, aerosol reduces reflectivity over very highly reflecting surfaces.

Calculation of absorption by the layer.



The total flux absorbed by the layer is $\Sigma in - \Sigma Out$, since the layer doesn't emit at solar wavelength.

Out gnd from Pg 3 is $t + tR_s b + tR_s^2 b^2 + \dots$

$$= \frac{t}{1 - R_s b}$$

$$In\ gnd = tR_s + tR_s^2 b + tR_s^3 b^2 + \dots$$

$$= \frac{tR_s}{1 - R_s b}$$

Thus

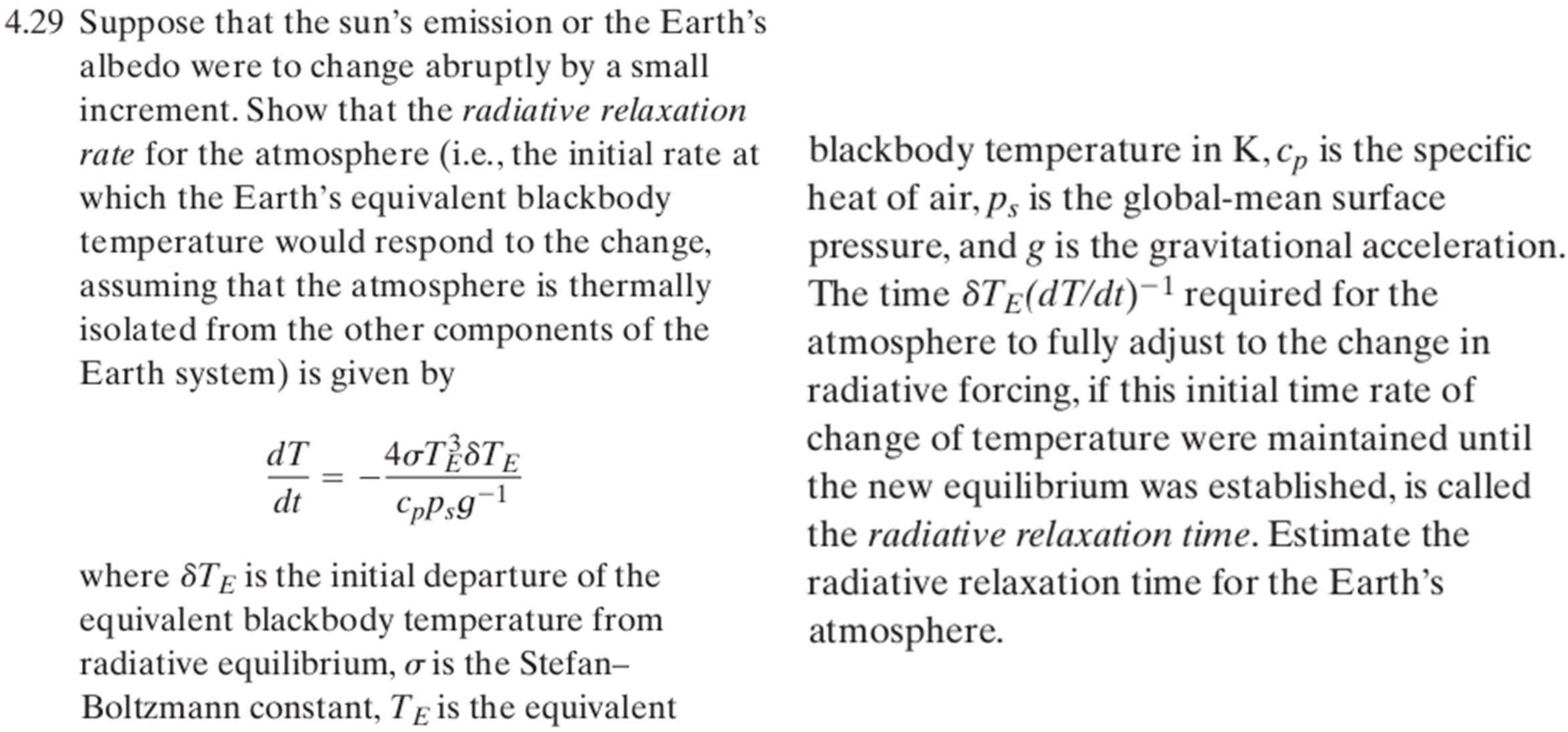
$$abs = \Sigma In - \Sigma Out =$$

$$1 + \frac{tR_s}{1-R_s b} - b - \frac{t^2 R_s}{1-R_s b} - \frac{t}{1-R_s b} =$$

$$\boxed{abs = \frac{(1-b)(1-R_s b) + t[R_s(1-t) - 1]}{1 - R_s b}}$$

Check: $R_s = 0 \Rightarrow abs = 1 - b - t$ ✓

Problem 4.29: Rate of change of temperature after land surface or solar radiation change.

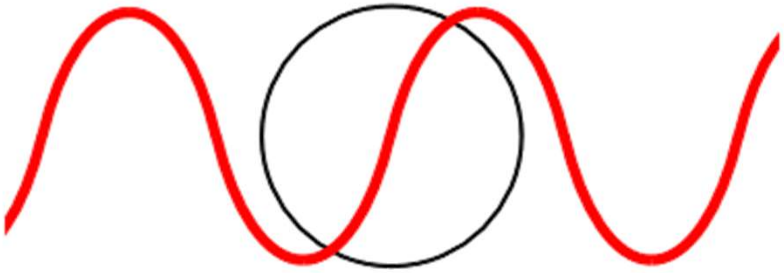


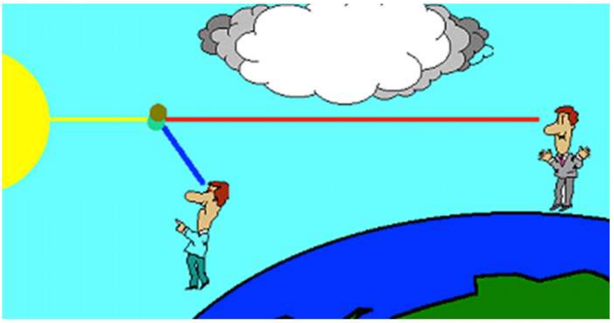
4.29 Suppose that the sun's emission or the Earth's albedo were to change abruptly by a small increment. Show that the *radiative relaxation rate* for the atmosphere (i.e., the initial rate at which the Earth's equivalent blackbody temperature would respond to the change, assuming that the atmosphere is thermally isolated from the other components of the Earth system) is given by

$$\frac{dT}{dt} = -\frac{4\sigma T_E^3 \delta T_E}{c_p p_s g^{-1}}$$

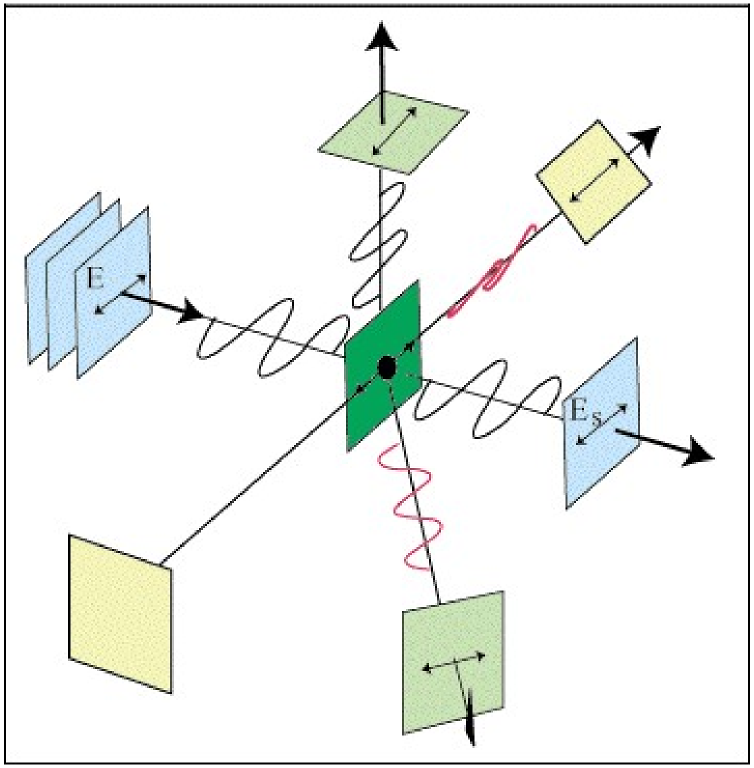
where δT_E is the initial departure of the equivalent blackbody temperature from radiative equilibrium, σ is the Stefan–Boltzmann constant, T_E is the equivalent blackbody temperature in K, c_p is the specific heat of air, p_s is the global-mean surface pressure, and g is the gravitational acceleration. The time $\delta T_E (dT/dt)^{-1}$ required for the atmosphere to fully adjust to the change in radiative forcing, if this initial time rate of change of temperature were maintained until the new equilibrium was established, is called the *radiative relaxation time*. Estimate the radiative relaxation time for the Earth's atmosphere.

Angular Dependence of Rayleigh Scattering (dipole)

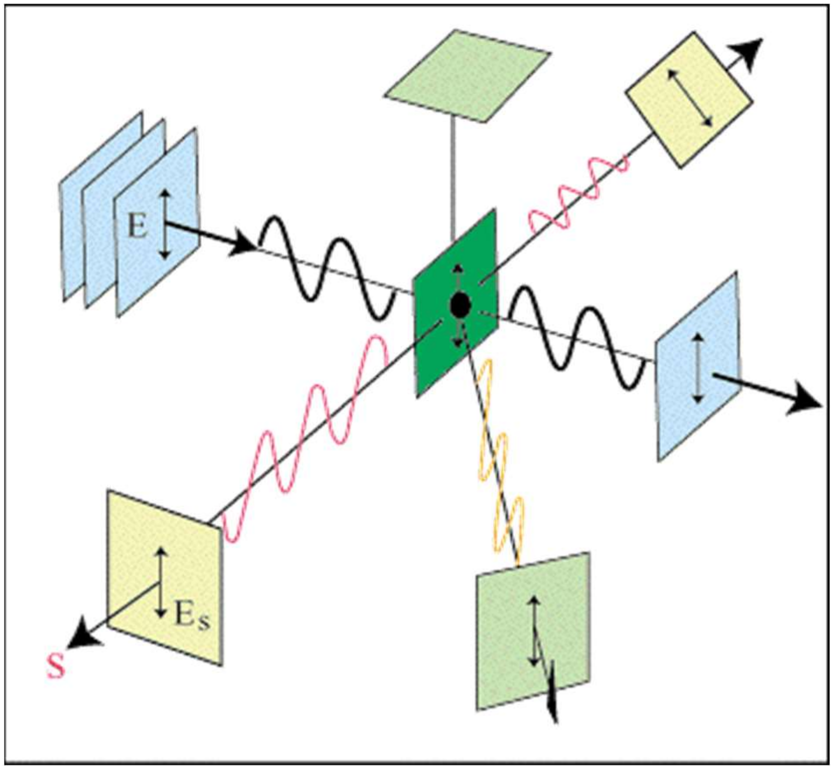




From:
http://qels.com/theory/rayleighscattering/mass.cfm
http://www.bo.astro.it/sait/spigolature/spigo402base.html



Horizontal E-field



Vertical E-field

Dipoles don’t radiate in the direction they are undergoing linear ocsillation.

From http://www.sparknotes.com/physics/optics/phenom/section3.rhtml

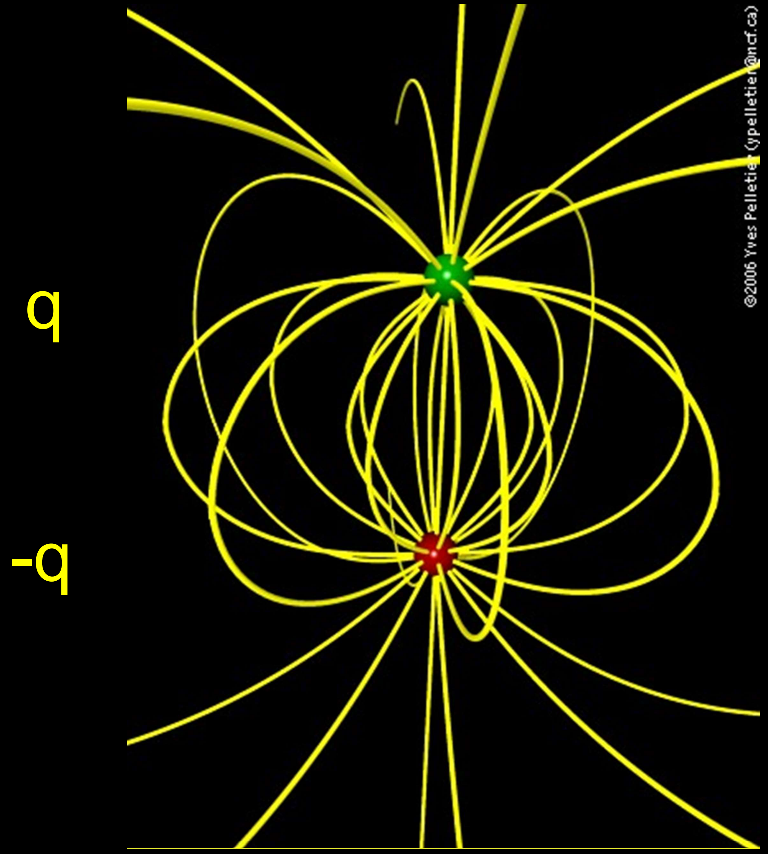
Selection Rules: Accelerated Charges are the Source, Sinks of Electromagnetic Radiation: Dipole Moment, **p.**

Selection rules specify the possible transitions among quantum levels due to absorption or emission of electromagnetic radiation. Incident electromagnetic radiation presents an oscillating electric field that interacts with a transition dipole p_z. A transition dipole moment is a transient dipolar polarization created by an interaction of electromagnetic radiation with a molecule. If p_z is zero then a transition is forbidden. The selection rule is a statement of when p_z is non-zero.

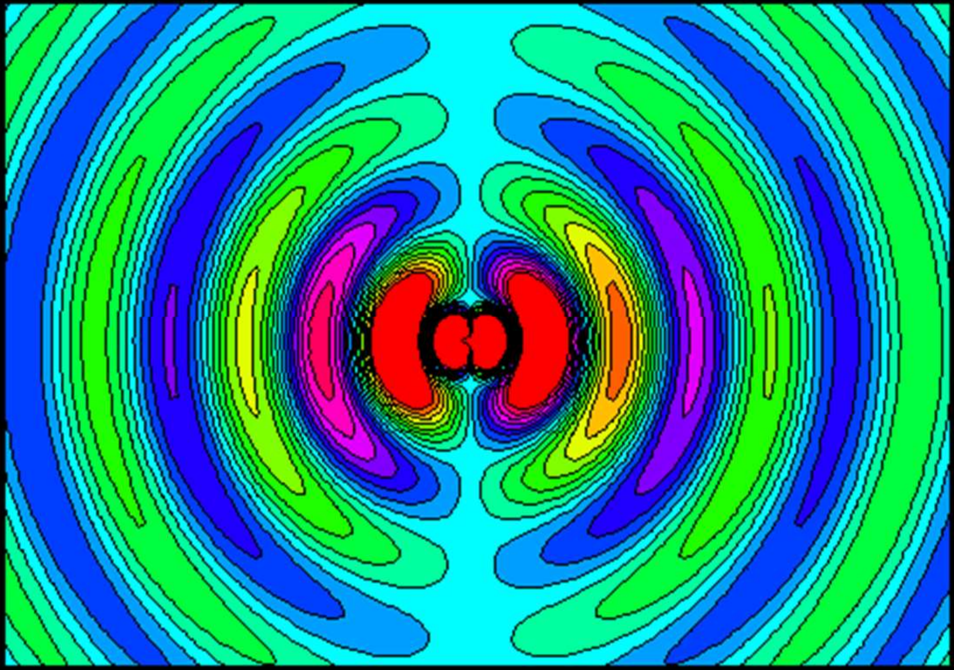
$$\vec{p} = q\,\vec{r}$$

$$\vec{p} = \sum_{i=1}^{N} q_i\,\vec{r}_i$$

General $\mathbf{p}(\mathbf{y}) = \int \rho(\mathbf{x})\,(\mathbf{x}-\mathbf{y})\,d\mathbf{x},$



©2006 Yves Pelletier (ypelletier@ncf.ca)

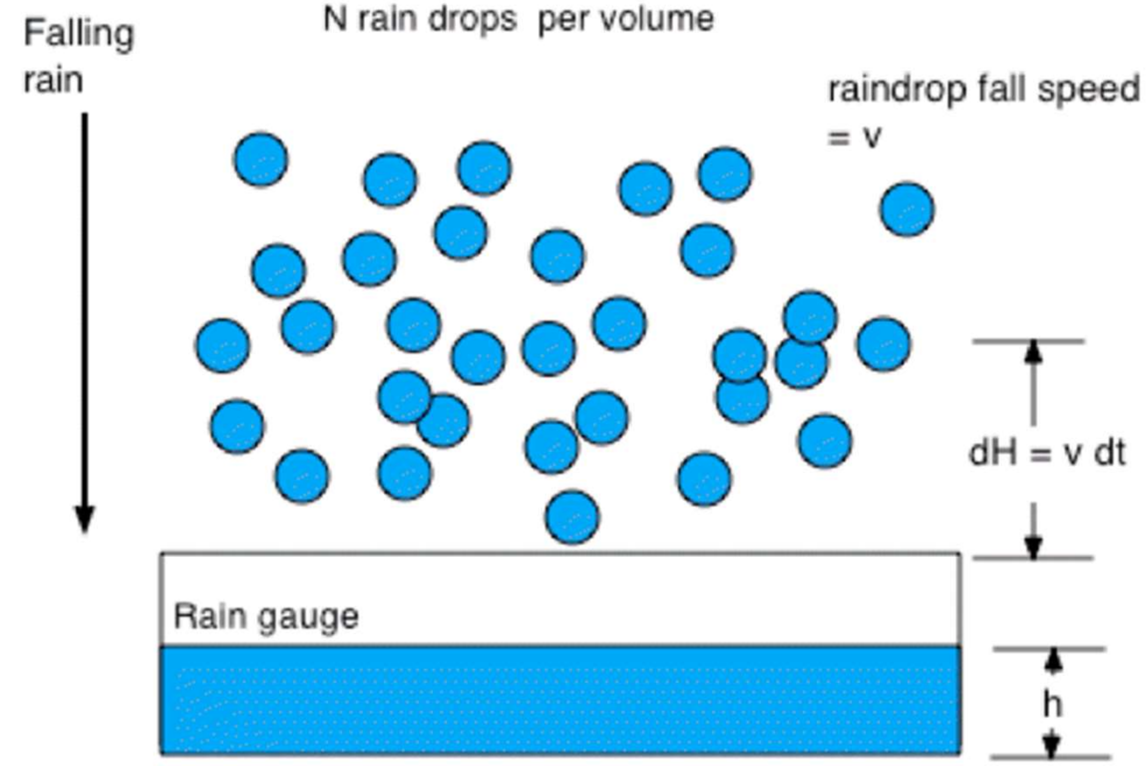


Rayleigh Scattering Cross Section Per Raindrop and Rainfall Rate: Mismatch of diameter dependence

$$\sigma_{sca} = \frac{2}{3}\pi^5 \frac{d^6}{\lambda^4}\left|\frac{m^2-1}{m^2+2}\right|^2$$

d=raindrop diameter
m=complex refractive index
λ=radar wavelength

Rain Fall Rate for Monodispersion

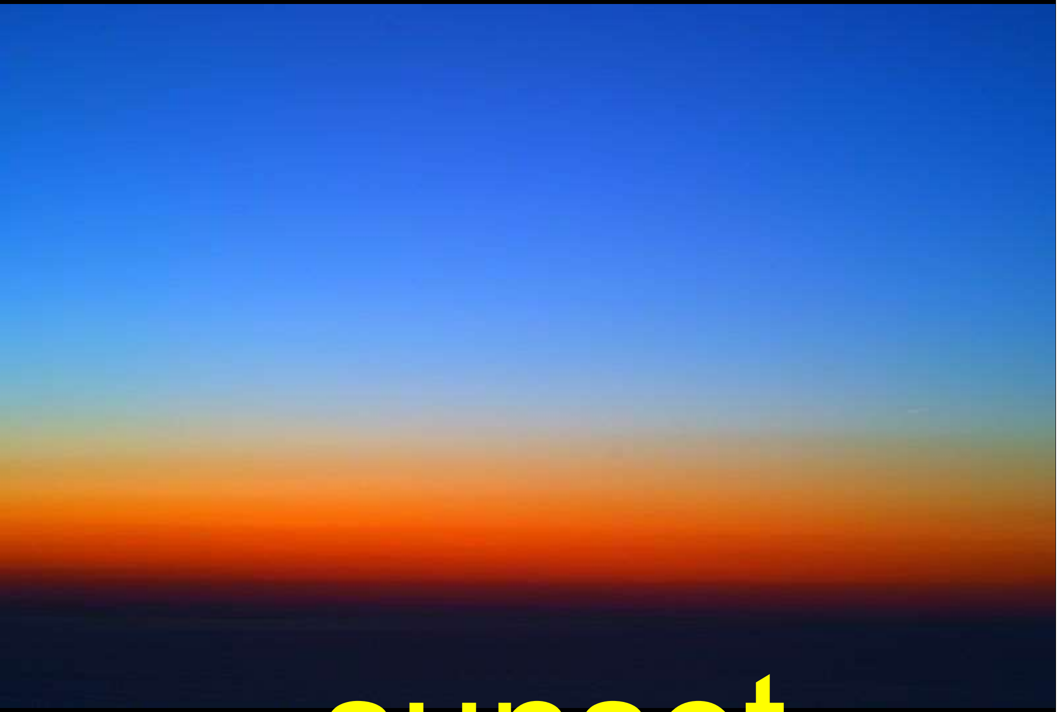


h = height of rain water in raing gauge.   Rainfall rate = dh/dt
In time dt, drops from a height dH = v dt will land in the rain gauge.
Drops have diameter D. A = area of the rain gauge opening.
Then total water volume added in time dt to the rain gauge is
N A dH $\pi D^3/6$ = # Rain Drops going into rain gauge * drop volume
= N A vdt $\pi D^3/6$ = A dh

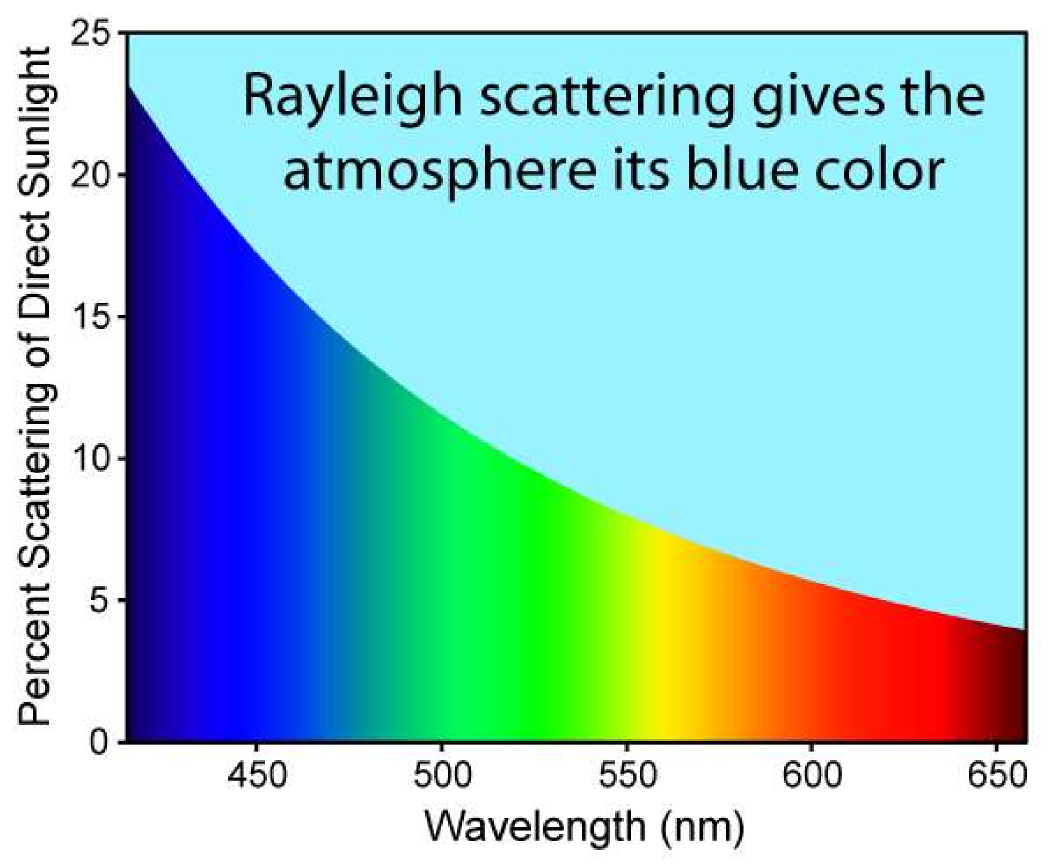
Then rain rate is dh/dt = N v $\pi D^3/6$.

SUNSET NEVADA

Rayleigh Scattering
Light Scattering by Electric Dipoles



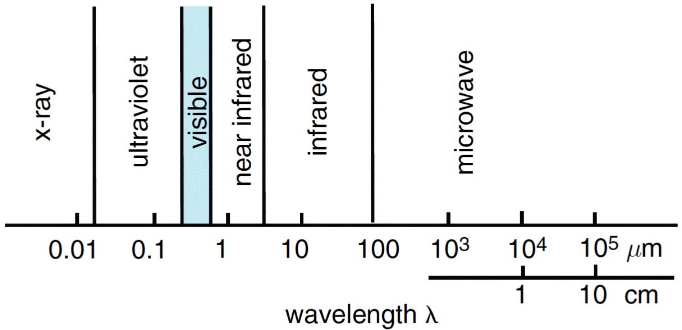
sunset



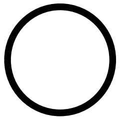
D = molecular diameter
λ= wavelength
$D << \lambda$

$$\beta_{sca} \propto \frac{D^6}{\lambda^4}$$

Light Scattering Basics (images from Wallace and Hobbs CH4).



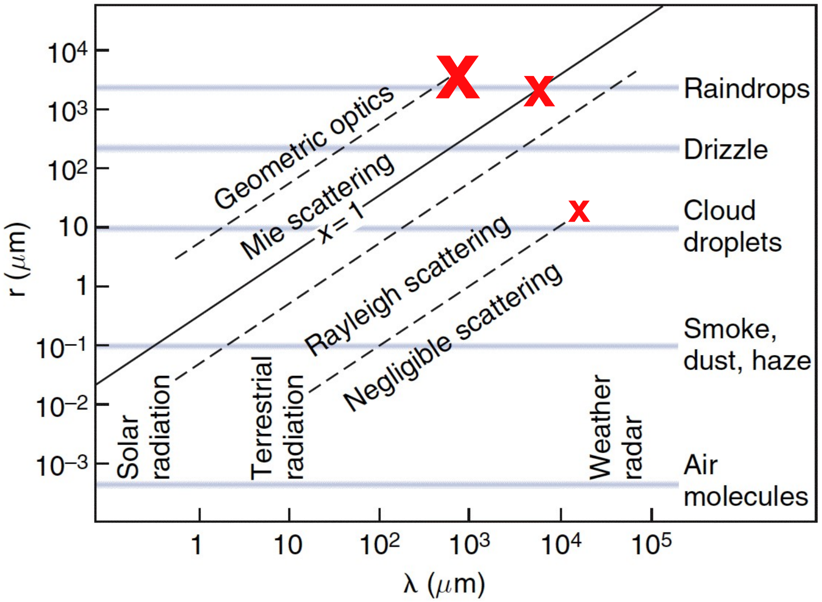
Sphere, radius r, complex refractive index $n=m_r + im_i$



Dimensionless Parameters

$$Size\ Parameter \equiv x = \frac{2\pi r}{\lambda}$$

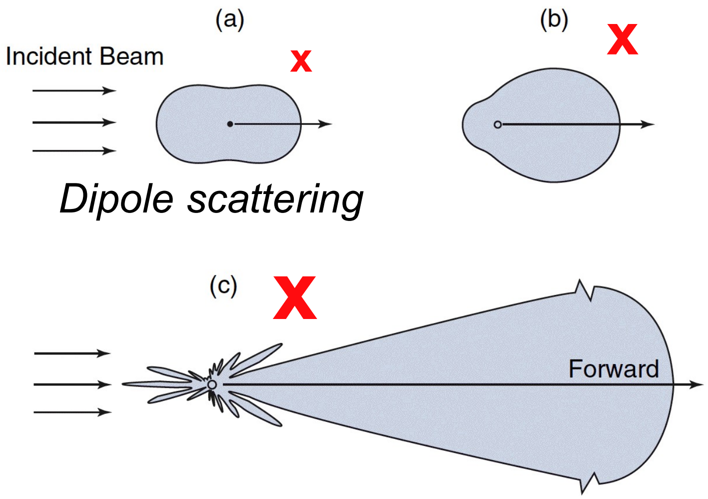
$$Scattering\ Efficiency \equiv Q_s = \frac{\sigma_{sca}}{\pi r^2}$$



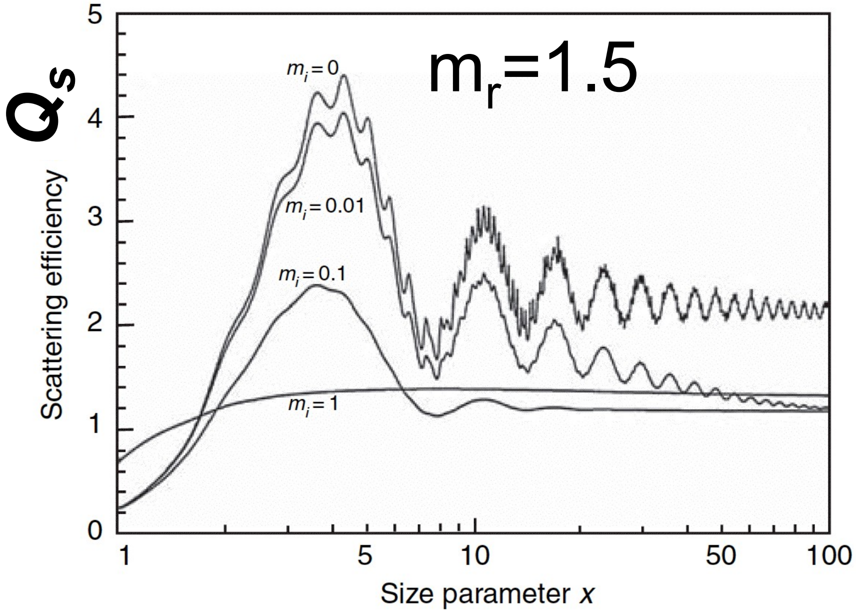
Lines:

$$r = \frac{x}{2\pi}\lambda$$

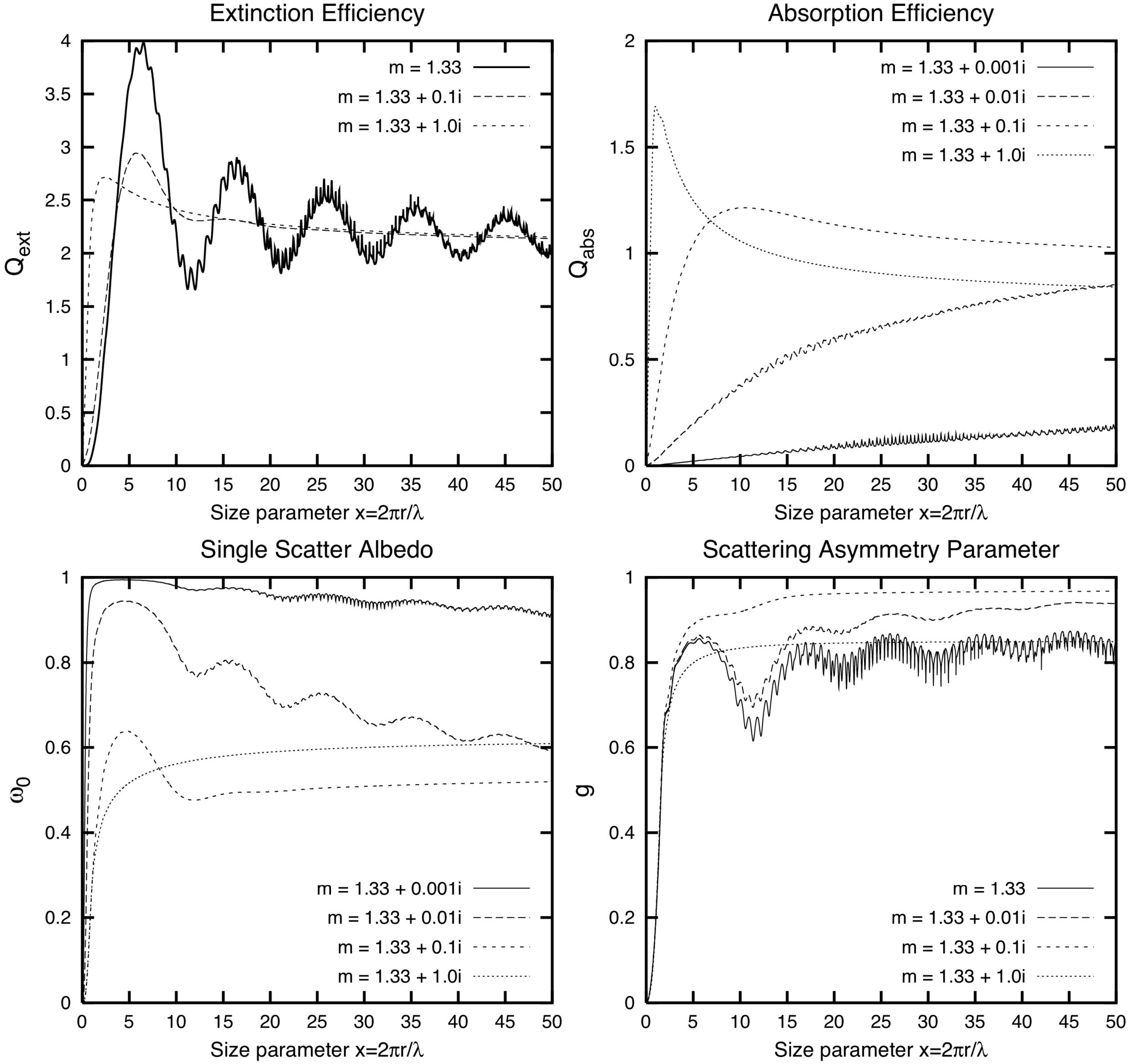
Angular Distribution of scattered radiation (phase function)



Dipole scattering



Mie Theory for Absorbing and Scattering Spheres



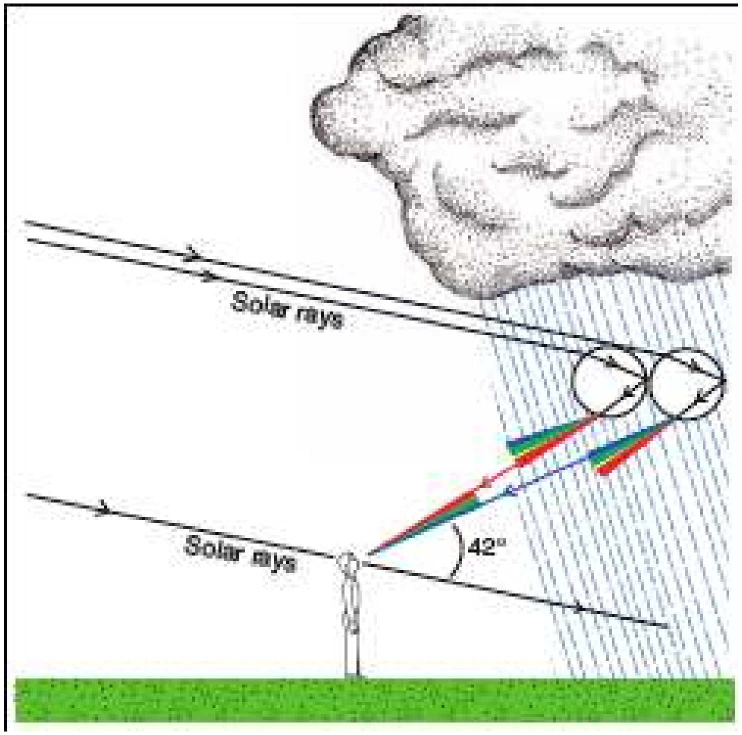
Extinction is the sum of absorption and scattering.

The single scattering albedo is $\omega_0 = Q_{sca}/Q_{ext}$.

The asymmetry parameter g. The probability of forward scatter is (1+g)/2. For backward scatter is (1-g)/2.

From Petty

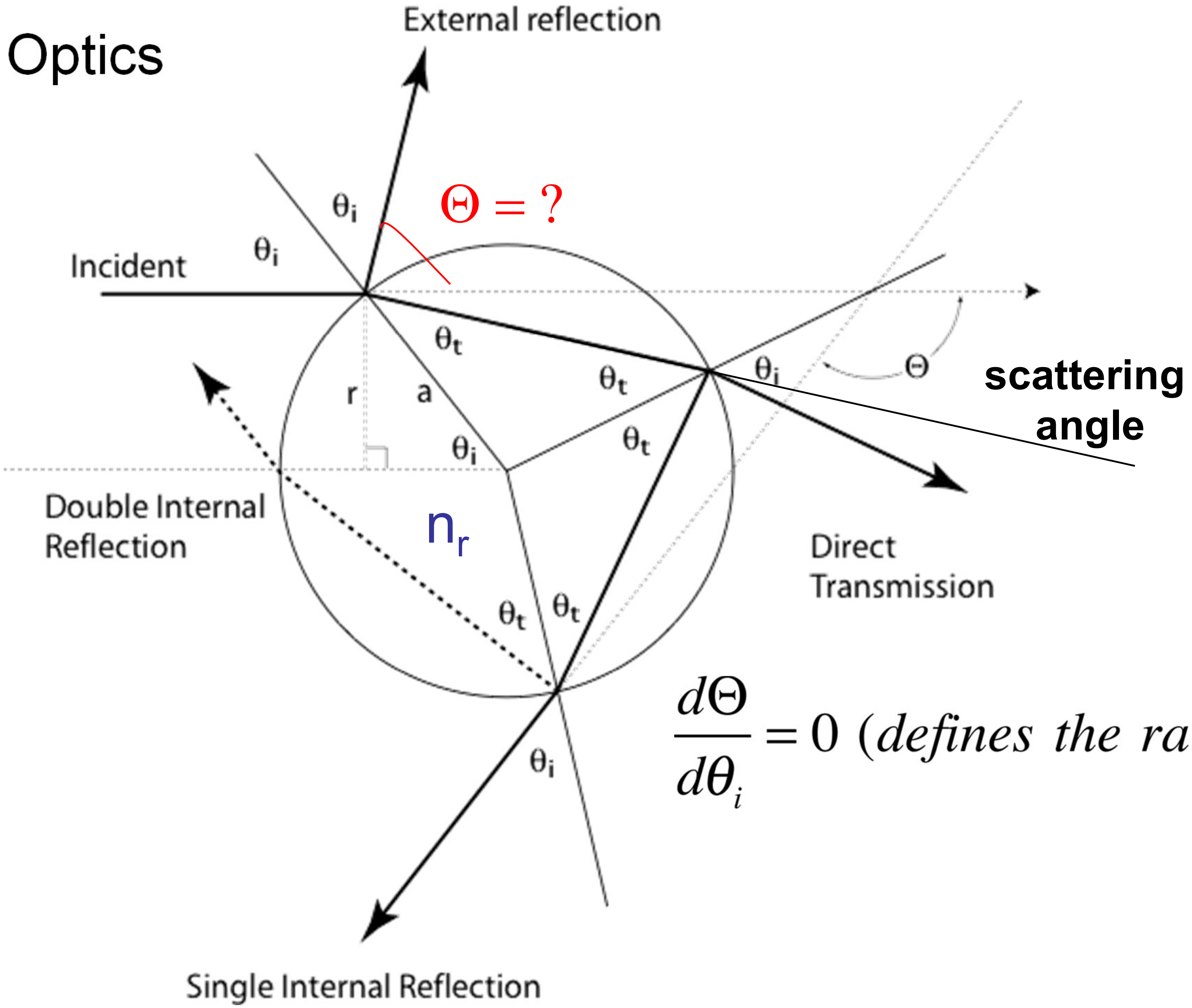
Geometrical Optics: Interpret Most Atmospheric Optics from **Raindrops** and lawn sprinklers (from Wallace and Hobbs CH4)



Primary Rainbow Angle: Angle of Minimum Deviation (turning point) for rays incident with 2 chords in raindrops.

Secondary Rainbow Angle: Angle of Minimum Deviation (turning point) for rays incident with 3 chords in raindrops.

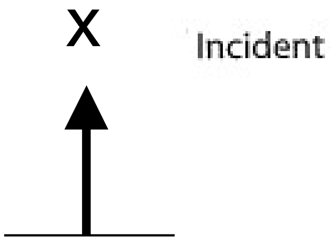
Rainbow Optics

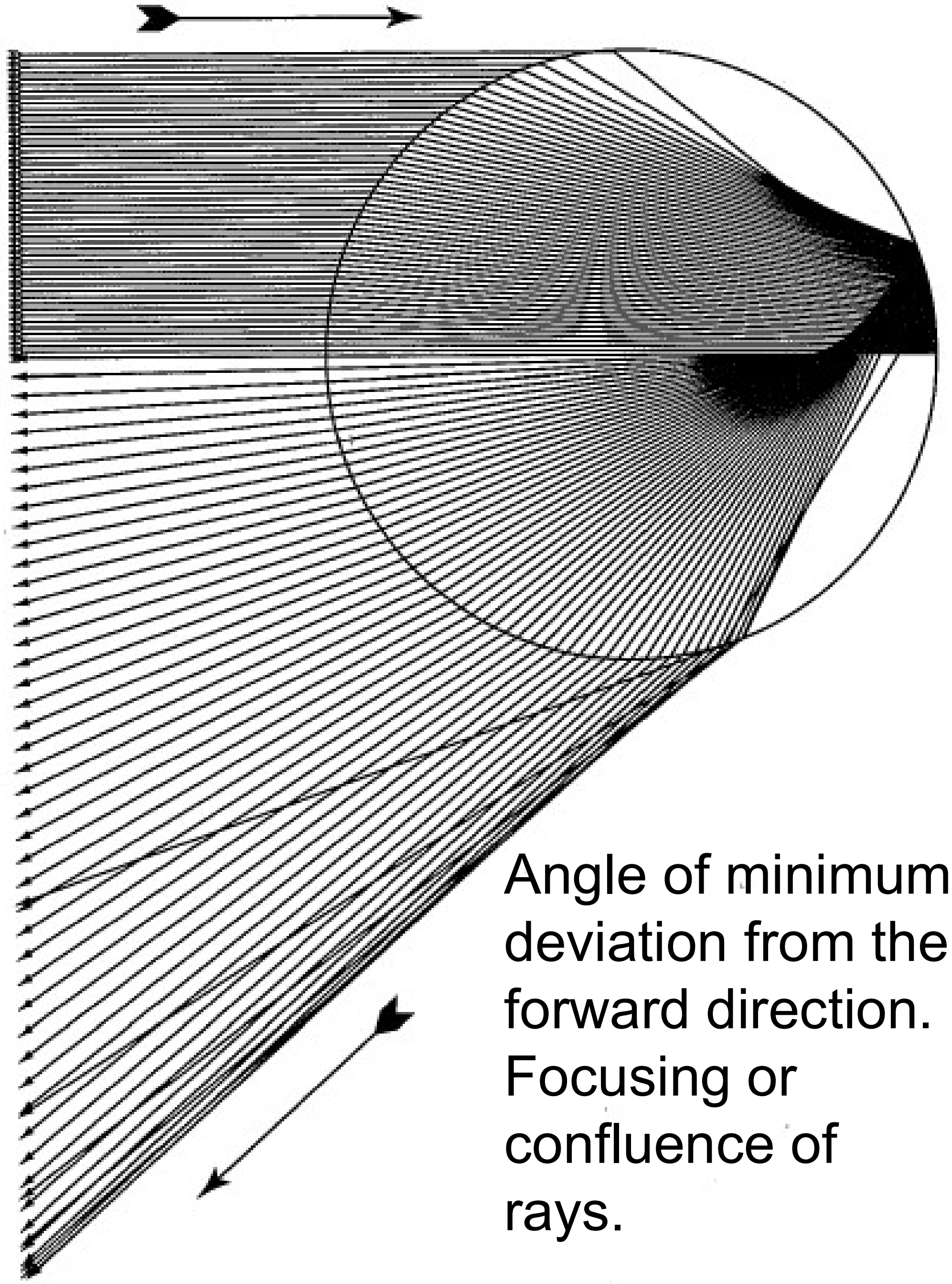


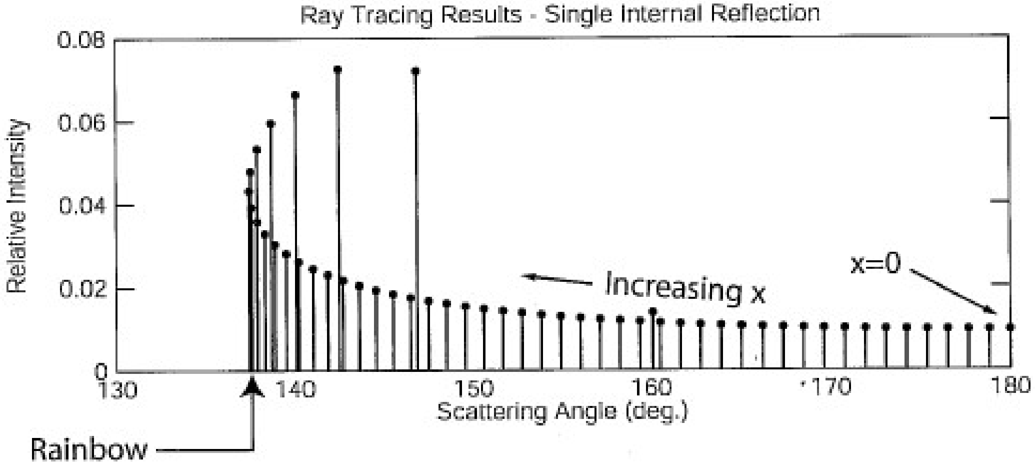
$$\frac{d\Theta}{d\theta_i} = 0 \ (\textit{defines the rainbow})$$

See http://www.philiplaven.com/p8e.html, and atmospheric optics.

Geometrical Optics: Rainbow (from Petty)



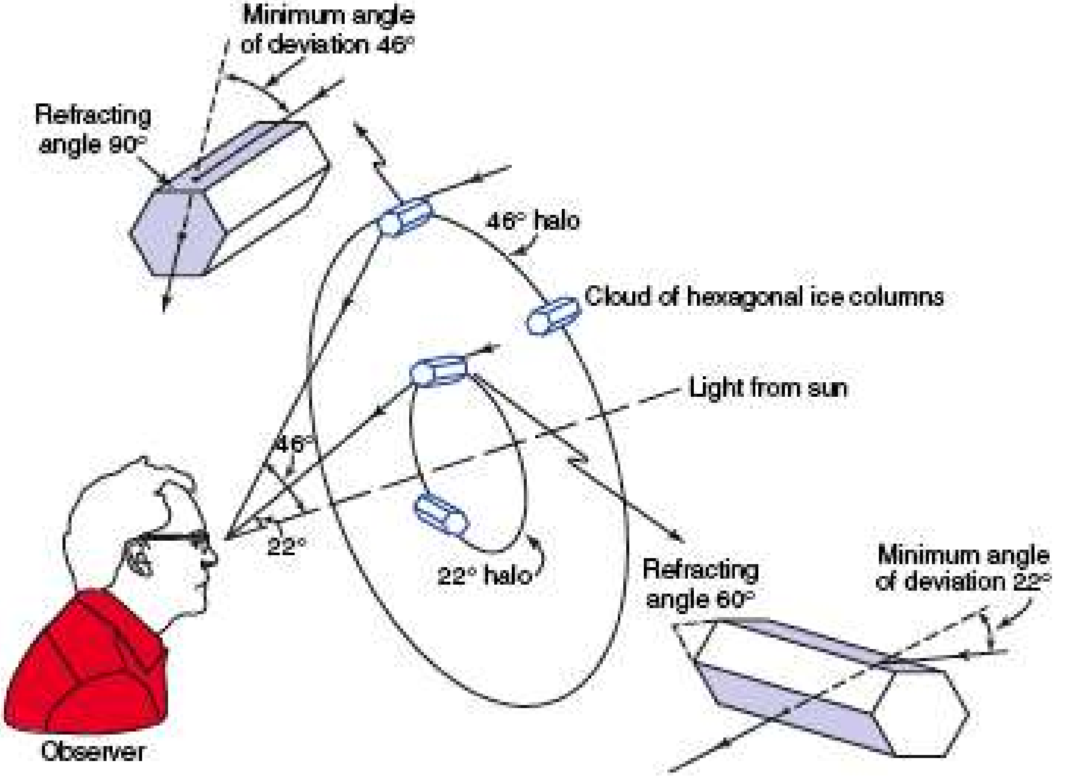




Distance x is also known as the impact parameter. (Height above the sphere center.)

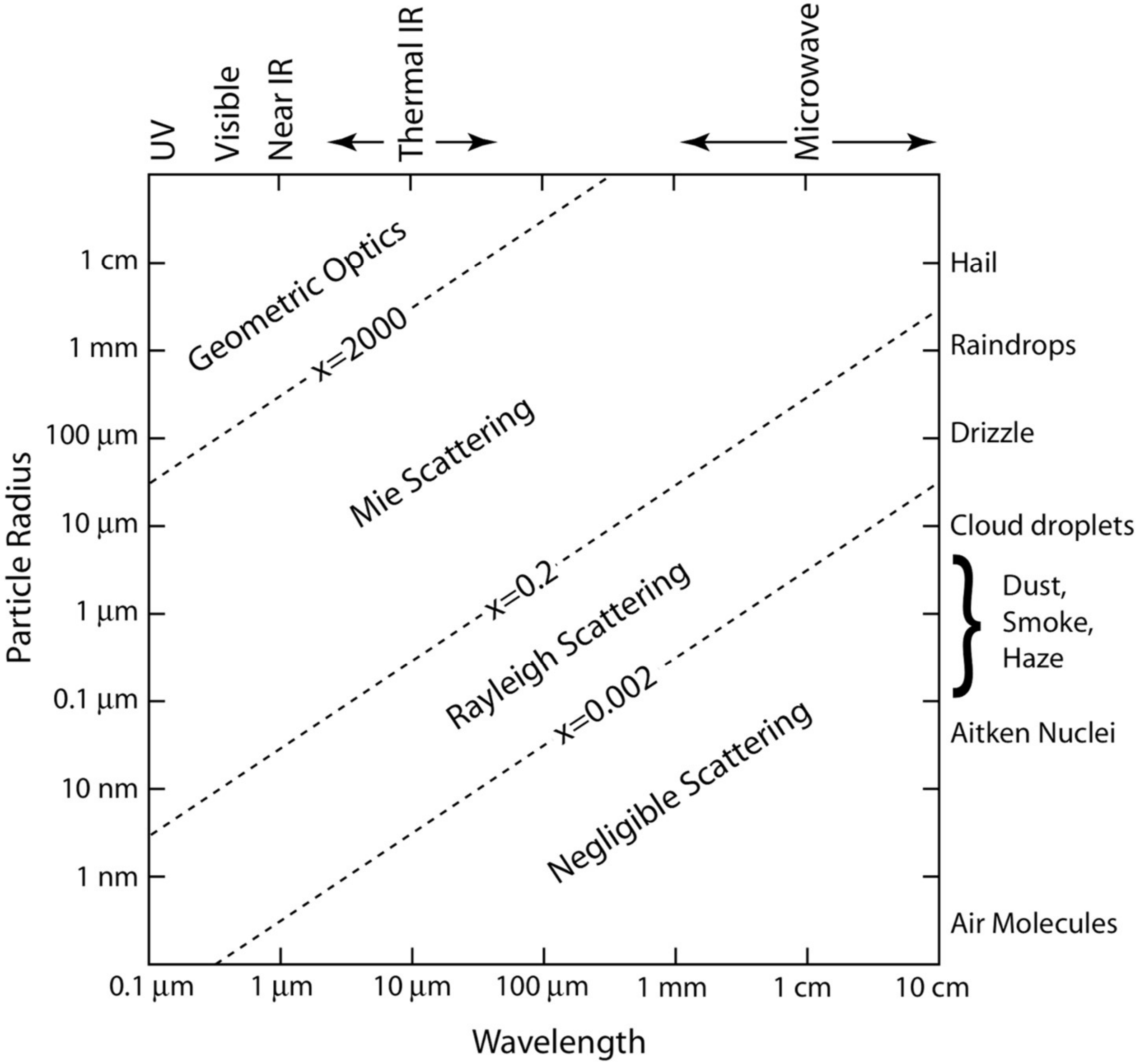
Angle of minimum deviation from the forward direction. Focusing or confluence of rays.

Geometrical Optics: Interpret Most Atmospheric Optics from Ice Crystals (from Wallace and Hobbs CH4)



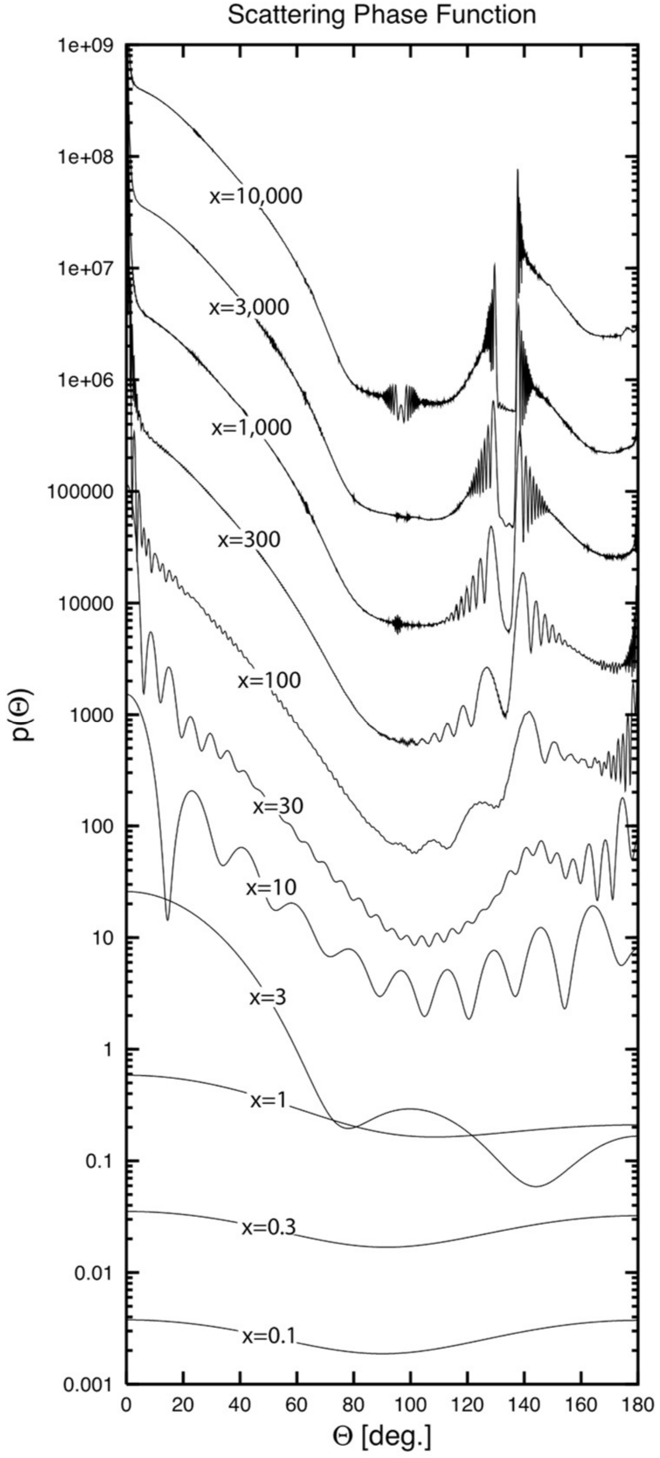
22 deg and 45 deg Halos from cirrus crystals of the column or rosette (combinations of columns) types. Both are angle of deviation phenomena like the rainbow. Crystal orientation important. 22 deg halo, more common, thumb rule to measure size of arc.

Size Parameter *x* Characterizes Scattering Regimes



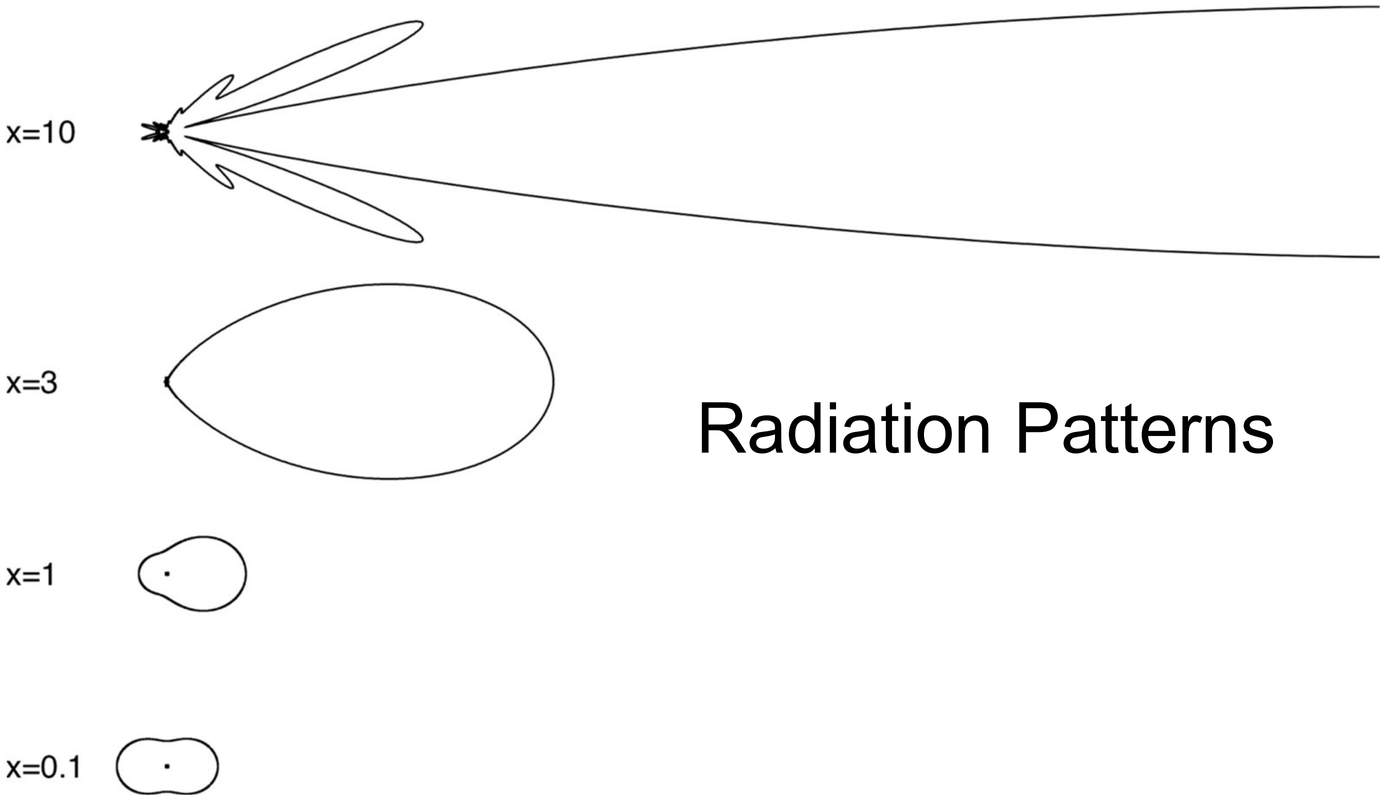
Summary of Scattering Regimes: Note Particle Sizes and wavelengths of radiation!!

Mie Phase Functions for Water Spheres



light in

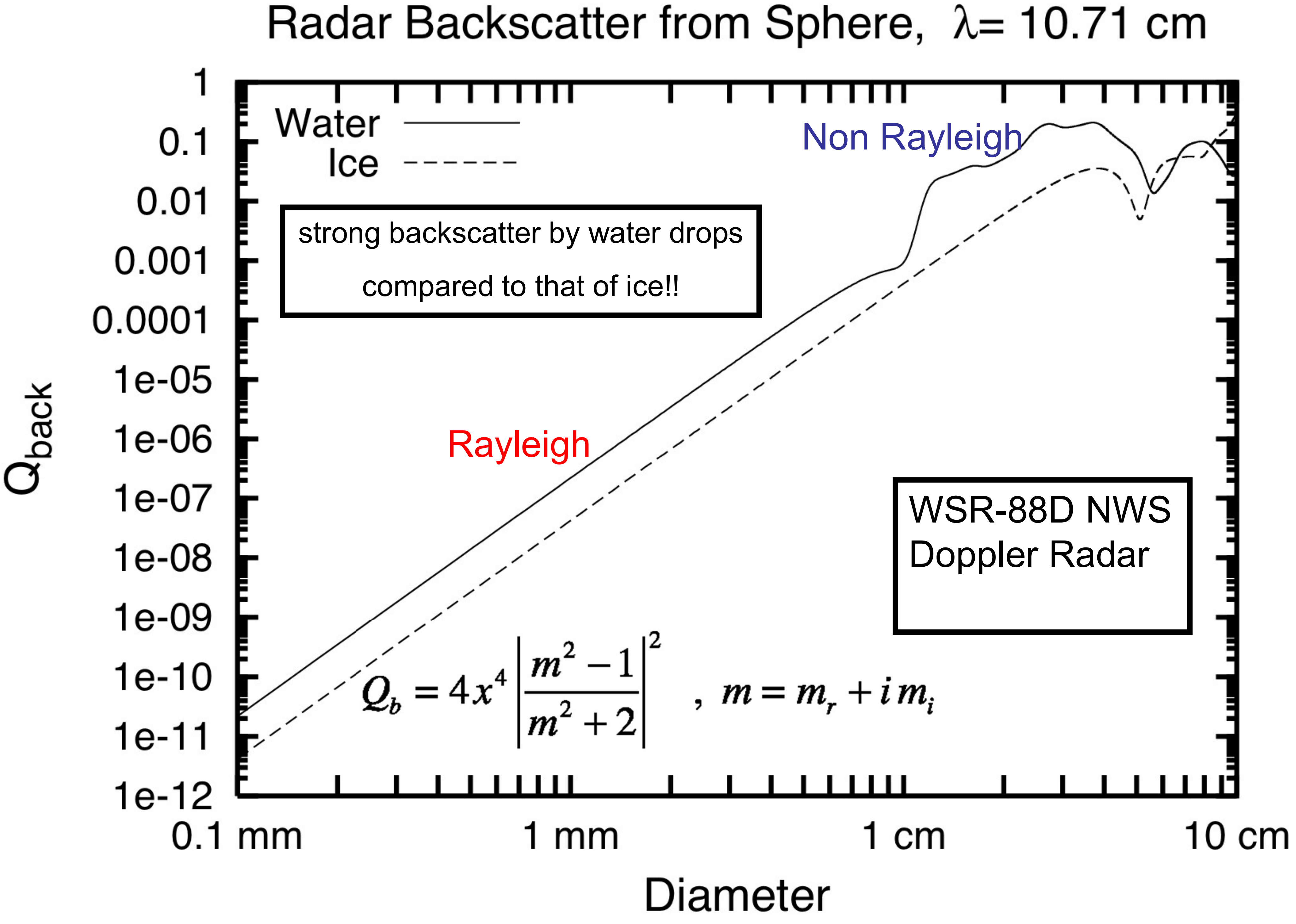
scattered light



Radiation Patterns

Note the dipole character for small x
and the primary and secondary rainbows
for large x.

Mie Radar Backscatter Efficiency for Water and Ice Spheres



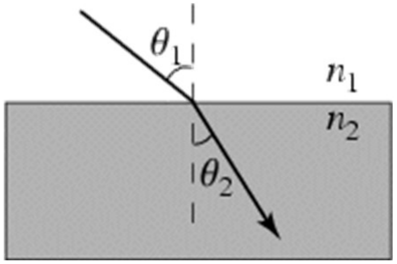
strong backscatter by water drops compared to that of ice!!

WSR-88D NWS Doppler Radar

$$Q_b = 4x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \quad , \; m = m_r + i\, m_i$$

Supplemental Material Follows This Point

Trace velocity matching principle: Snell's law (continuity of the wavefront at a boundary) *"slow is more normal"*



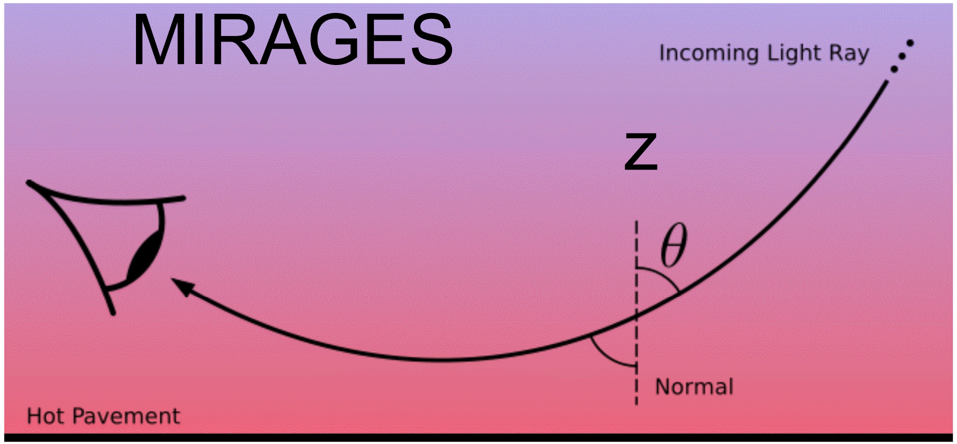
Here assume
$n_1=n_{1r}$, $n_{1i}=0$,
$n_2=n_{2r}$, $n_{2i}=0$.

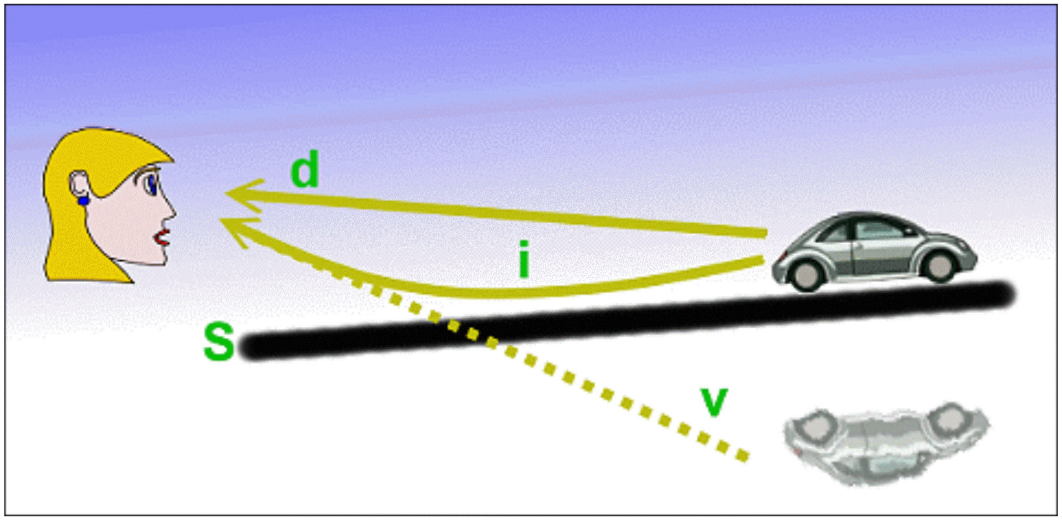
In which medium is the speed of light less?

Why do we sometimes see lightning but not hear thunder?

$$n_1\sin(\theta_1)= n_2\sin(\theta_2)$$

MIRAGES



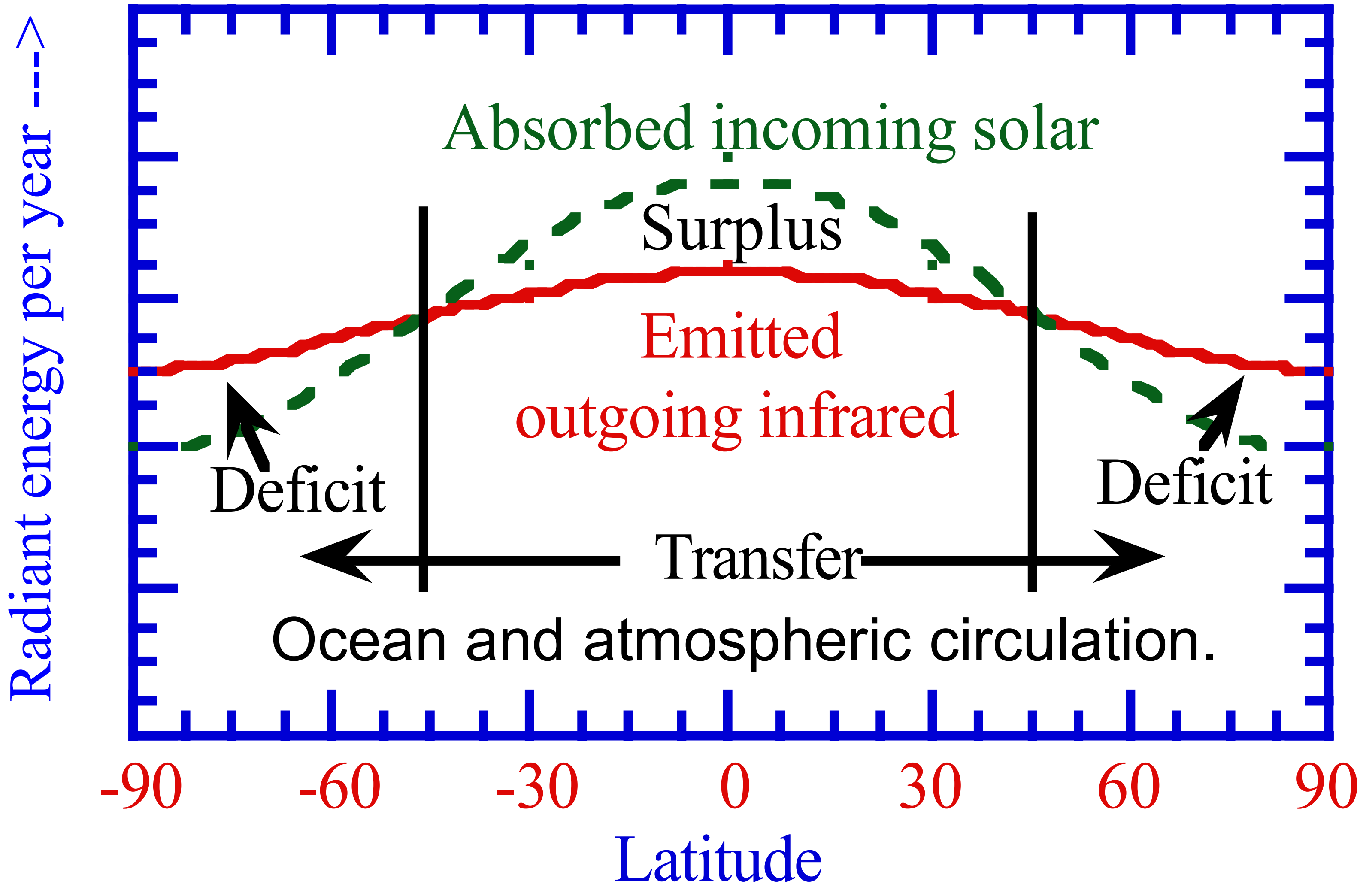


For a gas, $(n_r-1) \approx \rho$
ρ=gas density.

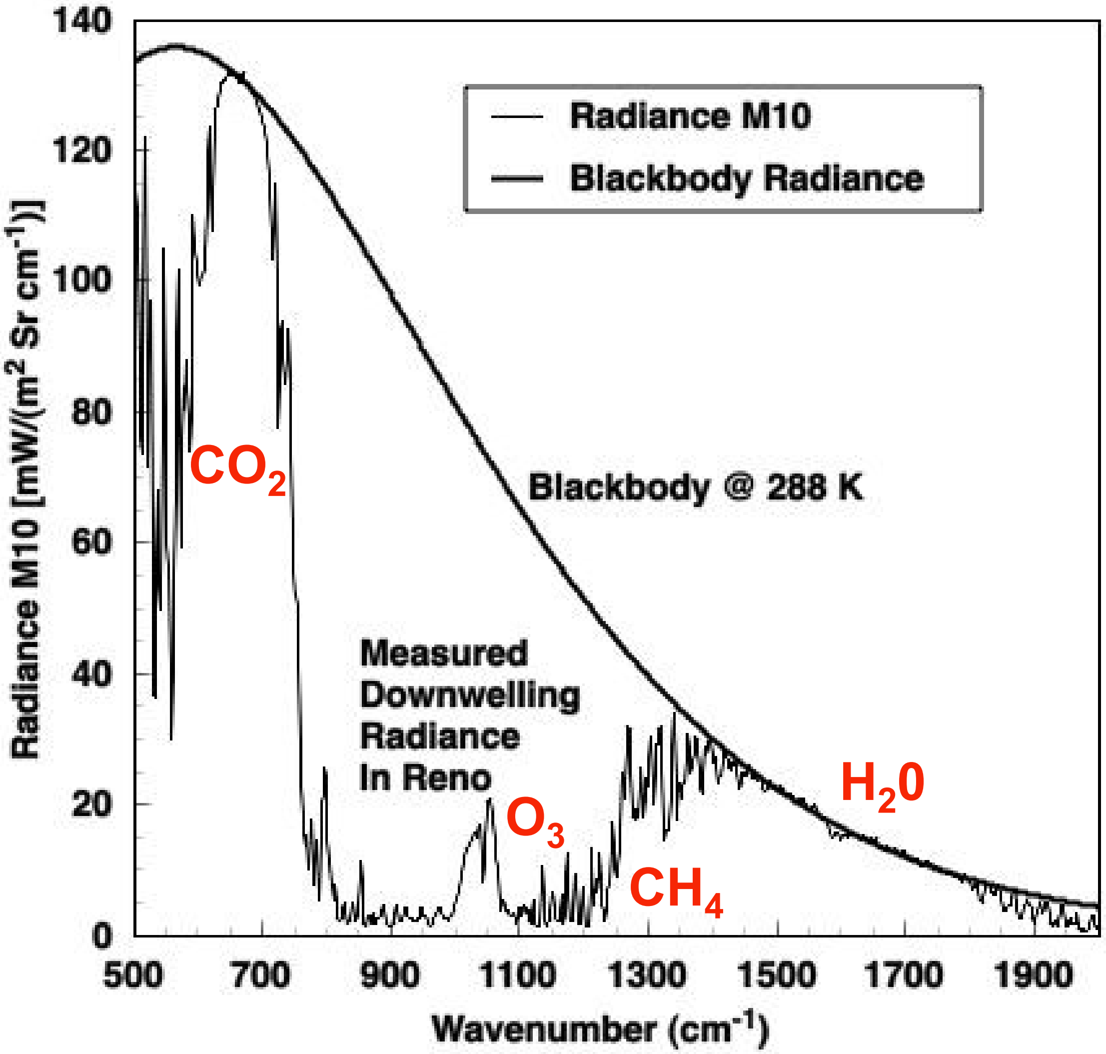
$d\rho/dz > 0$ for this type or mirage.

What does this say about the likelihood of convection?

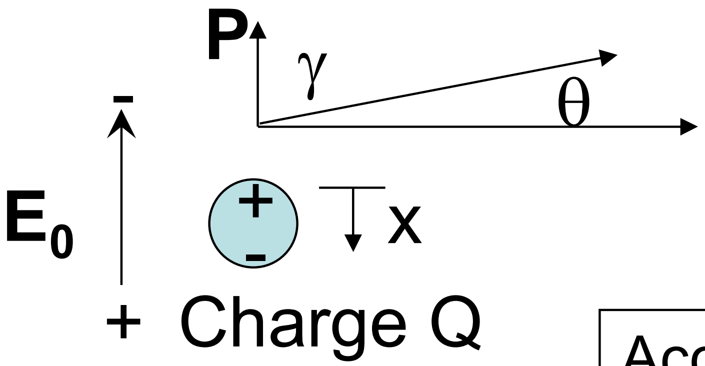
Energy Transfer From Equator to Poles



Infrared Spectrum from the Atmosphere to the Surface



Accelerating Dipole Moment Produces Rayleigh Scattering



dipole moment **=P**=Q**x=**α**E$_0$**

α=polarizability

Acceleration of dipole moment: Scattered E field!!!

$$\vec{E}_{sca}^{dipole} = \frac{1}{c^2}\frac{1}{r}\frac{\partial^2 \vec{P}}{\partial t^2}\sin\gamma \quad \text{(embodies the dipole radiation pattern!!)}$$

$$\frac{\partial \vec{P}}{\partial t} = -i\omega\vec{P},\ \frac{\partial^2 \vec{P}}{\partial t^2} = -\omega^2\vec{P} = -\omega^2\alpha\vec{E}_0 \text{ for time harmonic E fields, } \omega\text{=c}\frac{2\pi}{\lambda} = ck.$$

Then

$$\vec{E}_{sca}^{dipole} = -\frac{\vec{E}_0 \exp\left(i(kr-\omega t)\right)}{r}k^2\alpha\sin\gamma$$

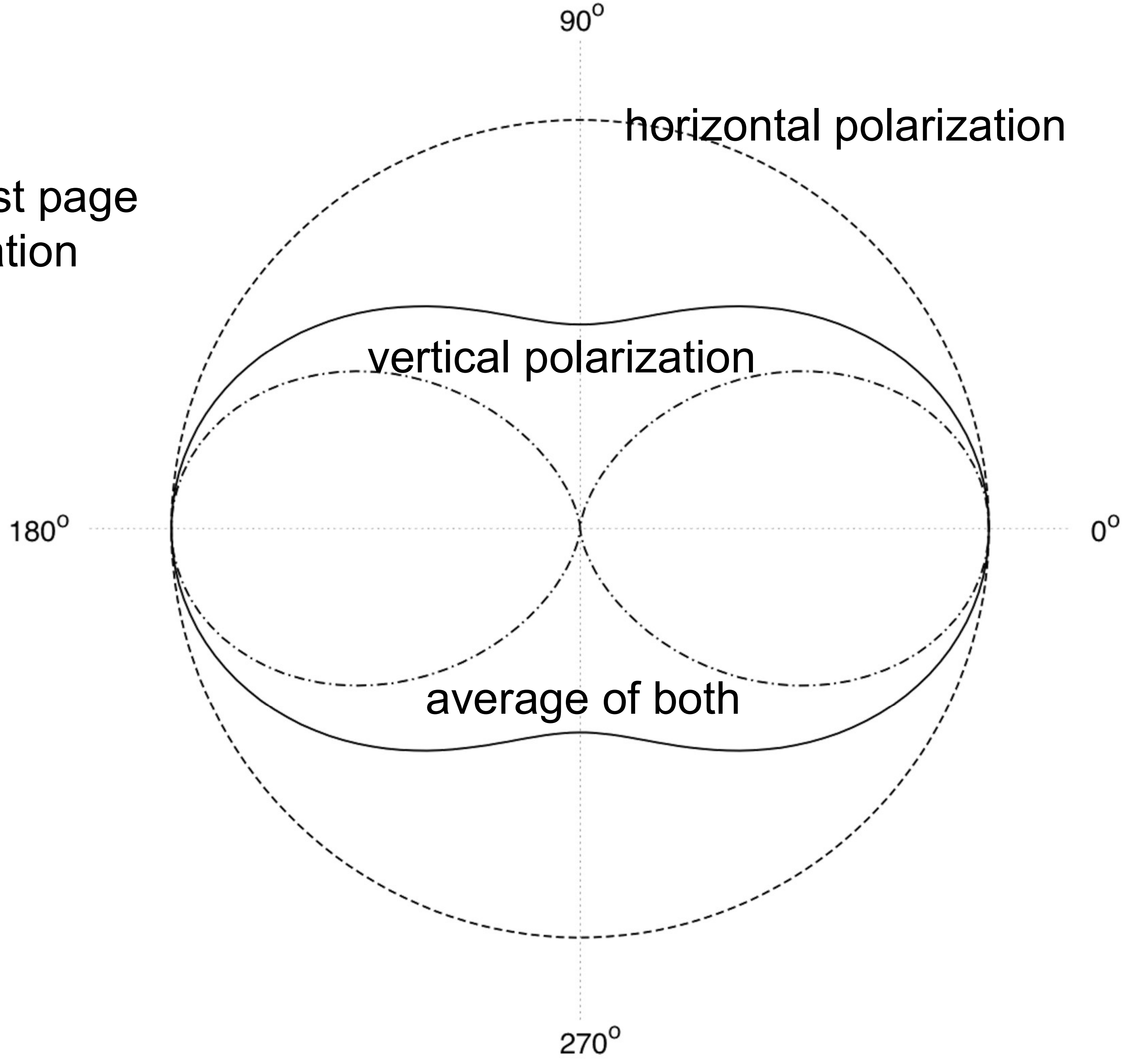
$$I_{sca}^{dipole} = \left|\vec{E}_{sca}^{dipole}\right|^2 = \frac{I_0 k^4\alpha^2\sin^2\gamma}{r^2}\text{, where } I_0 \text{ is the irradiance of the incident field.}$$

For random polarization incident,

$$I_{sca}^{dipole} = \frac{I_0}{r^2}\alpha^2\frac{128\pi^5}{3\lambda^4}\frac{p(\theta)}{4\pi}\text{, dipole phase function } p(\theta) = \frac{3}{4}(1+\cos^2\theta).$$

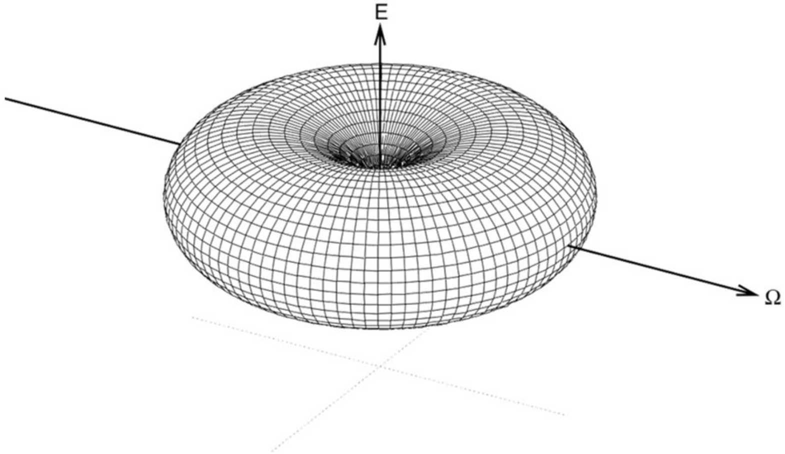
Rayleigh Scattering Phase Function: Angular Distribution of Light Scattered by a Dipole

Refer to last page for polarization states.

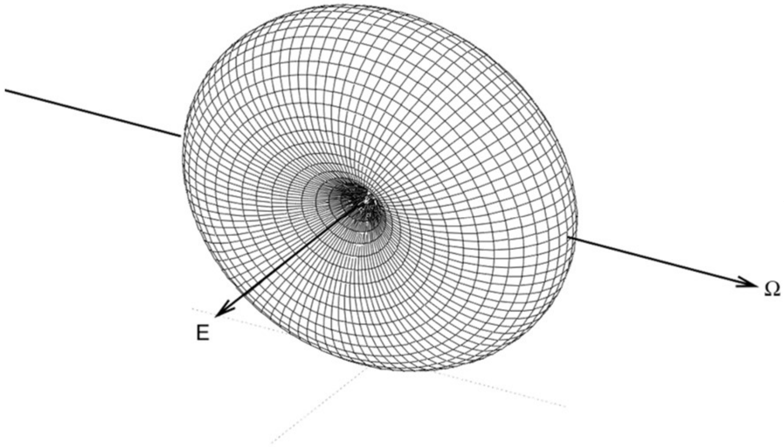


Rayleigh Scattering Phase Function: Angular Distribution of Light Scattered by a Dipole

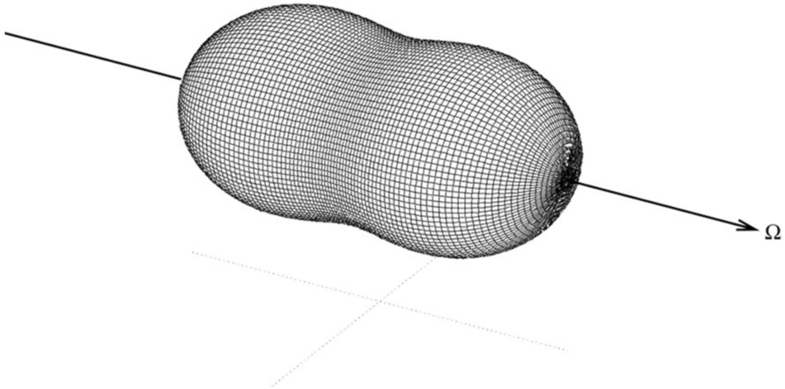
3D rendering



vertical polarization state

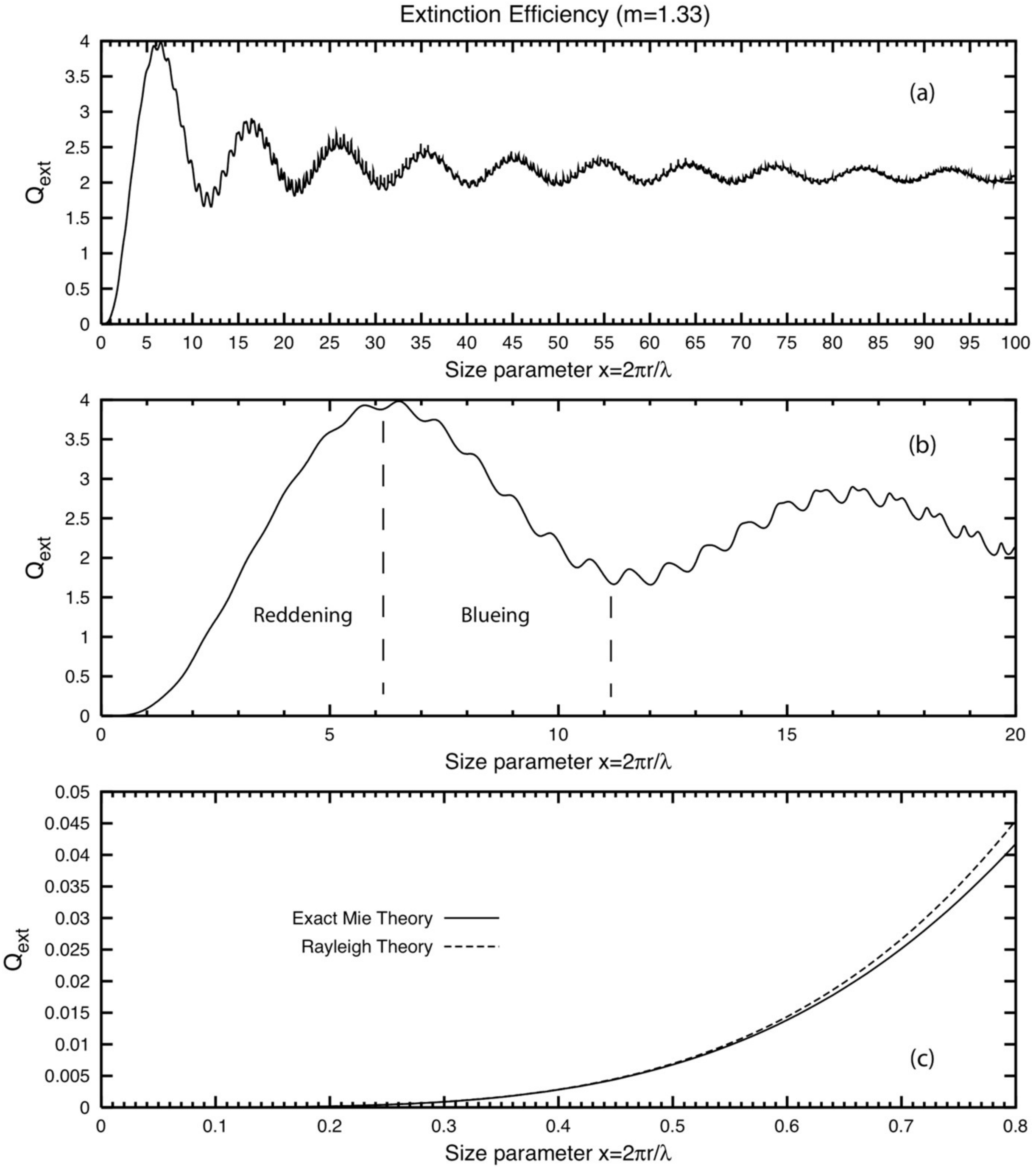


horizontal polarization state



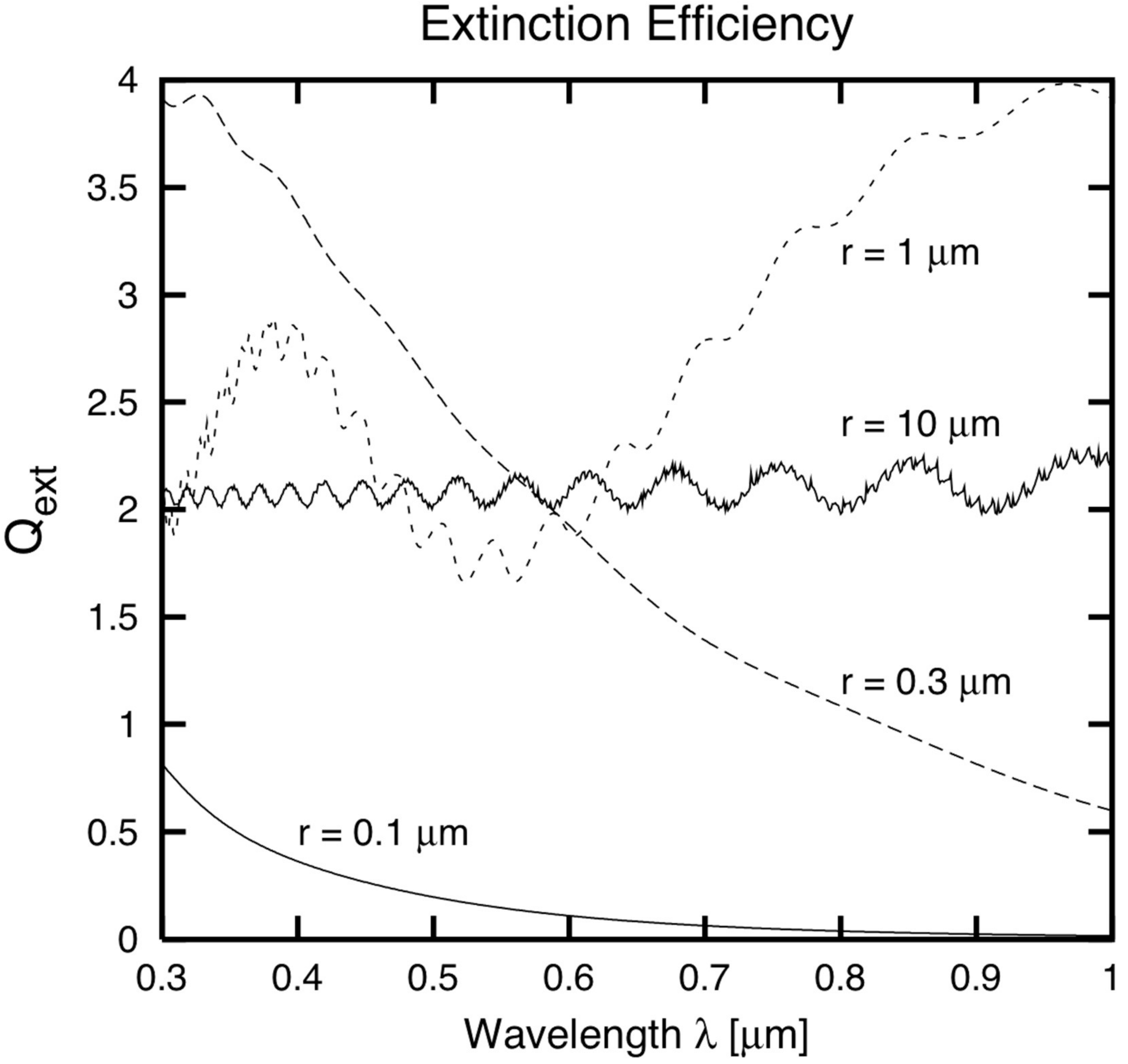
The Peanut!
Average of both polarization states.

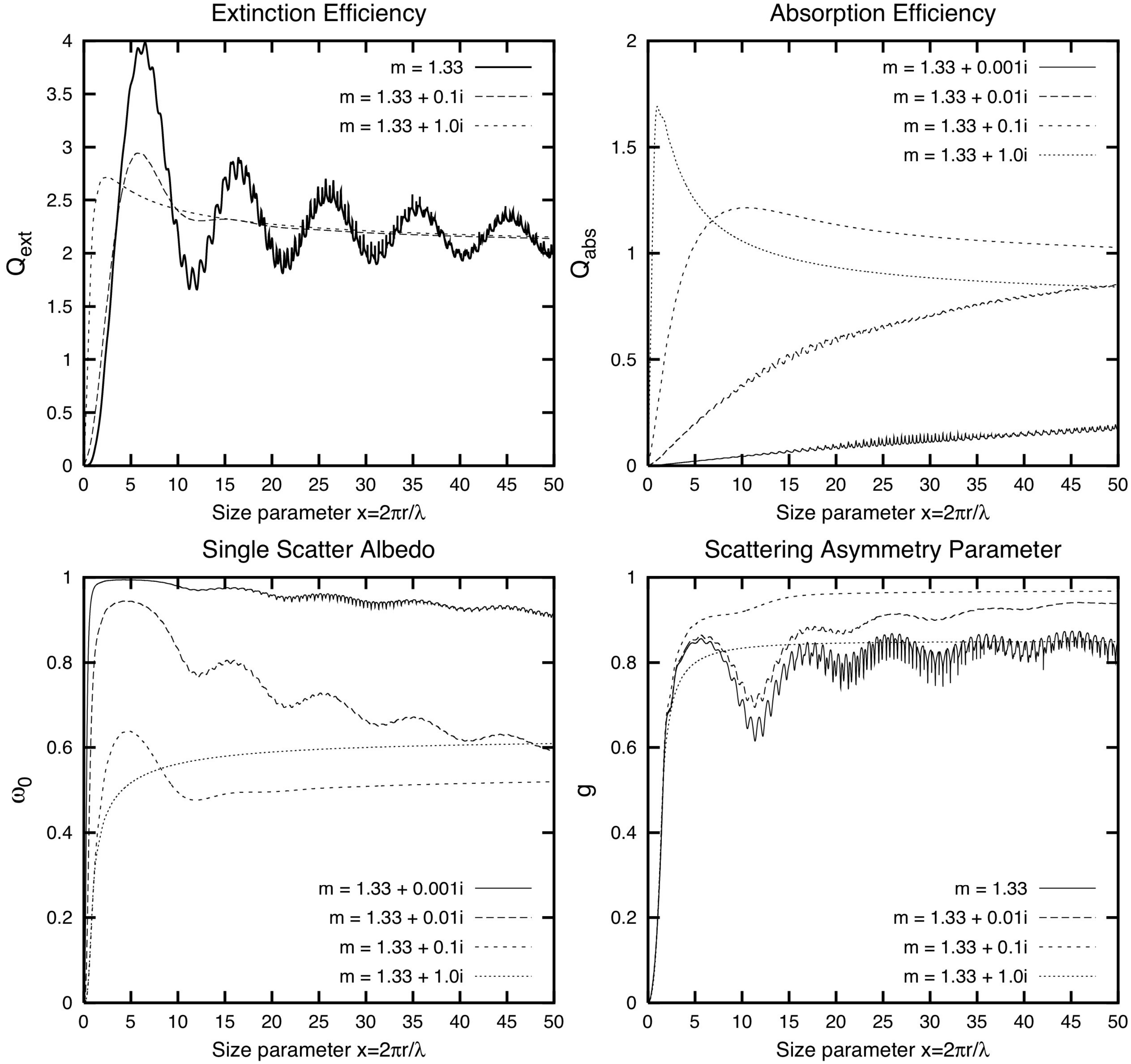
Mie Theory for Water Spheres (non absorbing)



Maximum, relative maxima and minima, approximate theor due to Rayleigh.

Mie Theory for Water Spheres: m=1.33, Visible Wavelengths

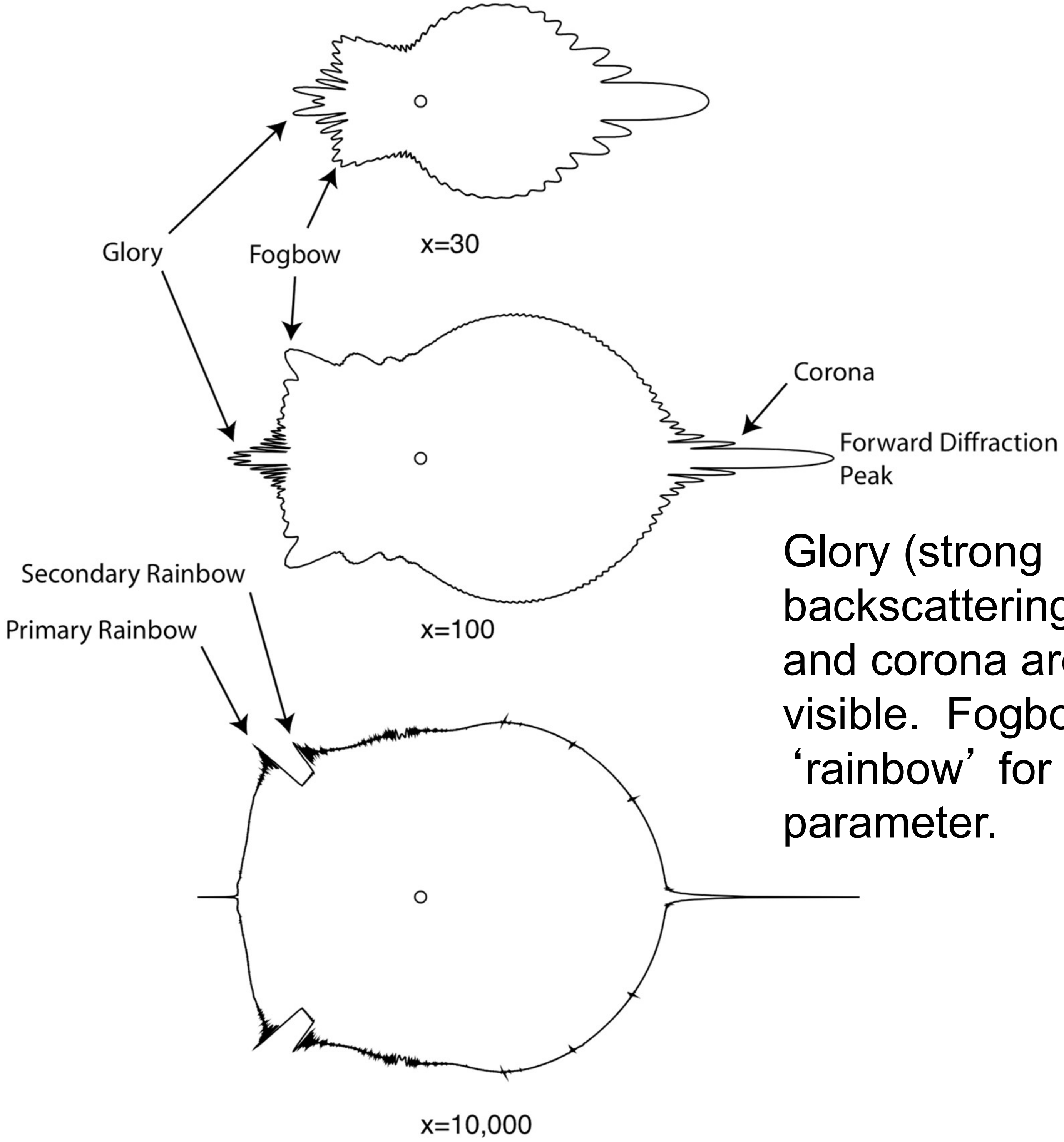




Spheres

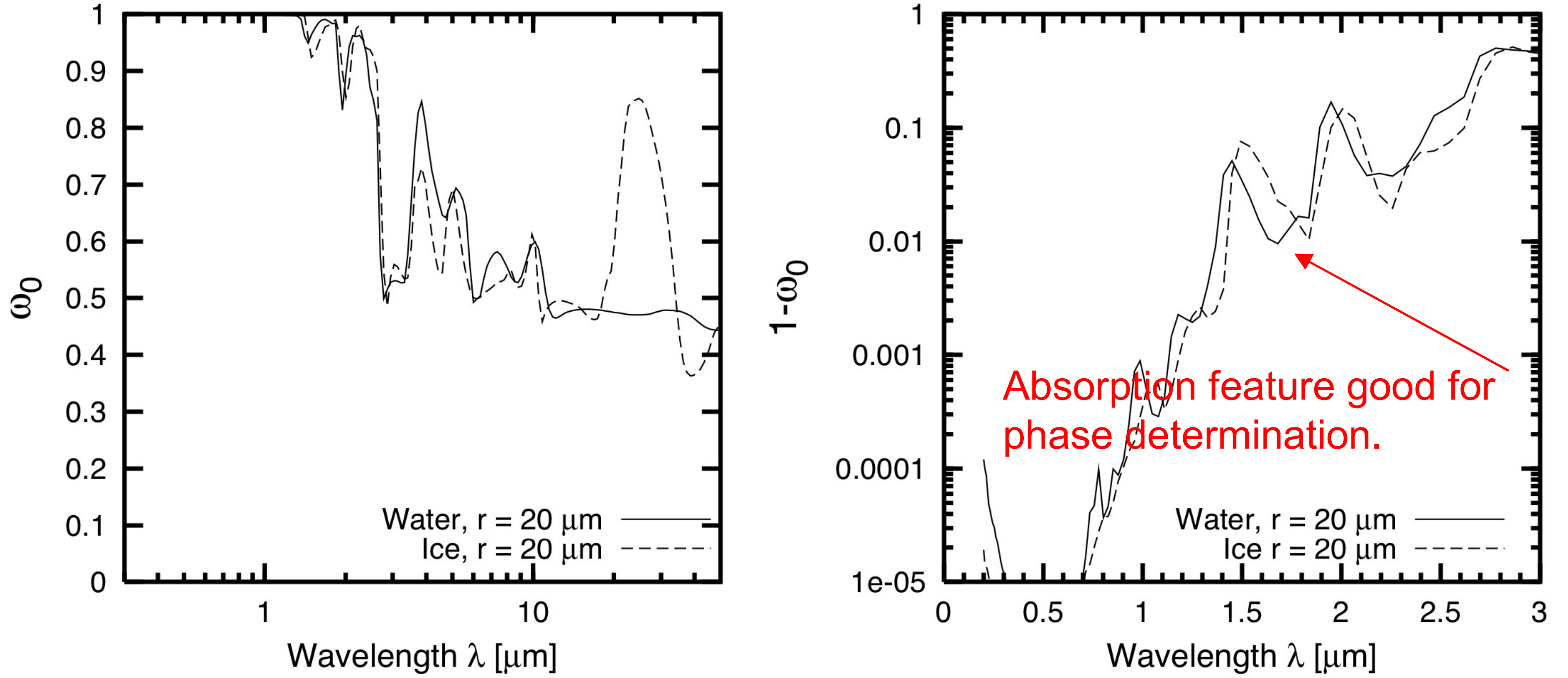
modest n_i has the largest absorption for modest size parameters.

Mie Phase Functions for Water Spheres: Log[$p(\theta)$] for details



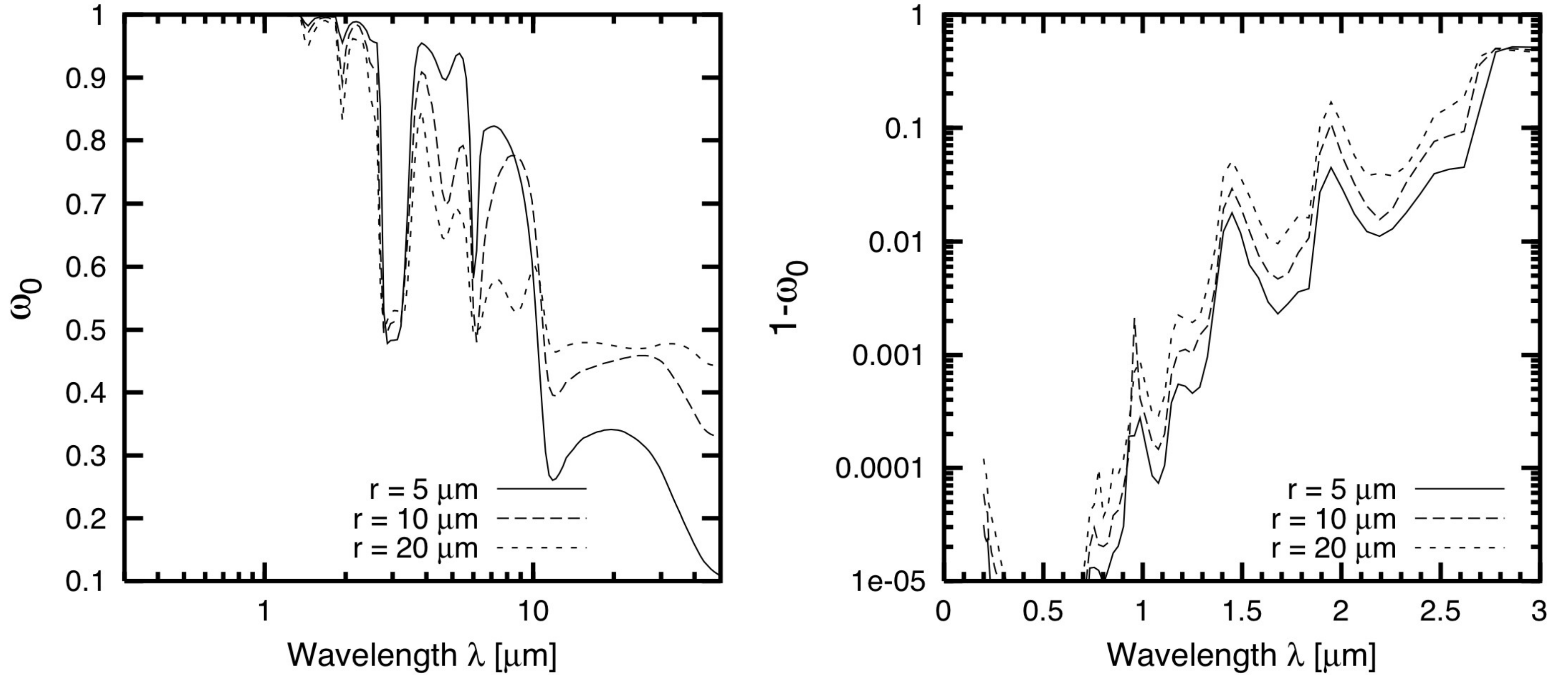
Glory (strong backscattering, rainbow, and corona are clearly visible. Fogbow is ‘rainbow’ for small size parameter.

Mie Theory for Water and Ice Spheres


(a) Single Scatter Albedo - Water vs Ice

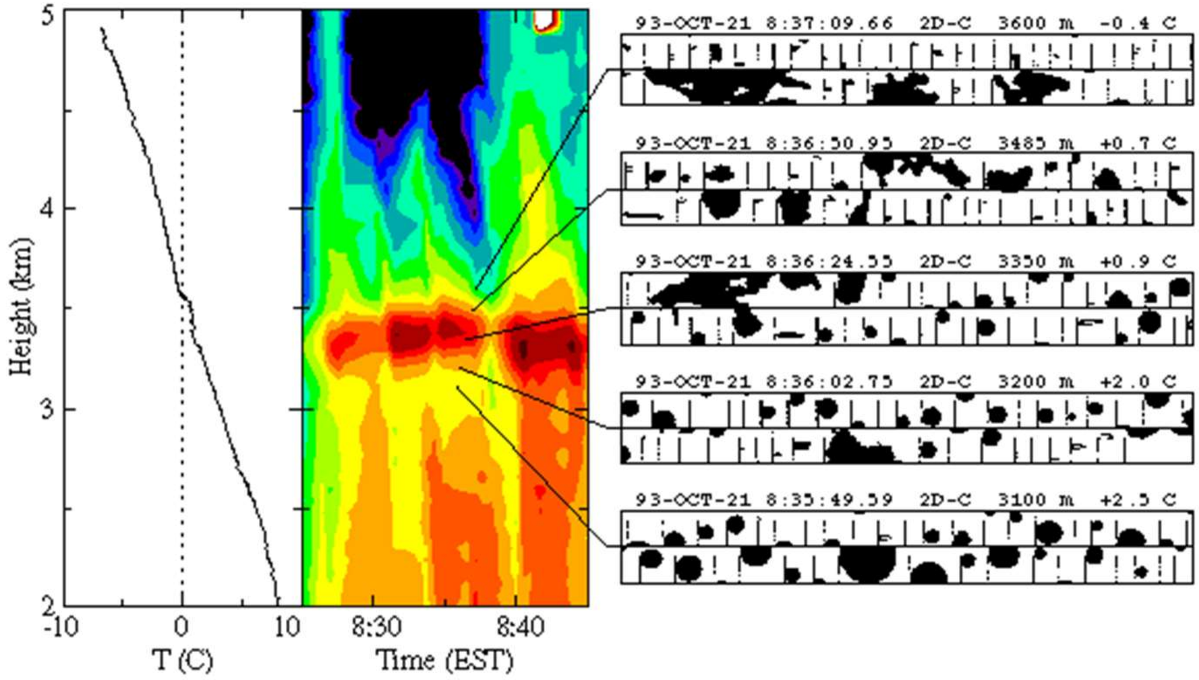
(b) Single Scatter Co-Albedo - Water vs Ice

Absorption feature good for phase determination.


(c) Single Scatter Albedo - Cloud Droplets

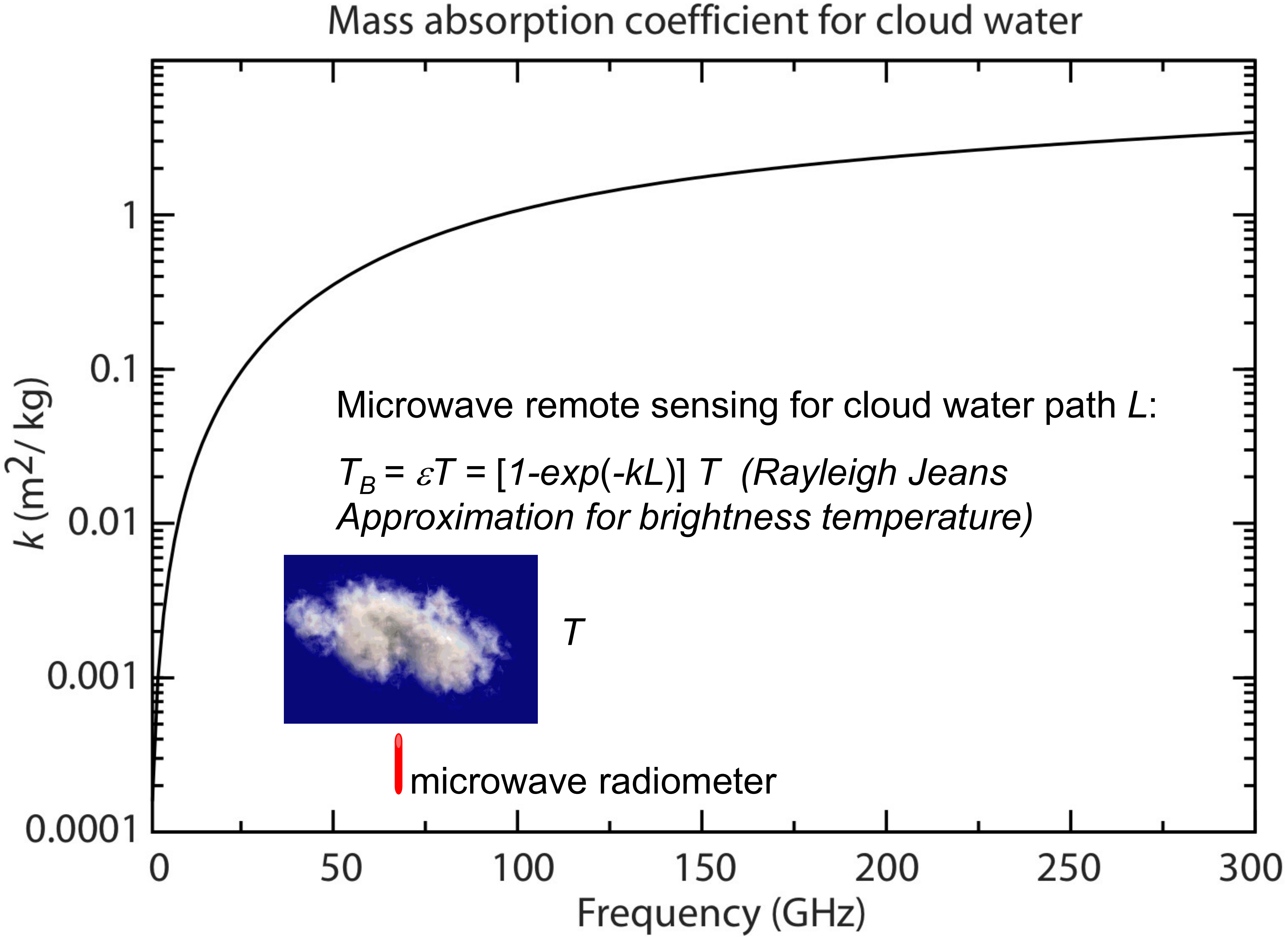
(d) Single Scatter Co-Albedo - Cloud Droplets

Radar Bright Band: Strong Scattering from Melting Hydrometeors



http://www.radar.mcgill.ca/bright_band.html

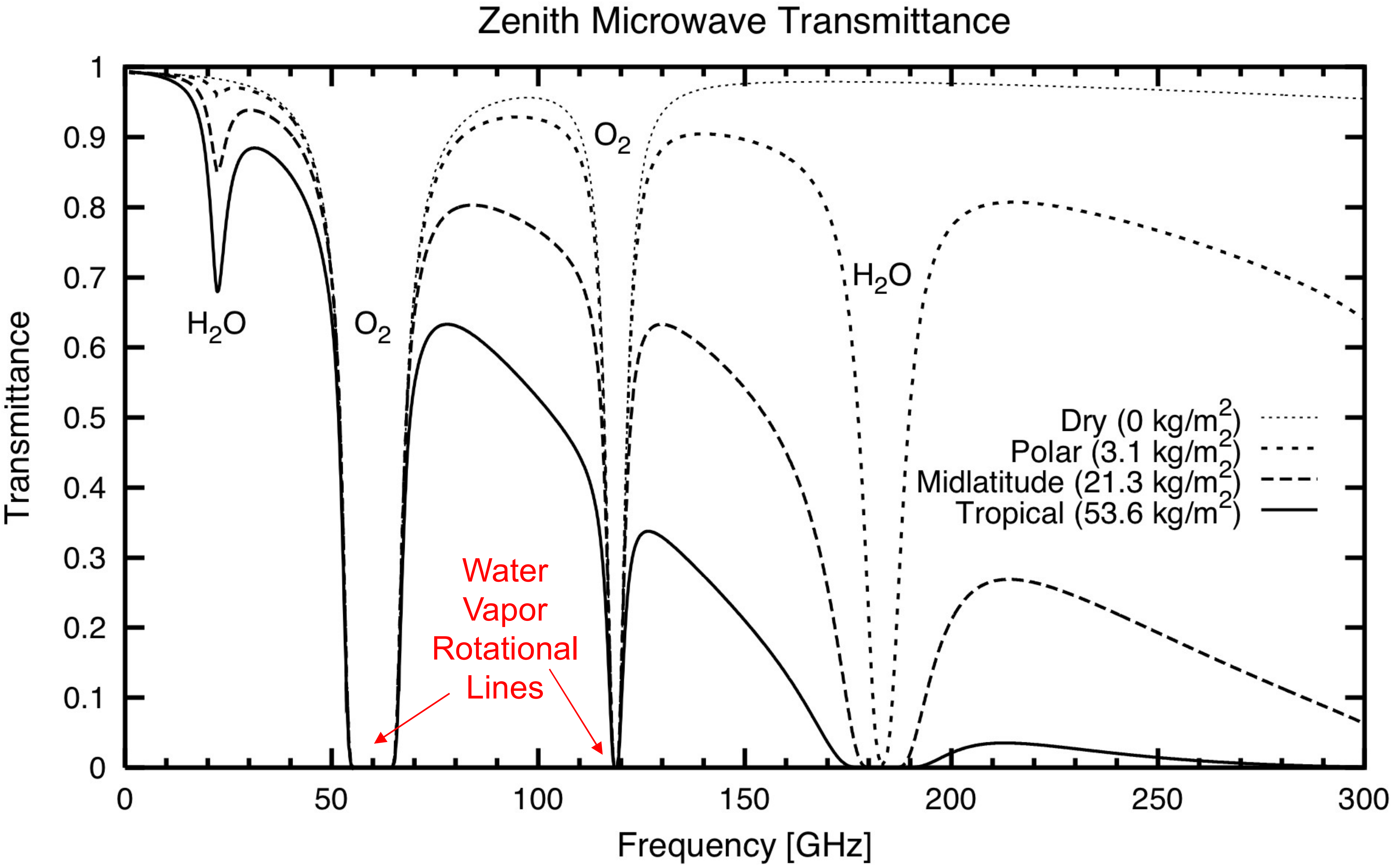
Mass Absorption Coefficient for Cloud Water Spheres, Microwaves



Microwave remote sensing for cloud water path *L*:

$T_B = \varepsilon T = [1\text{-}exp(\text{-}kL)]\ T$ *(Rayleigh Jeans Approximation for brightness temperature)*

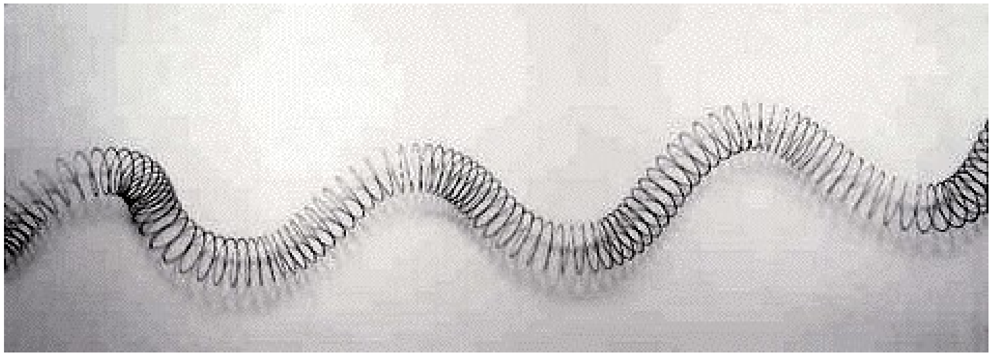
Zenith Microwave Transmittance: Cloud Free Atmospheres



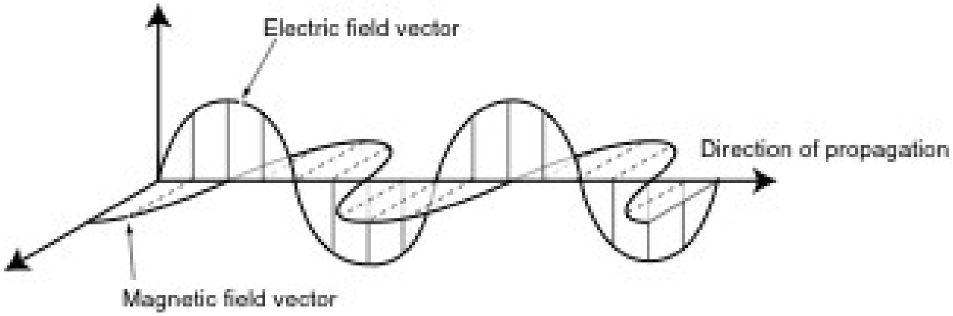
Zenith Microwave Transmittance

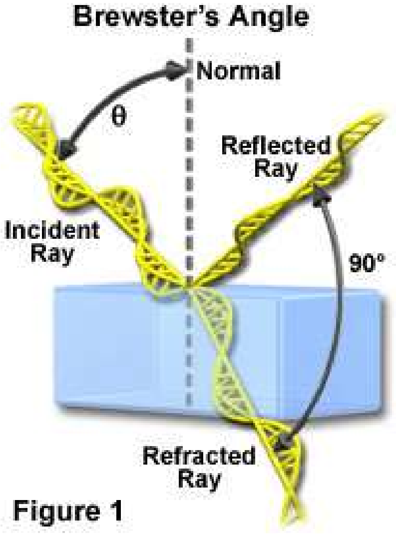
Water Vapor Rotational Lines

Choose microwave frequencies for cloud emissivity measurement where transmittance is high!!! Water vapor is variable; choose low frequency.



v



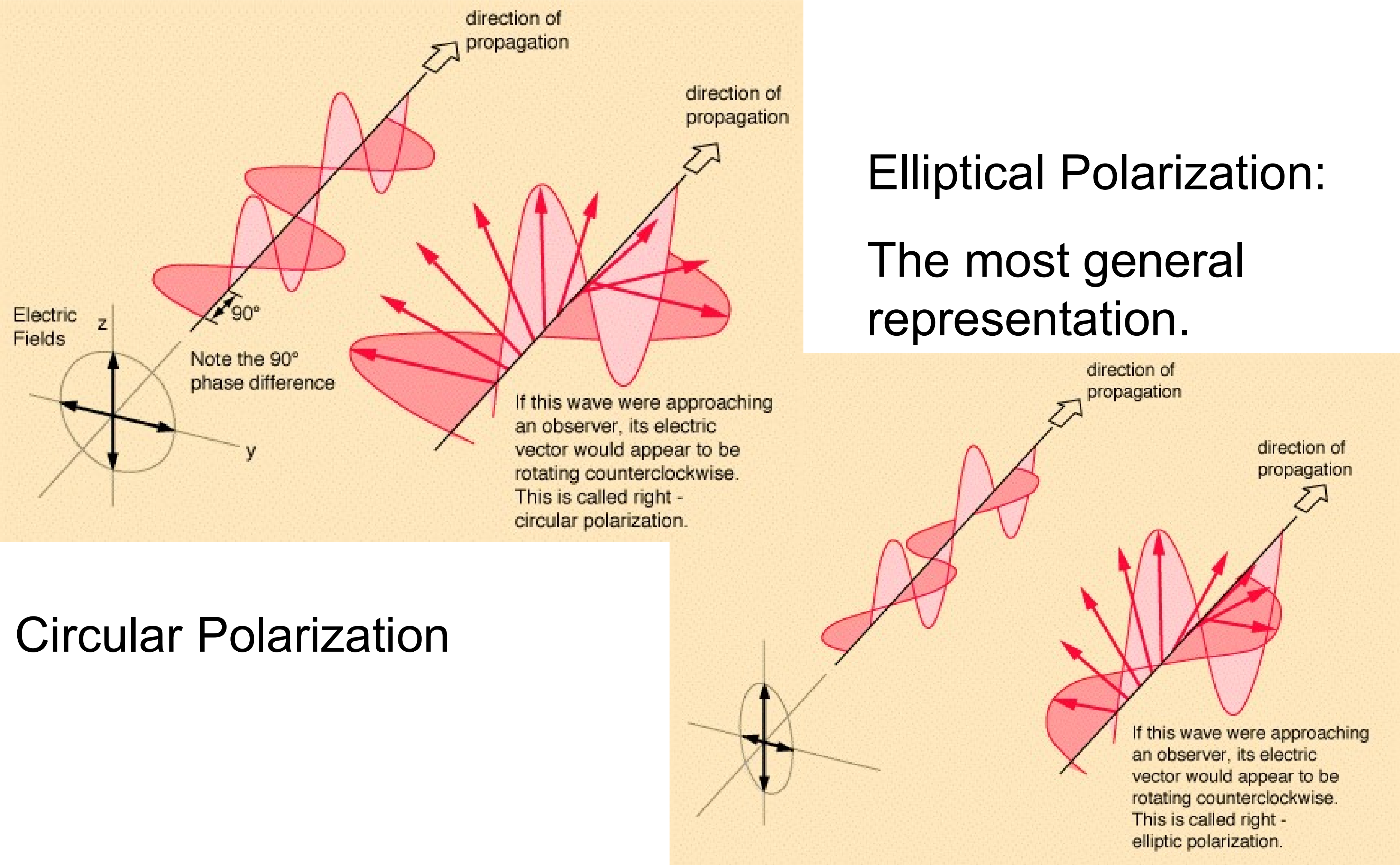


Slinky illustrates longitudinal and transverse waves. Polarization cards illustrate Brewster angle reflection from floor tiles with ‘natural light’ incident.

More Details on the Polarization States

Elliptical Polarization:

The most general representation.

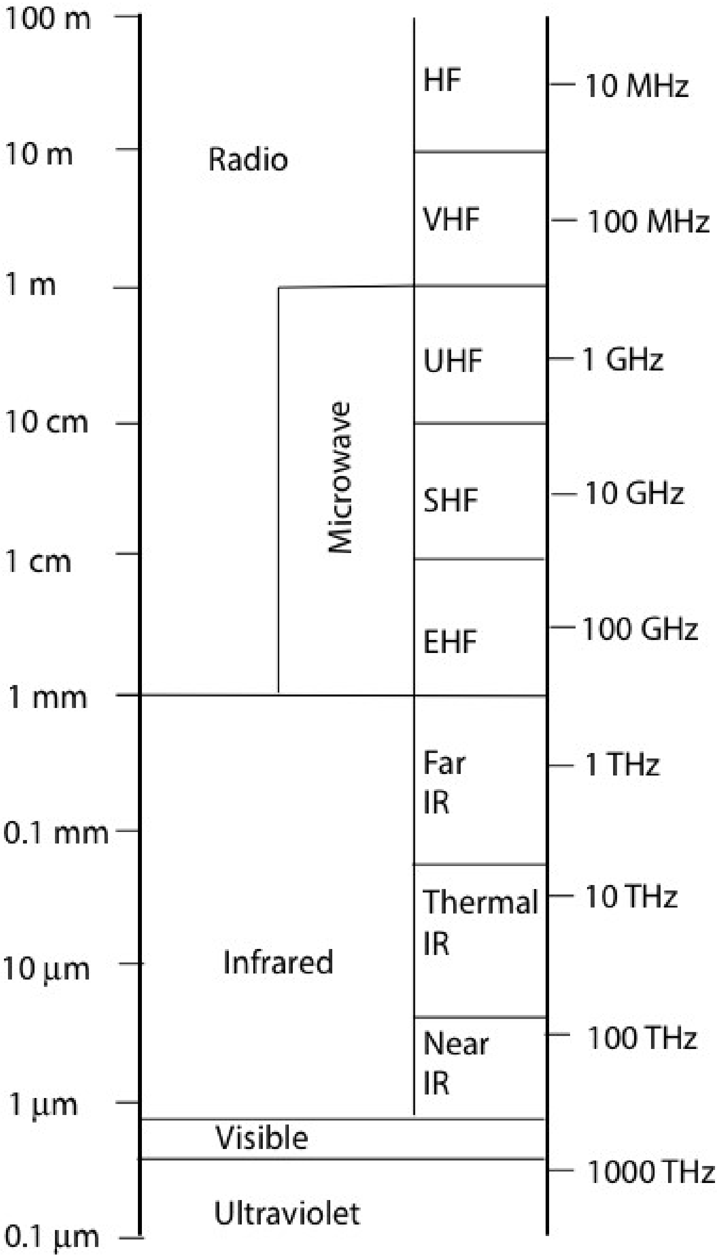


If this wave were approaching an observer, its electric vector would appear to be rotating counterclockwise. This is called right - circular polarization.

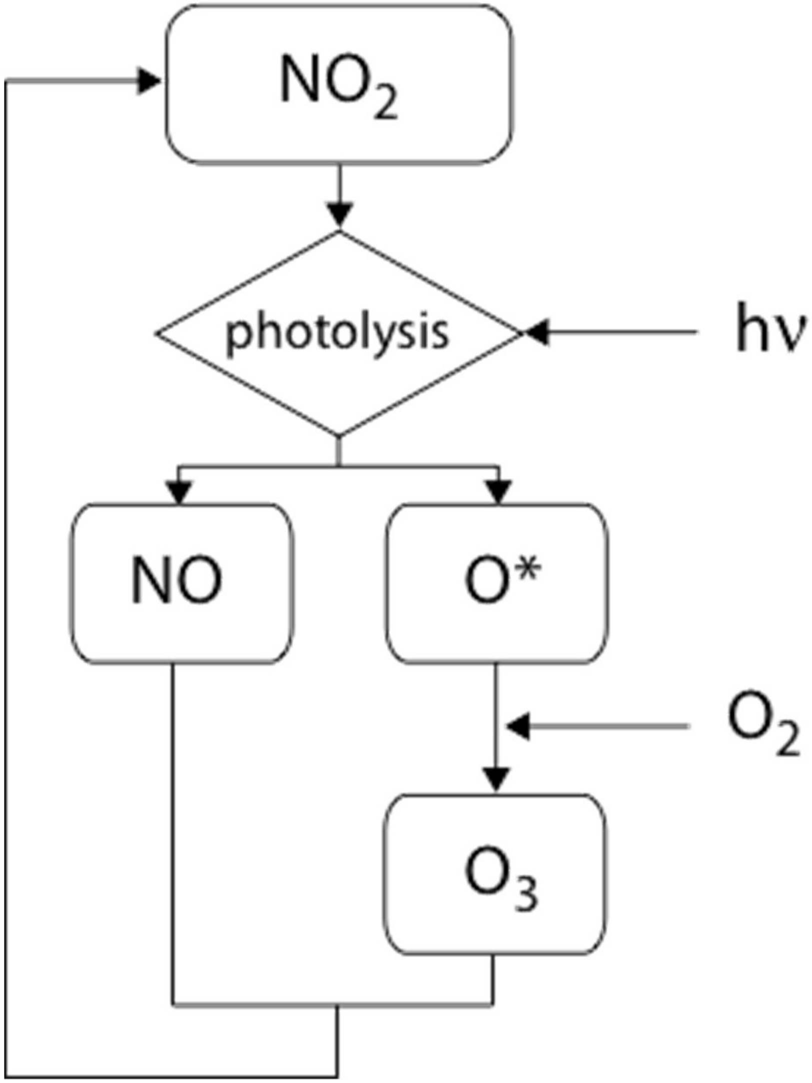
If this wave were approaching an observer, its electric vector would appear to be rotating counterclockwise. This is called right - elliptic polarization.

Circular Polarization

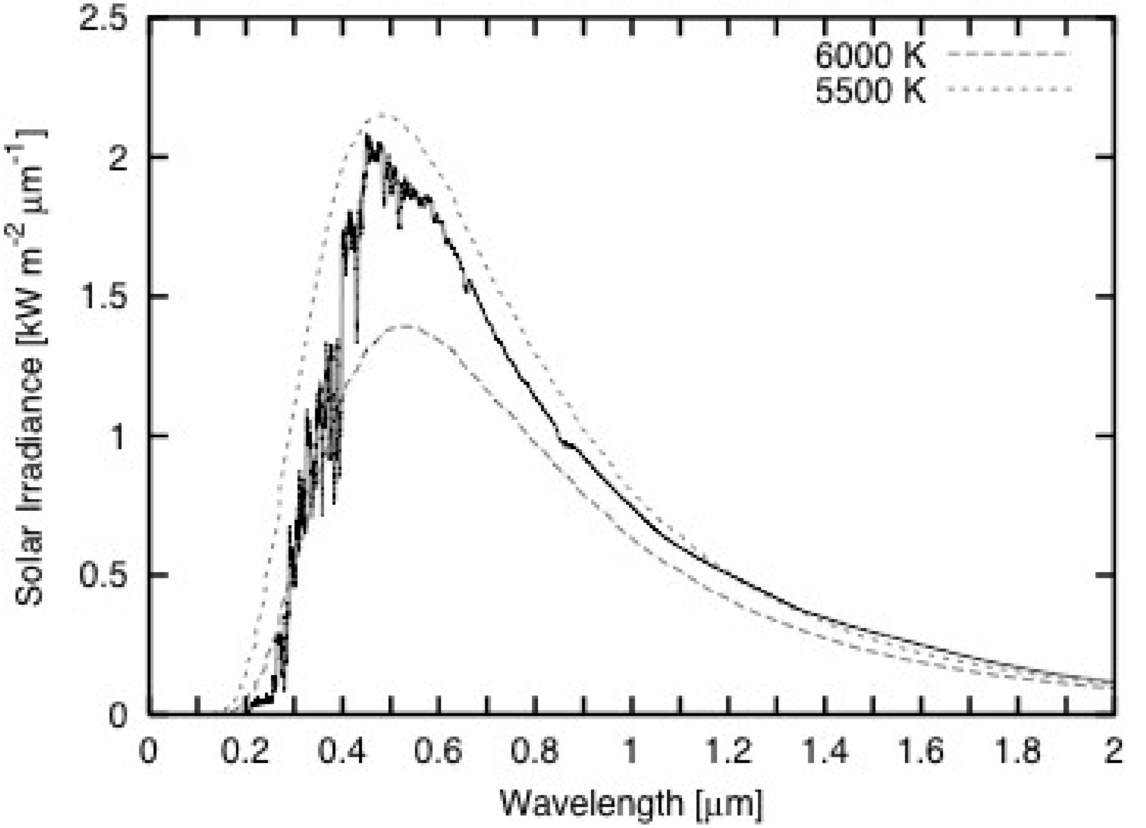
CH 3: The Electromagnetic Spectrum



What wavelengths are associated with sunburns?
What wavelengths 'break' NO_2?
What types of transitions are important for UV, VIS, IR, Microwave?

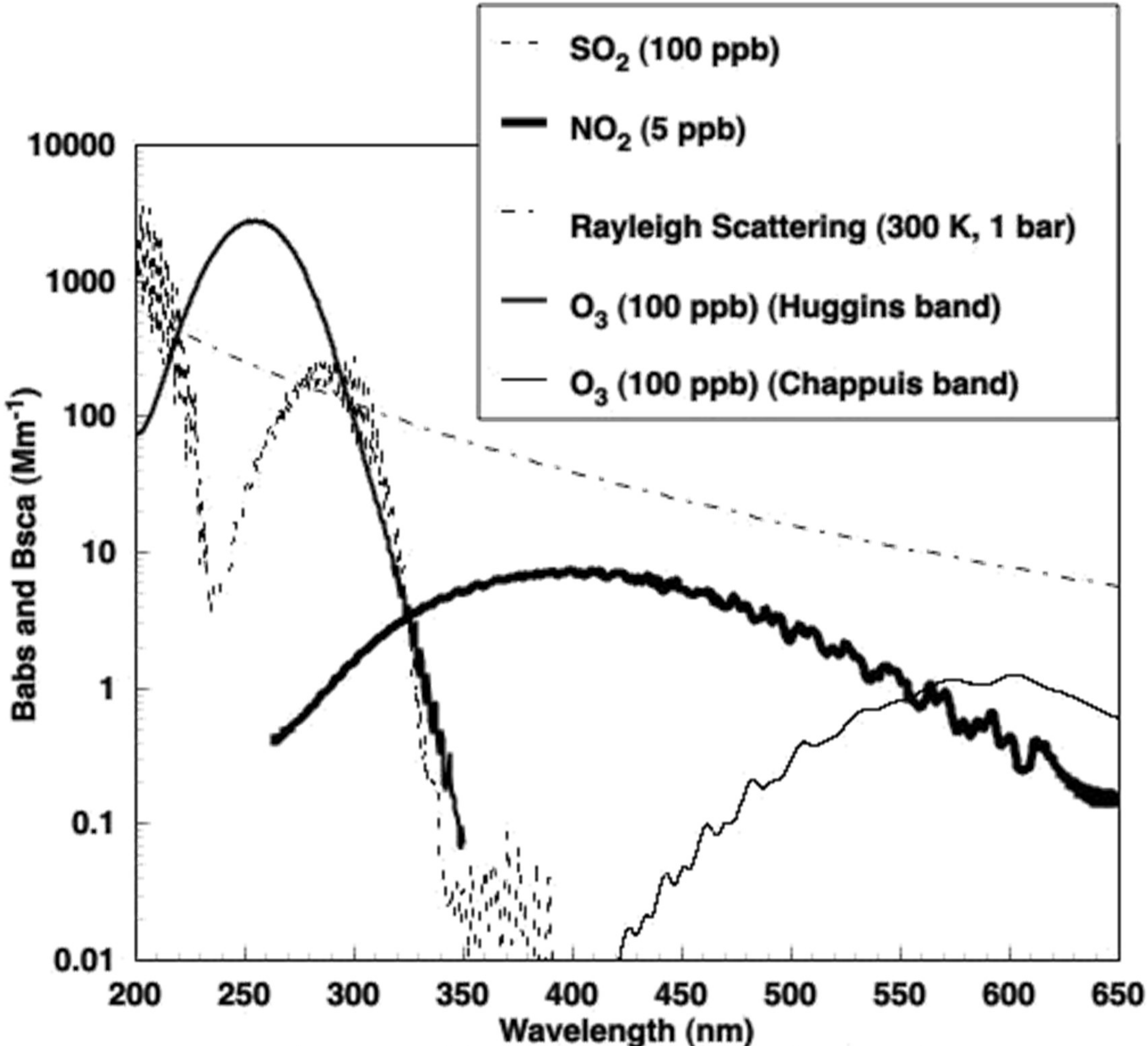


CH 3: The Electromagnetic Spectrum



Top of the Atmosphere Solar Radiation:
Fits to a black body between 5500 K and 6000 K.

Rayleigh Scattering In Perspective Relative to Absorption



Gaseous Absorption and Scattering Spectra

Absorption cross sections of O_3 and O_2 in the UV and Visible.

Strongly affects atmospheric chemistry, thermal structure, and amount of deadly UV that *doesn't* make it to the surface.

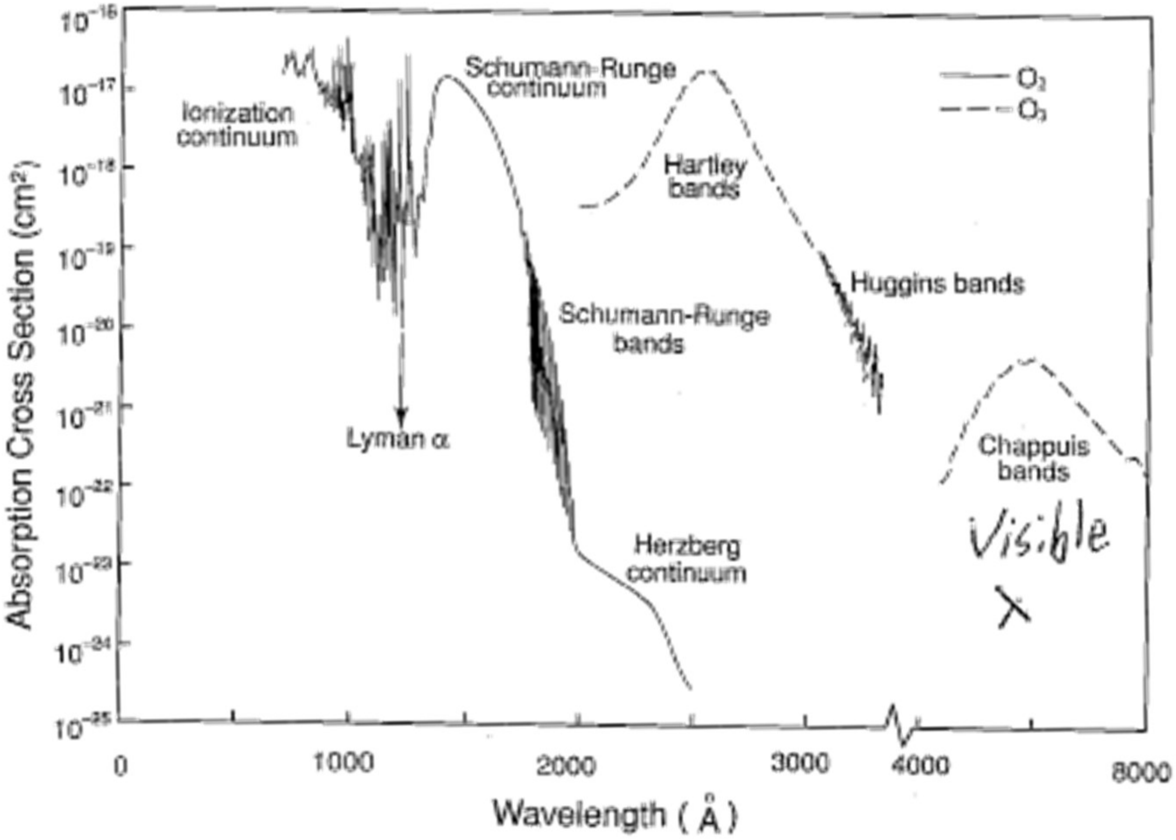
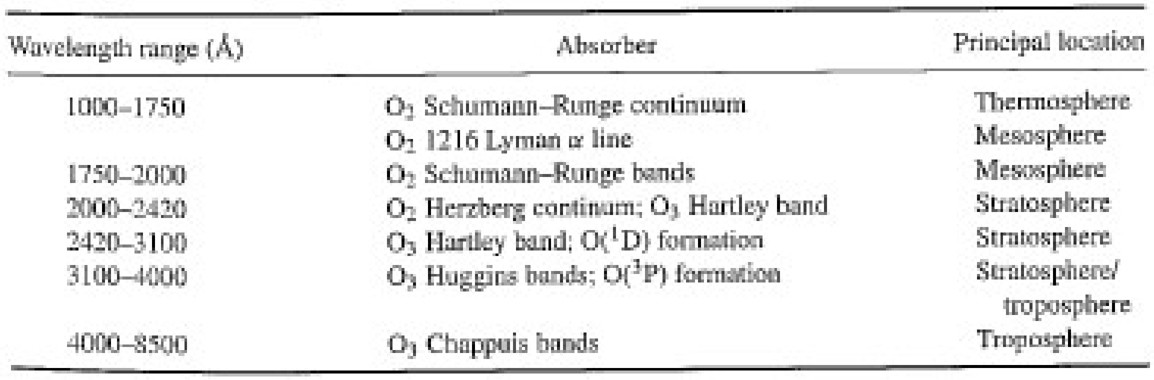


Figure 3.5 Absorption cross section of ozone and molecular oxygen in the ultraviolet spectral region. Data taken from Brasseur and Solomon (1986), Vigroux (1953), and Griggs (1968).

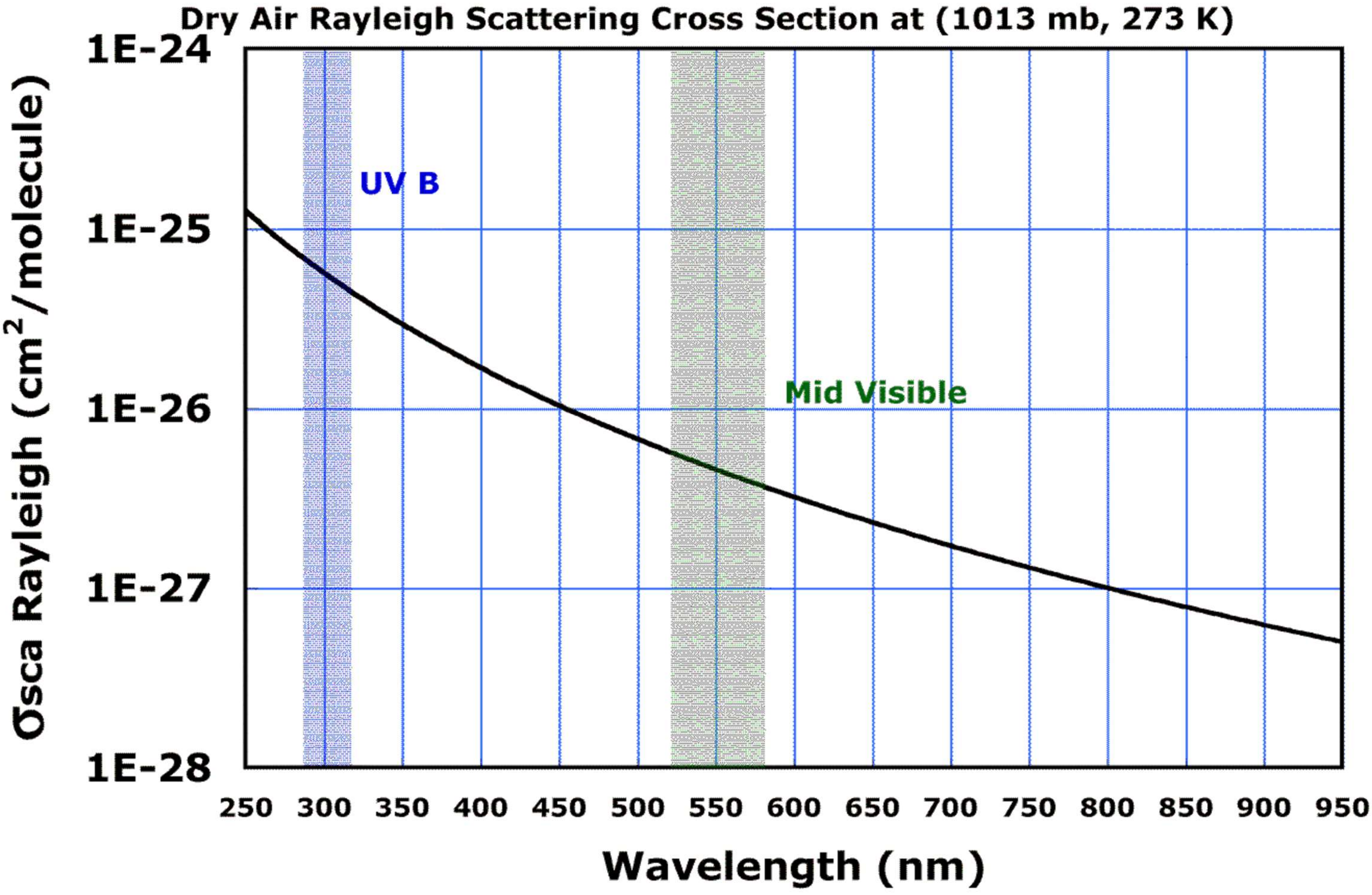
Table 3.2

Important Absorption Spectral Regions Associated with Photochemistry in the Atmosphere



Wavelength range (Å)	Absorber	Principal location
1000–1750	O_2 Schumann–Runge continuum	Thermosphere
	O_2 1216 Lyman α line	Mesosphere
1750–2000	O_2 Schumann–Runge bands	Mesosphere
2000–2420	O_2 Herzberg continuum; O_3 Hartley band	Stratosphere
2420–3100	O_3 Hartley band; O(^{1}D) formation	Stratosphere
3100–4000	O_3 Huggins bands; O(^{3}P) formation	Stratosphere/ troposphere
4000–8500	O_3 Chappuis bands	Troposphere

Rayleigh Scattering (light scattering by air as dipole radiation)



Atmospheric Temperature Profile: US “Standard” Atmosphere.

66 **3 Absorption and Scattering of Solar Radiation in the Atmosphere**

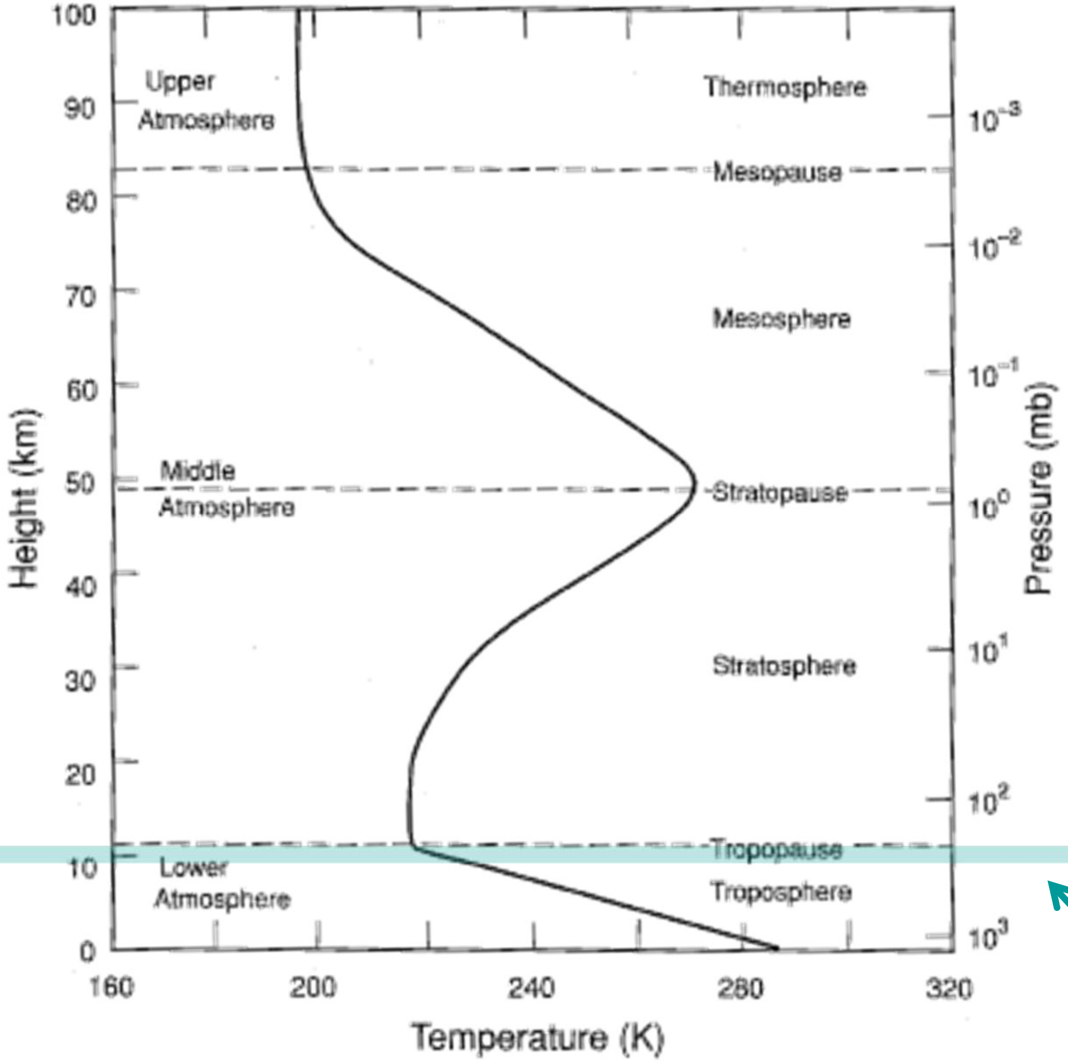
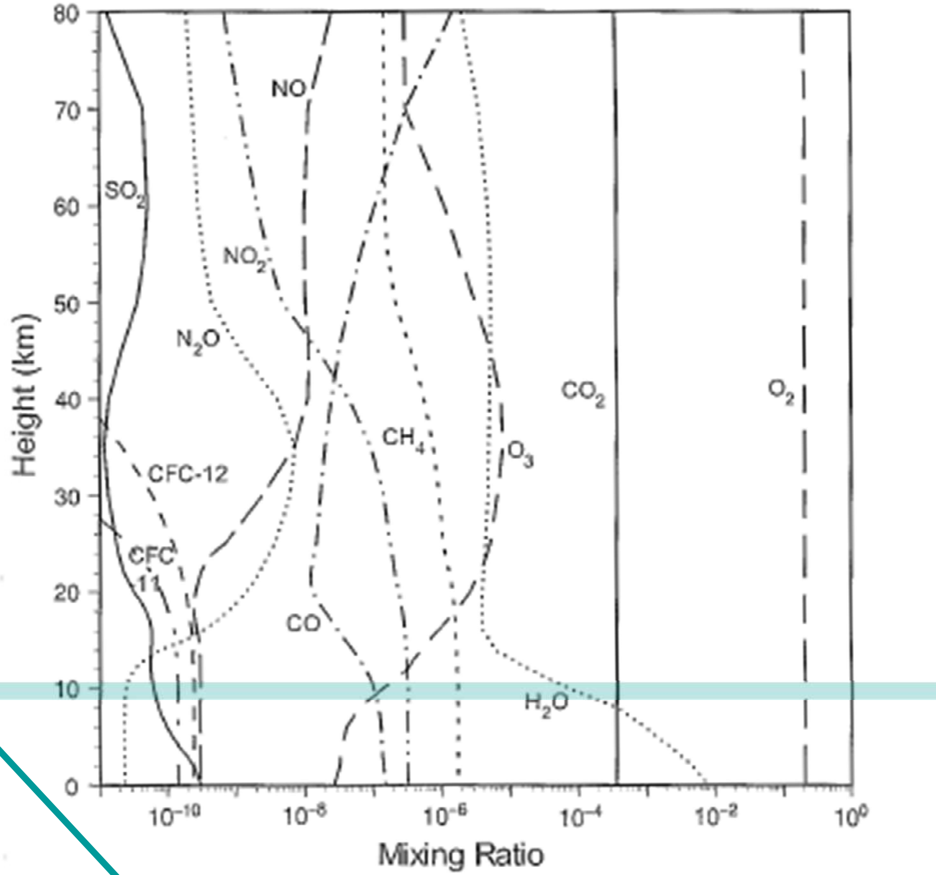


Figure 3.1 Vertical temperature profile after the U.S. Standard Atmosphere and definitions of atmospheric nomenclature.



From Liou

Cirrus cloud level.
High cold clouds, visible optical depth range 0.001 to 10, emits IR to surface in the IR window.

Solar Spectrum, Top of the Atmosphere and at the Surface

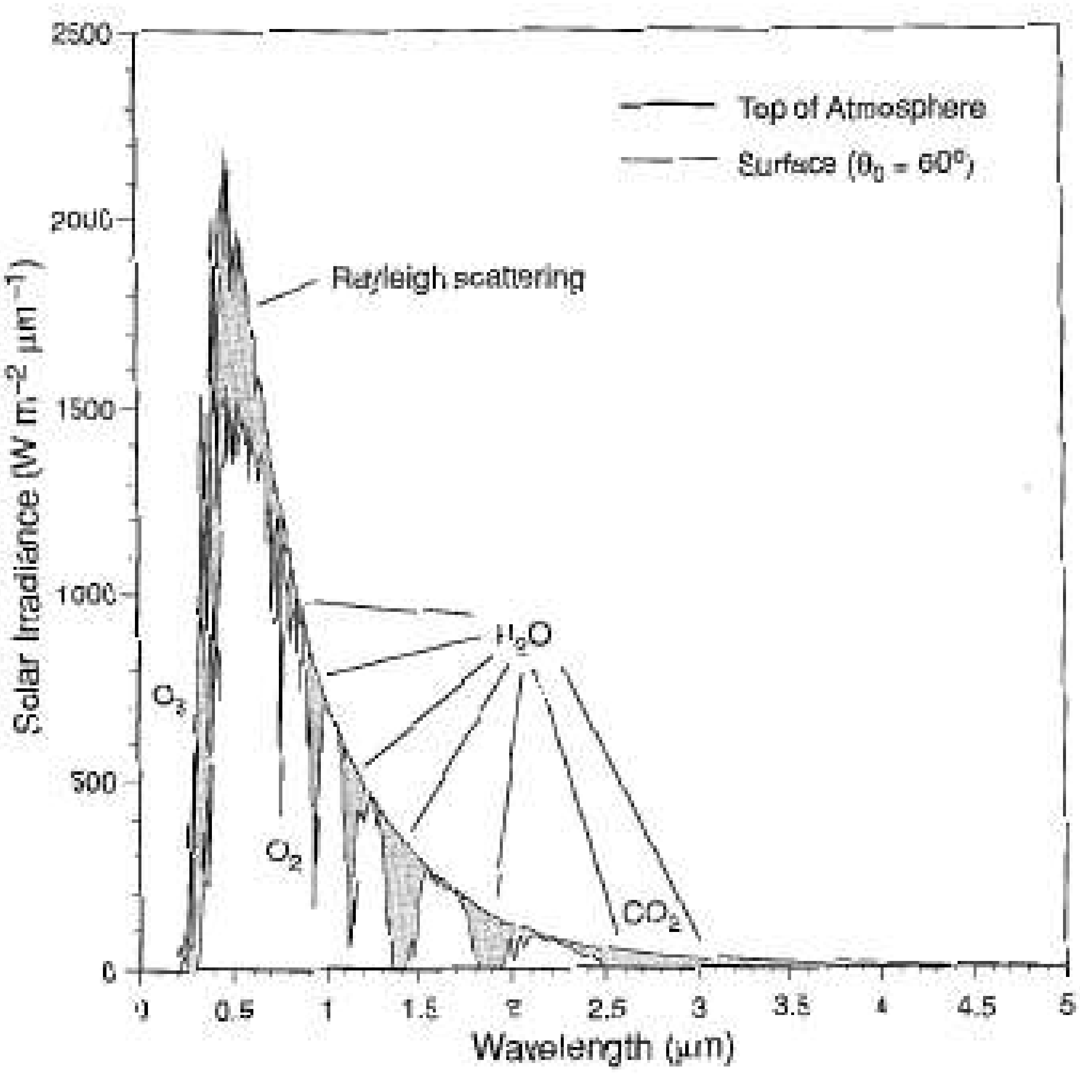


Figure 3.9 Solar irradiance curve for a 50 cm^{-1} spectral interval at the top of the atmosphere (see Fig. 2.9) and at the surface for a solar zenith angle of 60° in an atmosphere without aerosols or clouds. Absorption and scattering regions are indicated. See also Table 3.3 for the absorption of N_2O, CH_4, CO and NO_2.

Shaded region is solar irradiance removed by Rayleigh scattering and absorption by gases as indicated. (from Liou).

Electronic, Vibrational, energy levels and the big break up (dissociation level)

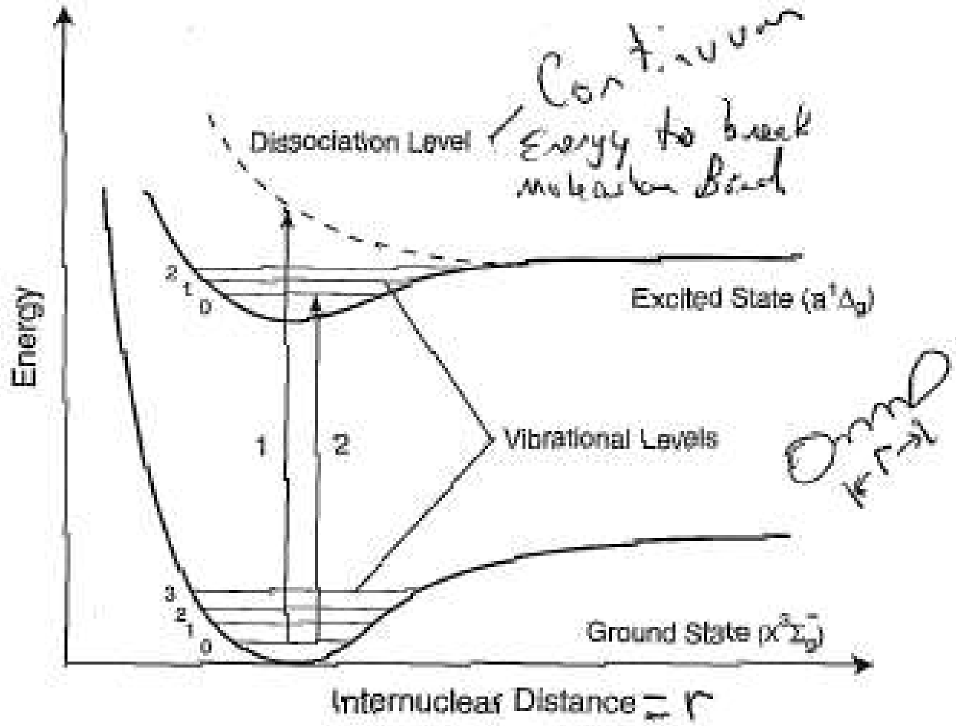
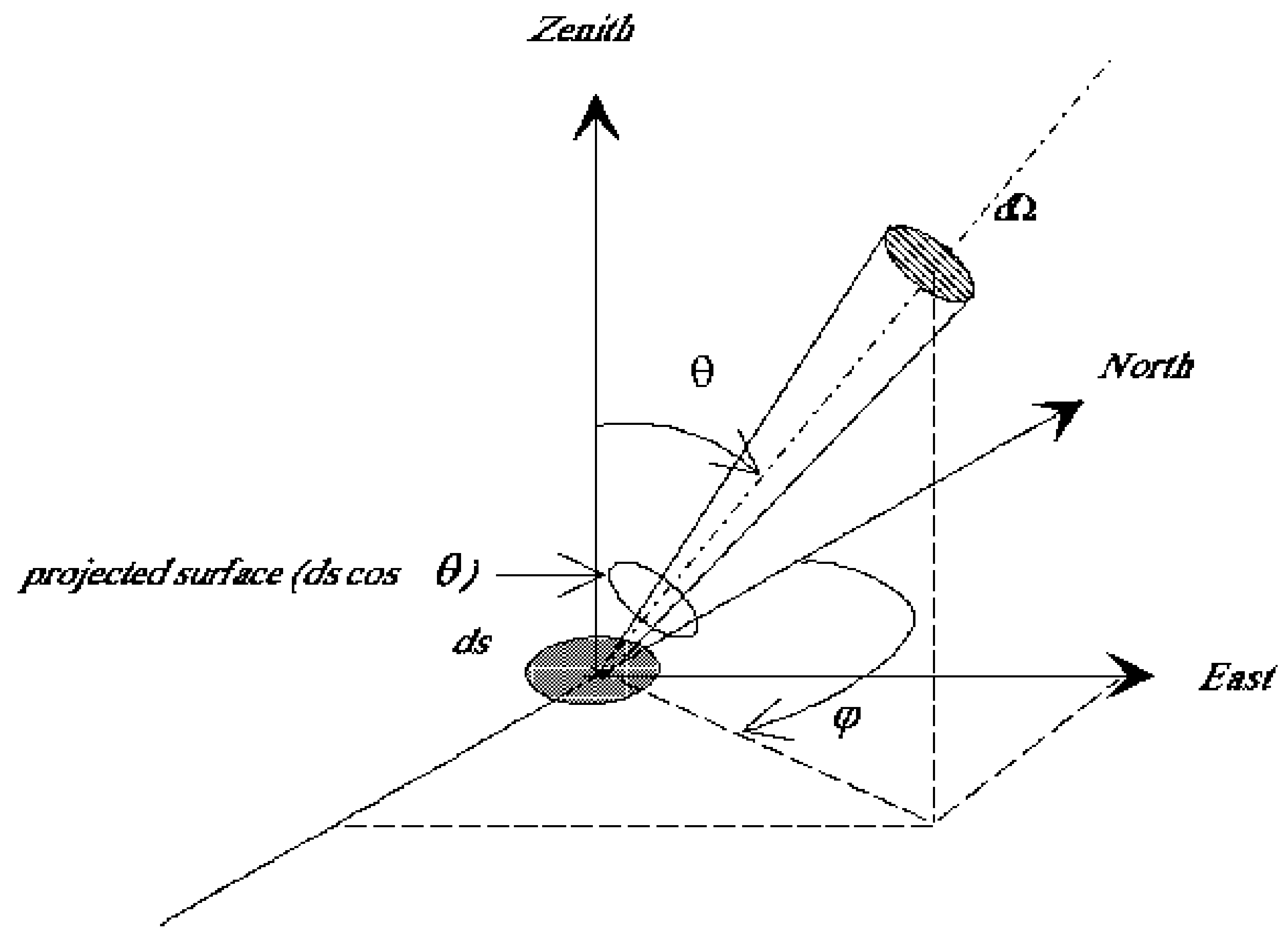


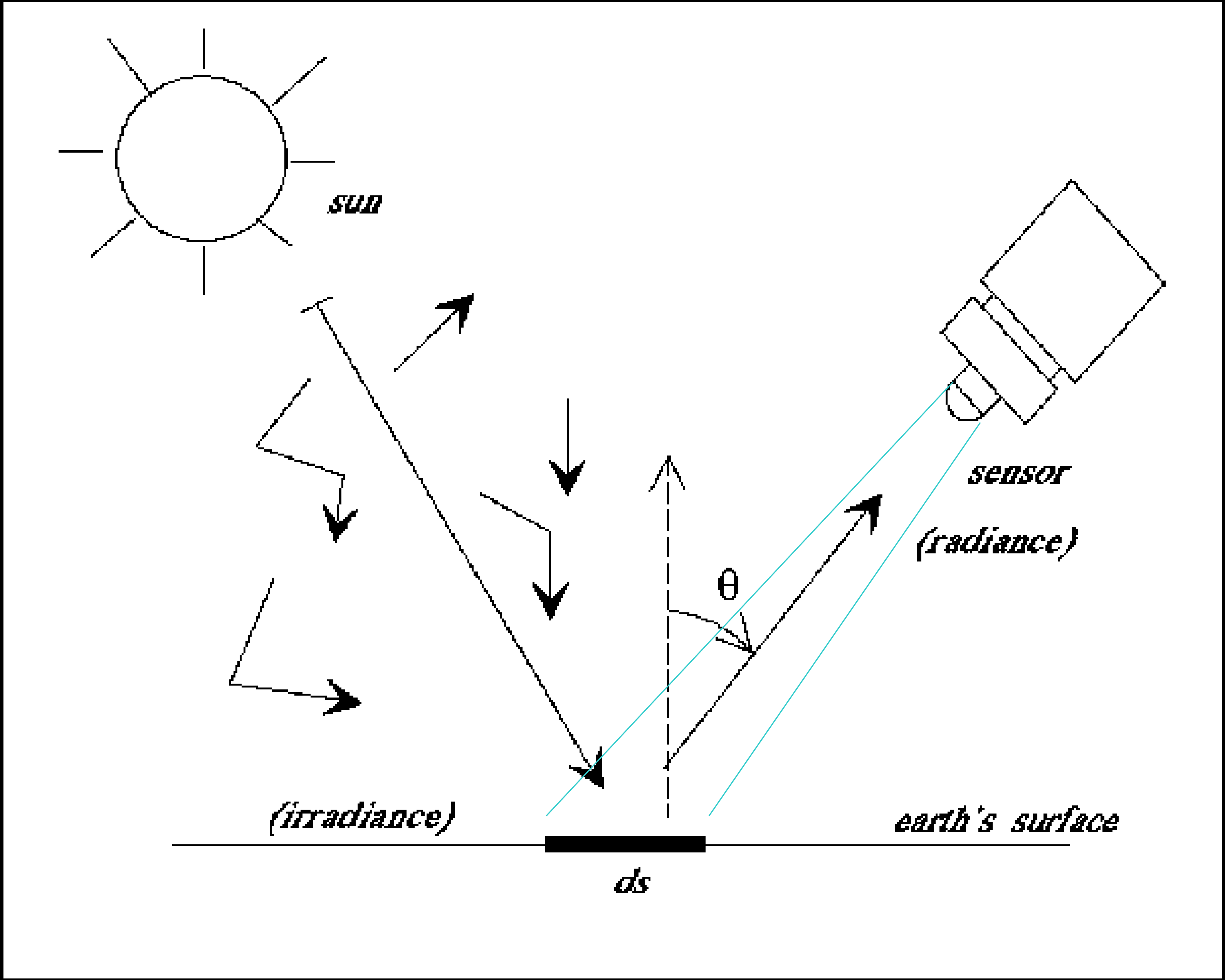
Figure 3.4 Illustrative potential energy curves for two electronic states of a diatomic molecule. The horizontal lines in the potential well represent vibrational energy levels.

From Liou

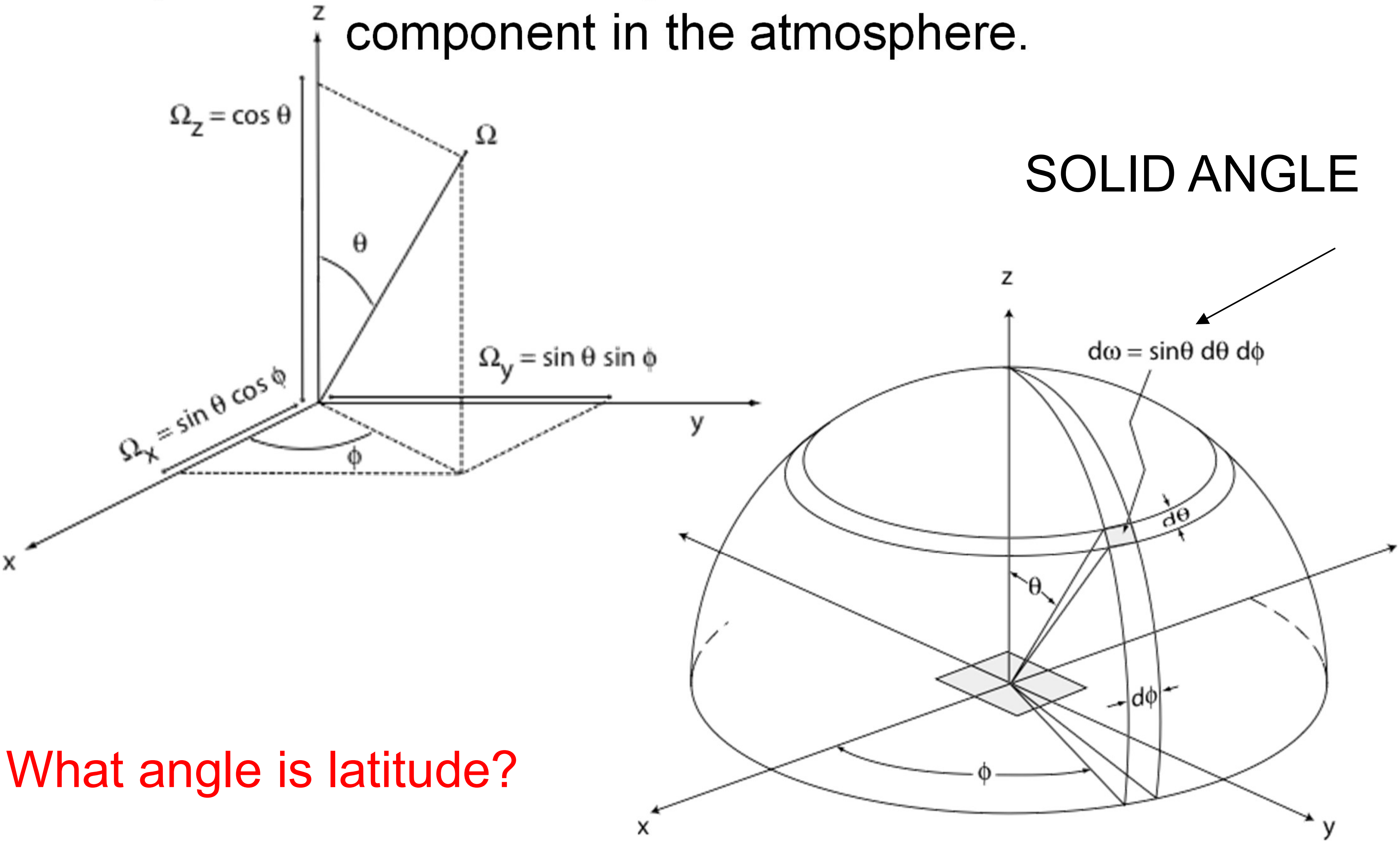
Radiant Intensity or Radiance: Watts / (m^2 Sr)



Flux (also Irradiance) and Radiant Intensity (Radiance)



Spherical Coordinate System: z axis is the vertical component in the atmosphere.



SOLID ANGLE

What angle is latitude?

Spherical Coordinate System: z axis is the vertical component in the atmosphere: Another view.

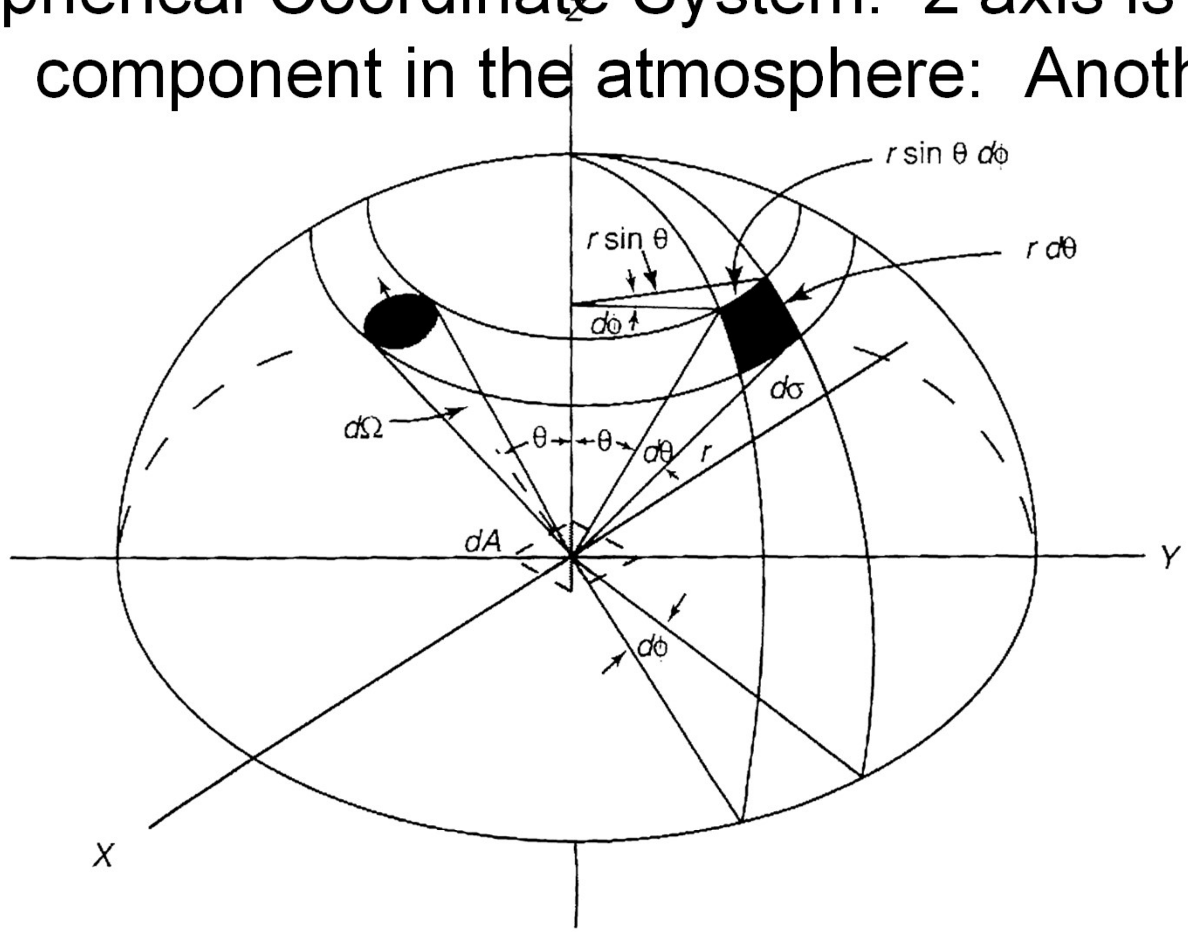


Figure 1.3 Illustration of a differential solid angle and its representation in polar coordinates. Also shown for demonstrative purposes is a pencil of radiation through an element of area dA in directions confined to an element of solid angle $d\Omega$. Other notations are defined in the text.

Hence, the differential solid angle is

$$d\Omega = d\sigma / r^2 = \sin\theta \, d\theta \, d\phi, \qquad (1.1.5)$$

Flux (irradiance) as a distribution function and broadband quantity. Purpose: Describe radiation in particular direction such as net downward, net upward, etc.

$$F(\lambda_1, \lambda_2) = \int_{\lambda_1}^{\lambda_2} F_\lambda \; d\lambda$$

$$SI\ Dimensions:$$

$$\frac{W}{m^2} = \frac{W}{m^2 \; nm} \; nm$$

Radiant Intensity Definition (also known as Radiance)

Purpose: Describe radiation from all and any direction.
It is also a distribution function with respect to wavelength (or frequency, or wavenumber, depending on the orientation).

$$I(\widehat{\Omega}, \lambda) = \frac{\delta F(\lambda)}{\delta \omega}$$

$$SI\ UNITS:$$

$$\frac{Watts}{m^2\ nm\ steradian}$$

Flux and Radiant Intensity Relationships

UPWARD FLUX (example, outgoing flux at the top of the atmosphere)

$$F^{\uparrow}(\lambda) = \int_{2\pi} I^{\uparrow}(\widehat{\Omega},\lambda)\ \hat{n} \bullet \widehat{\Omega}\ d\omega = \int_{0}^{2\pi}\int_{0}^{\pi/2} I^{\uparrow}(\theta,\phi,\lambda)\cos(\theta)\sin(\theta)d\theta d\phi$$

DOWNWARD FLUX (example, downward flux at the ground)

$$F^{\downarrow}(\lambda) = \int_{2\pi} I^{\downarrow}(\widehat{\Omega},\lambda)\ \hat{n} \bullet \widehat{\Omega}\ d\omega = -\int_{0}^{2\pi}\int_{\pi/2}^{\pi} I^{\downarrow}(\theta,\phi,\lambda)\cos(\theta)\sin(\theta)d\theta d\phi$$

NET FLUX DEFINITION (divergence of this quantity gives heating rate)

$$F^{net}(\lambda) = F^{\uparrow}(\lambda) - F^{\downarrow}(\lambda) = \int_{0}^{2\pi}\int_{0}^{\pi} I(\theta,\phi,\lambda)\cos(\theta)\sin(\theta)d\theta d\phi = \int_{4\pi} I^{\uparrow}(\widehat{\Omega},\lambda)\ \hat{n} \bullet \widehat{\Omega}\ d\omega$$

SPECIAL CASE : ISOTROPIC RADIATION LIKE BLACKBODY RADIATION

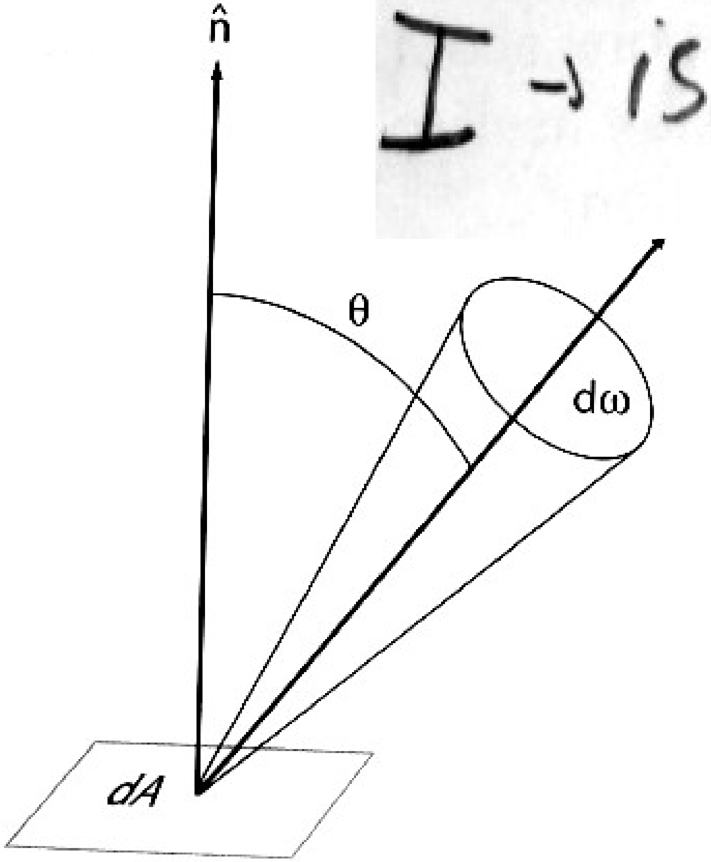
$$F^{\uparrow}(\lambda) = I^{\uparrow}(\lambda) \int_{0}^{2\pi}\int_{0}^{\pi/2} \cos(\theta)\sin(\theta)d\theta d\phi = \pi\ I^{\uparrow}(\lambda)$$

←

Prove this relation…

$I \rightarrow$ isotropic

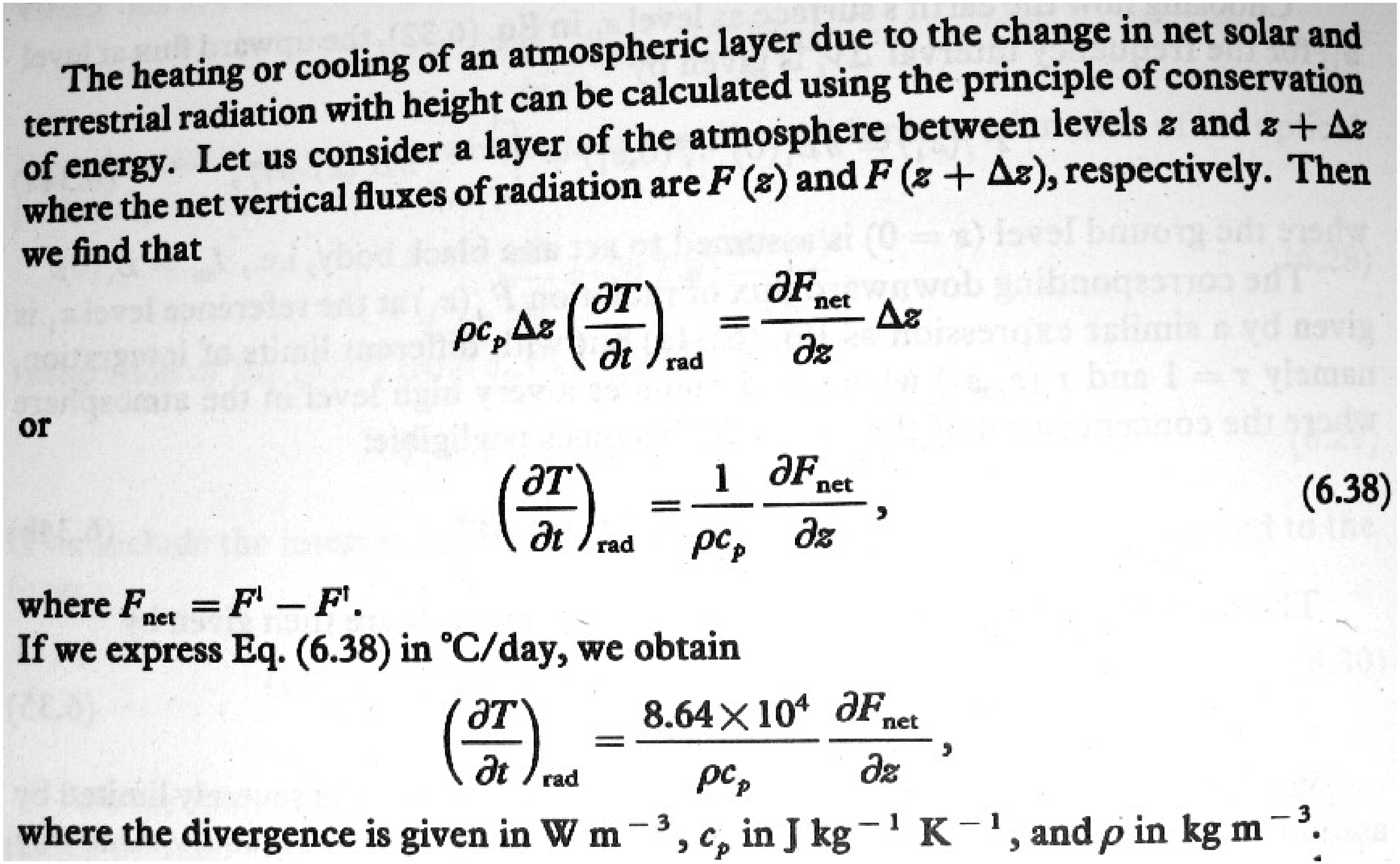
$$\boxed{F = \pi I \, .}$$



Special case: I isotropic, same in all directions, like black body radiation from a surface.

$$\boxed{F^{\uparrow} = \int_0^{2\pi} \int_0^{\pi/2} I^{\uparrow}(\theta, \phi) \cos\theta \sin\theta \, d\theta d\phi \, ,}$$

THE BIG PICTURE: Radiation Heating of the Atmosphere



The heating or cooling of an atmospheric layer due to the change in net solar and terrestrial radiation with height can be calculated using the principle of conservation of energy. Let us consider a layer of the atmosphere between levels z and $z + \Delta z$ where the net vertical fluxes of radiation are $F(z)$ and $F(z + \Delta z)$, respectively. Then we find that

$$\rho c_p \Delta z \left(\frac{\partial T}{\partial t}\right)_{\text{rad}} = \frac{\partial F_{\text{net}}}{\partial z} \Delta z$$

or

$$\left(\frac{\partial T}{\partial t}\right)_{\text{rad}} = \frac{1}{\rho c_p} \frac{\partial F_{\text{net}}}{\partial z}, \tag{6.38}$$

where $F_{\text{net}} = F^{\downarrow} - F^{\uparrow}$.

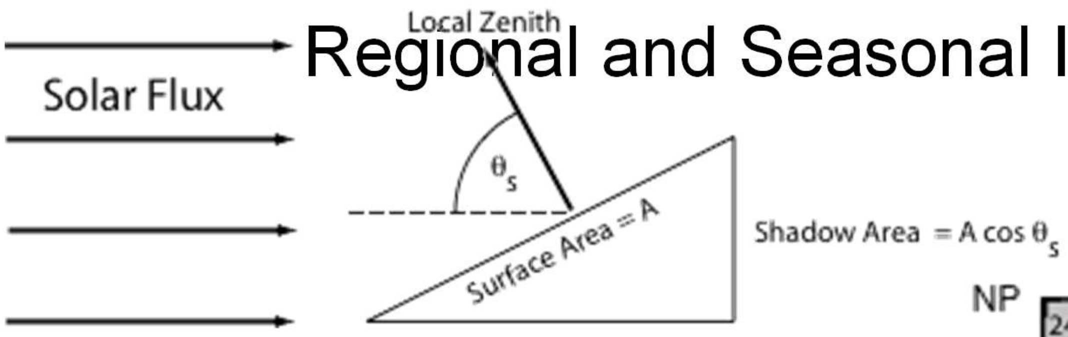
If we express Eq. (6.38) in °C/day, we obtain

$$\left(\frac{\partial T}{\partial t}\right)_{\text{rad}} = \frac{8.64 \times 10^4}{\rho c_p} \frac{\partial F_{\text{net}}}{\partial z},$$

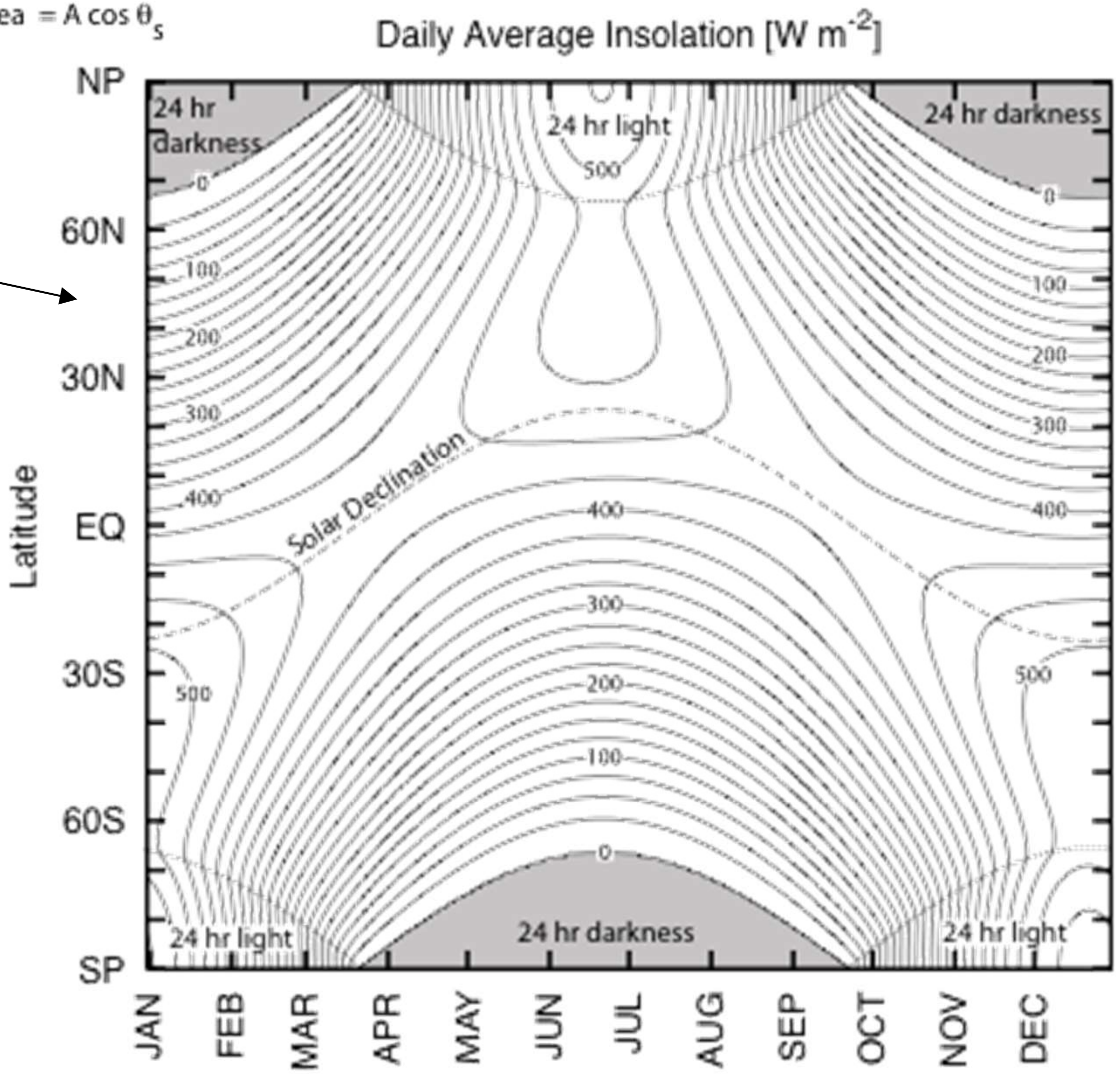
where the divergence is given in W m^{-3}, c_p in J kg^{-1} K^{-1}, and ρ in kg m^{-3}.

From Oort and Peixoto

Regional and Seasonal Insolation at the TOA



Normal Flux:



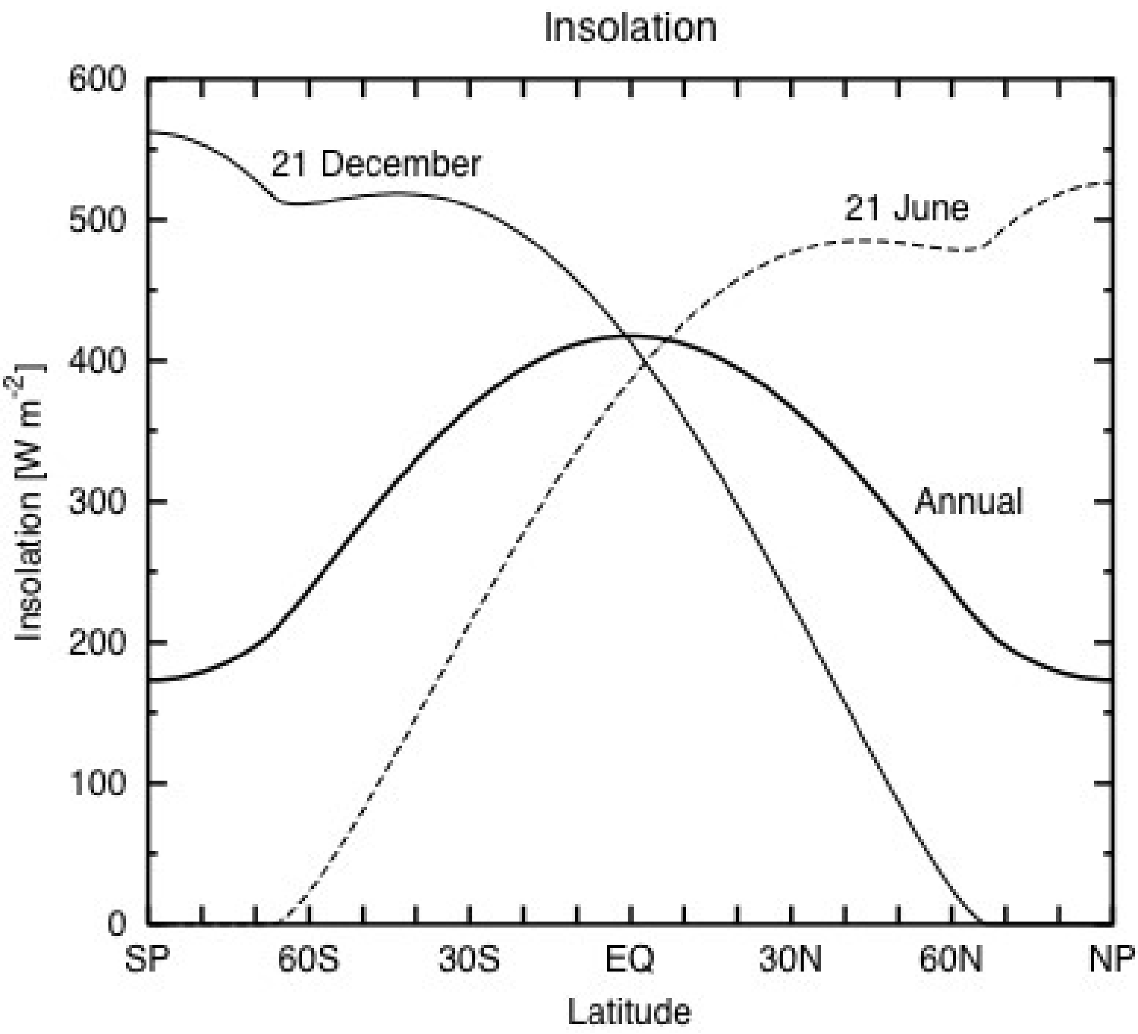
What is the range in Reno?

In Mexico City?

In Barrow Alaska?

Where is the peak? Why?

Insolation … l Average



What is the average insolation over all latitudes?

Sun Cross Section, Sunspots, and Nuclear Fusion

A sunspot is a region on the Sun's surface (photosphere) that is marked by a lower temperature than its surroundings and has intense magnetic activity, which inhibits convection, forming areas of reduced surface temperature. They can be visible from Earth without the aid of a telescope. **Although they are at temperatures of roughly 4000-4500 K, the contrast with the surrounding material at about 5800 K leaves them clearly visible as dark spots, as the intensity of a heated black body (closely approximated by the photosphere) is a function of T (temperature) to the fourth power.** If a sunspot was isolated from the surrounding photosphere it would be brighter than an electric arc. Source: Wikipedia.

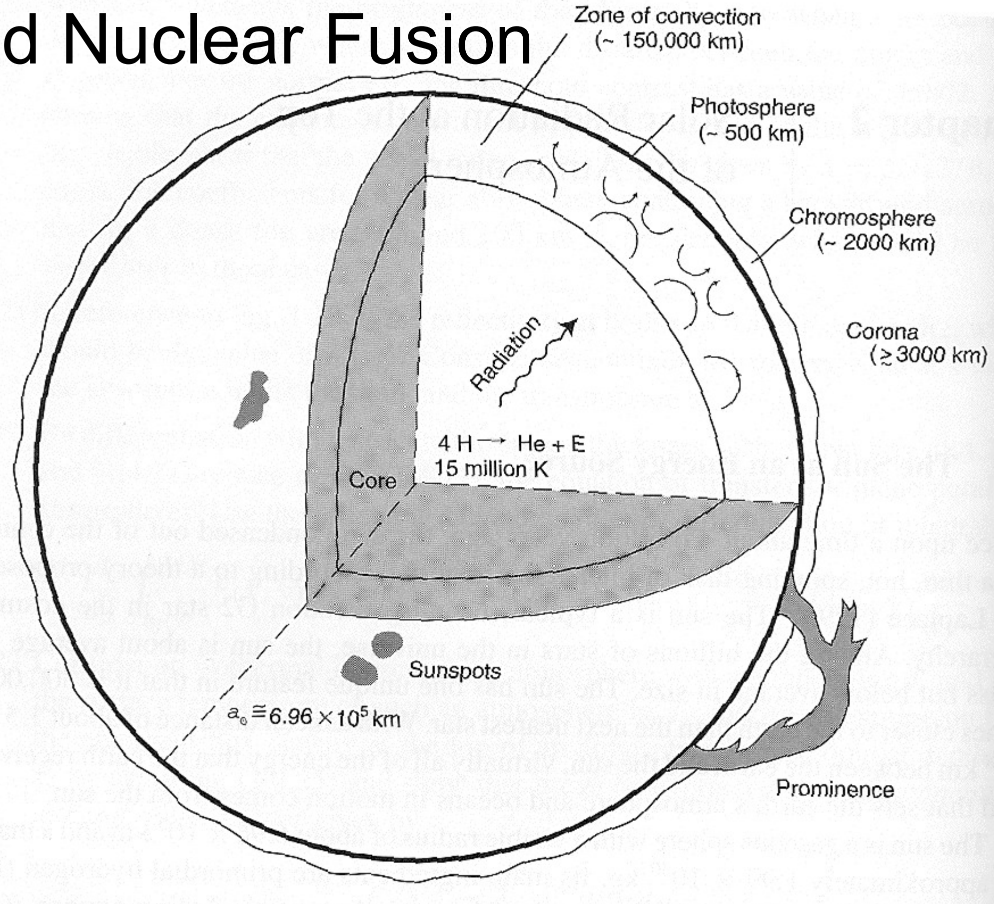
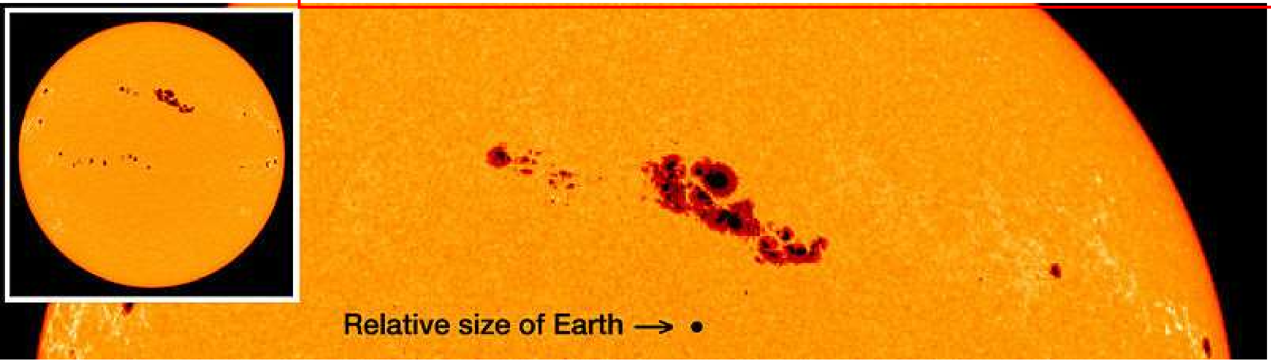
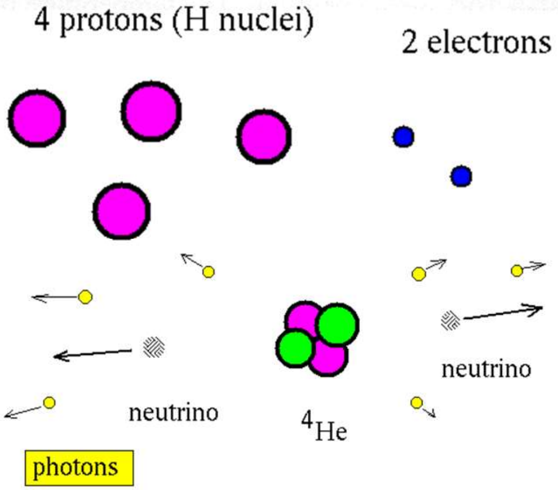


Figure 2.1 A cross section of the sun illustrating the solar interior and atmosphere. The solar interior includes the core with a temperature of about 1.5×10^7 K, the radiation zone, and the convective zone. The solar atmosphere includes the photosphere, the chromosphere, and the corona. The former two layers are exaggerated for illustration purposes.

4 ^{1}H + 2 e --> ^{4}He + 2 neutrinos + 6 photons





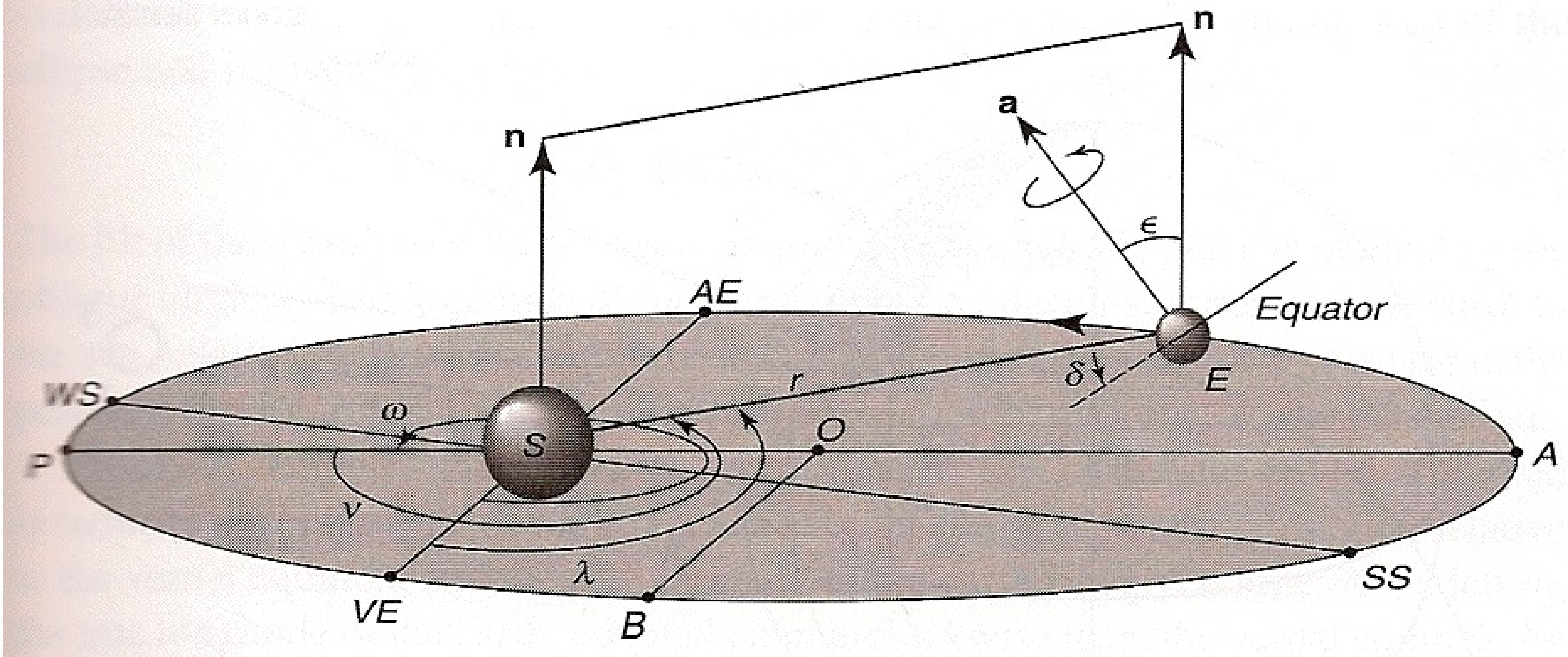
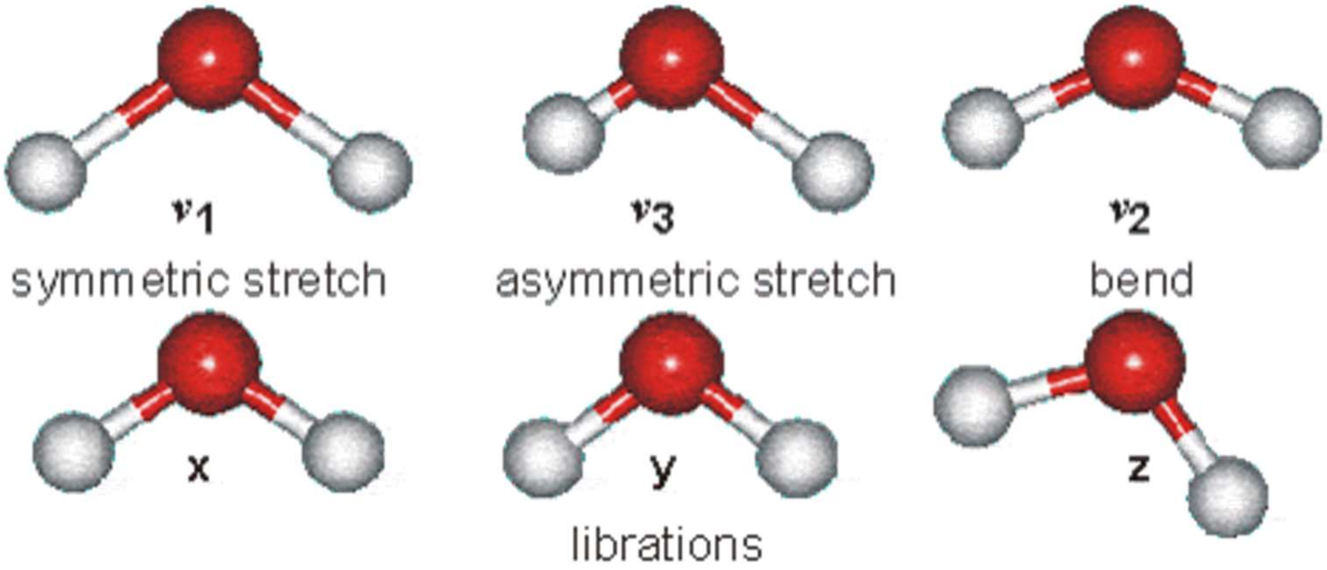


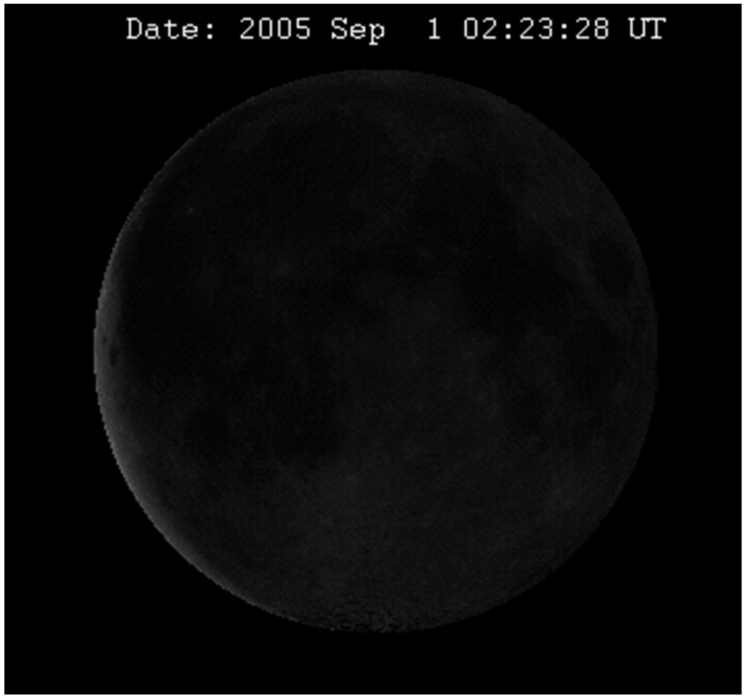
Figure 2.5 The earth–sun geometry. *P* denotes the perihelion, *A* the aphelion, *AE* the autumnal equinox, *VE* the vernal equinox, *WS* the winter solstice, and *SS* the summer solstice, **n** is normal to the ecliptic plane, **a** is parallel to the earth's axis, δ is the declination of the sun, ϵ the oblique angle of the earth's axis, ω the longitude of the perihelion relative to the vernal equinox, ν the true anomaly of the earth at a given time, λ the true longitude of the earth, *O* the center of the ellipse, *OA* (or $OP = a$) the semimajor axis, *OB* ($= b$) the semiminor axis, *S* the position of the sun, *E* the position of the earth, and *ES* ($= r$) the distance between the earth and the sun.

Some Energy States of Water Molecules

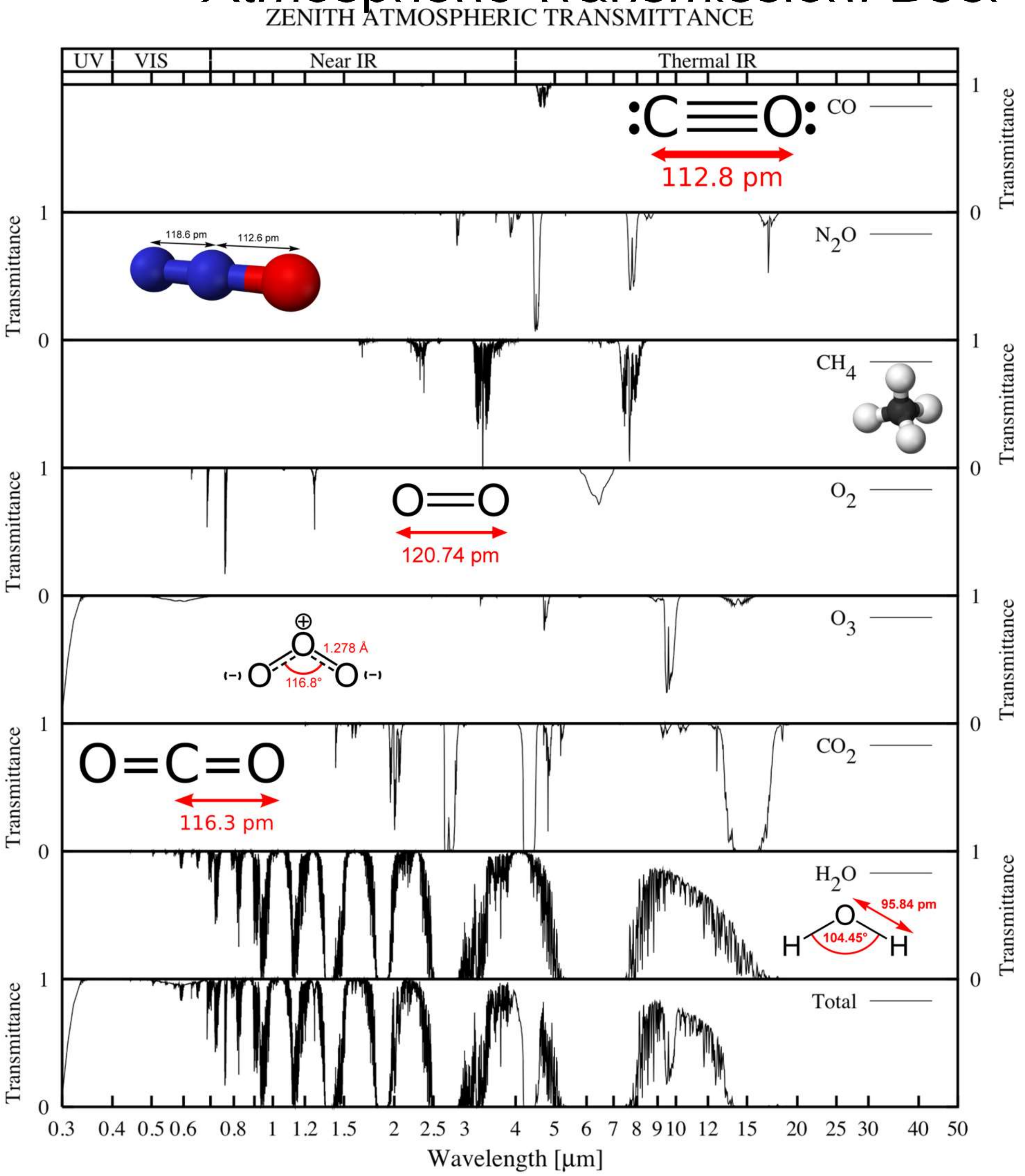


http://www.lsbu.ac.uk/water/vibrat.html

http://en.wikipedia.org/wiki/Libration



Atmospheric Transmission: Beer’s Law: $I(x)=I_0e^{(-\beta_{abs}\,x)}$



What are the main sources for each gas?

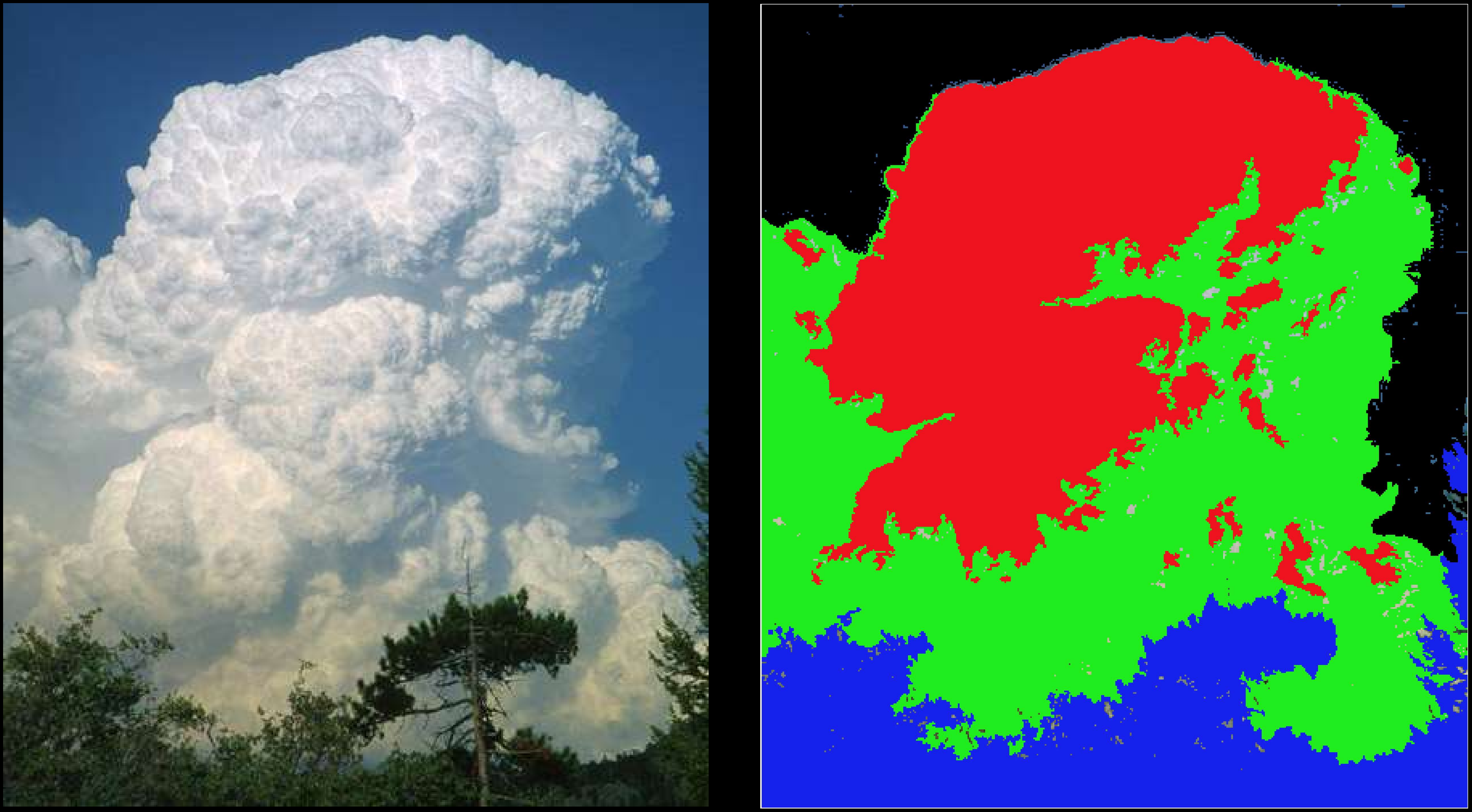
Which gases are infrared active and contribute to greenhouse warming?

Which gases significantly absorb solar radiation?

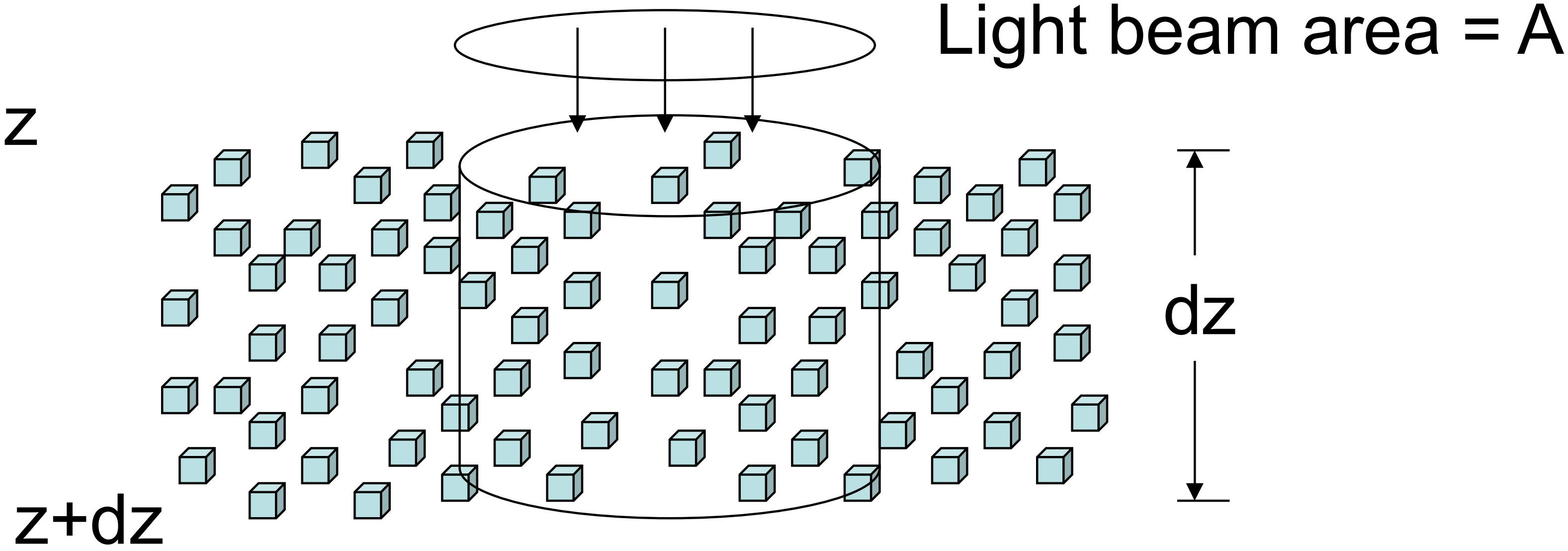
Gas concentrations from ‘typical’ midlatitude summer atmosphere.

Nitrous oxide is emitted by bacteria in soils and oceans, and thus has been a part of Earth's atmosphere for eons. Agriculture is the main source of human-produced nitrous oxide: cultivating soil, the use of nitrogen fertilizers, and animal waste handling can all stimulate naturally occurring bacteria to produce more nitrous oxide. The livestock sector (primarily cows, chickens, and pigs) produces 65% of human-related nitrous oxide. [1] Industrial sources make up only about 20% of all anthropogenic sources, and include the production of nylon and nitric acid, and the burning of fossil fuel in internal combustion engines. Human activity is thought to account for somewhat less than 2 teragrams of nitrogen oxides per year, nature for over 15 teragrams.

Clouds at Visible and IR (e.g. 10 um) Wavelengths



Optics of N identical (particles / volume)



Power removed in dz: = I(z) N A dz σ_{ext}

$$\left(I(z)-I(z+dz)\right)A = I(z)\ N\ \ A\ dz\ \sigma_{ext}$$

$$-dI = I(z)\ N\ \sigma_{ext}\ \ dz$$

$$\int_{I_0}^{I(z)} \frac{dI}{I} = -\int_0^z N\ \sigma_{ext}\ \ dz'\ ,\ \ln\left(\frac{I(z)}{I_0}\right) = -N\ \sigma_{ext}\ \ z$$

$$I(z) = I_0 \exp(-N\ \sigma_{ext}\ \ z) = I_0 \exp(-\beta_{ext}\ \ z)$$

Bouger-Beer
“law”
(direct beam only!)

CH 8: ATMOSPHERIC EMISSION: PRACTICAL CONSEQUENCES OF THE **SCHWARZSCHILD EQUATION** FOR RADIATION TRANSFER WHEN SCATTERING IS NEGLIGIBLE

What process subtracts radiation?
What process adds radiation?

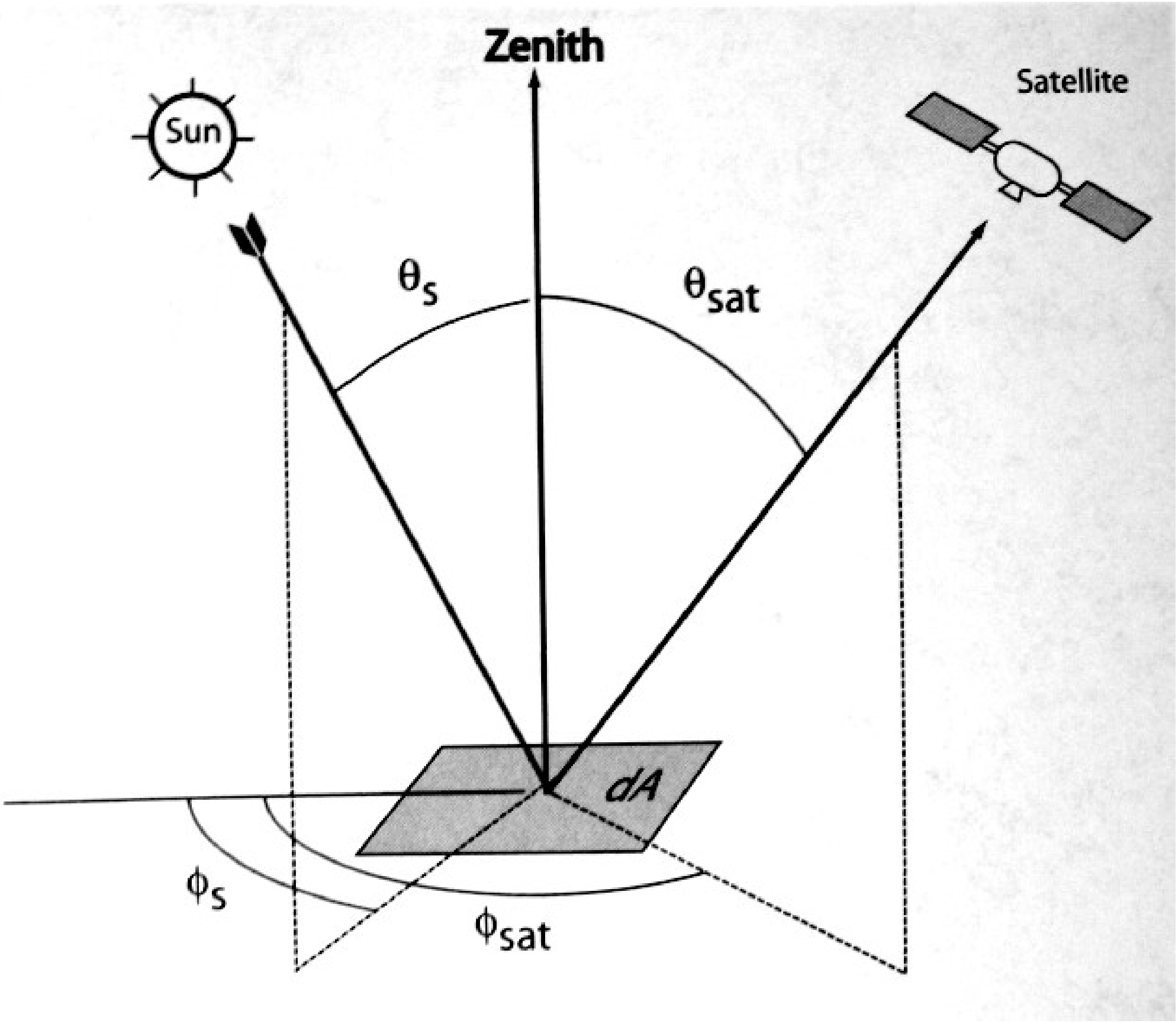
$$\boxed{\frac{dI}{ds} = \beta_a(B - I)\,.} \tag{8.4}$$

What equation is used to calculate optical depth for a gaseous atmosphere?

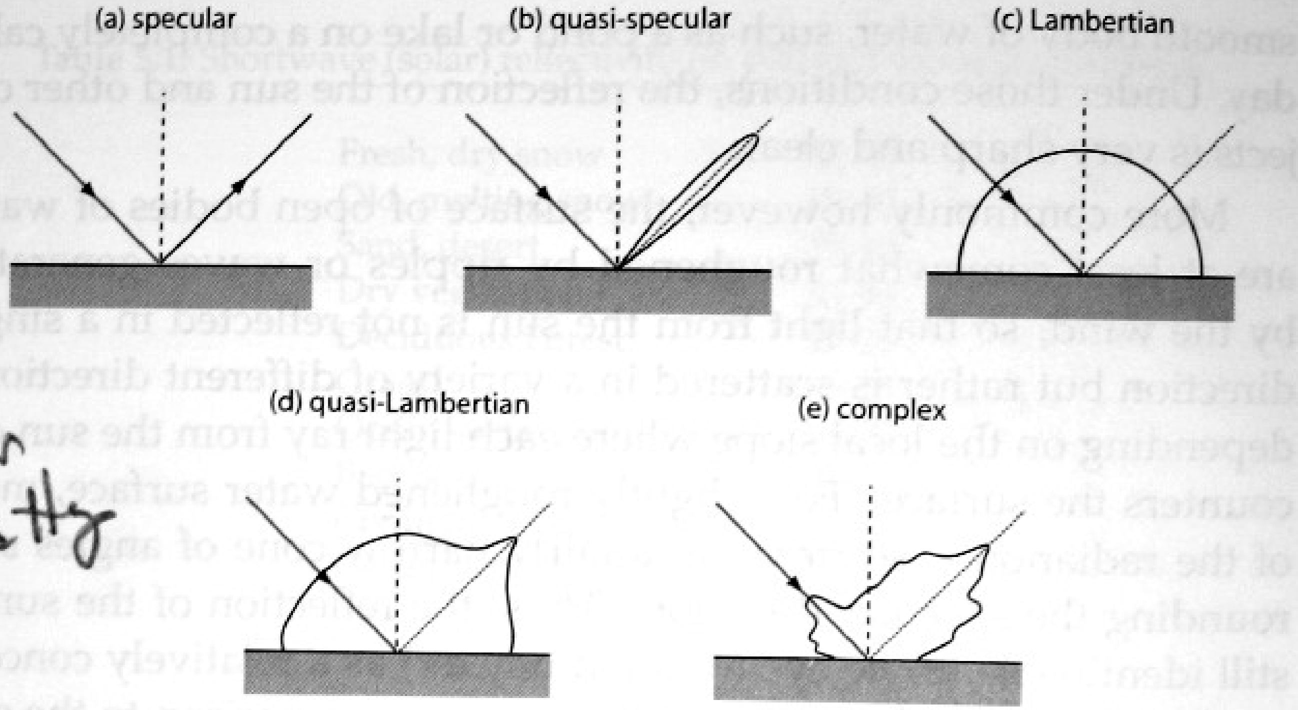
$$\boxed{I(0) = I(\tau')e^{-\tau'} + \int_0^{\tau'} Be^{-\tau}\,d\tau\,.} \tag{8.13}$$

Key Point: *Almost all common radiative transfer problems involving emission and absorption in the atmosphere (without scattering) can be understood in terms of (8.13)!*[2]

Sun and Satellite Perspective: How do the properties of the surface affect what we see?



Radiance and Irradiance: How do we define radiation?

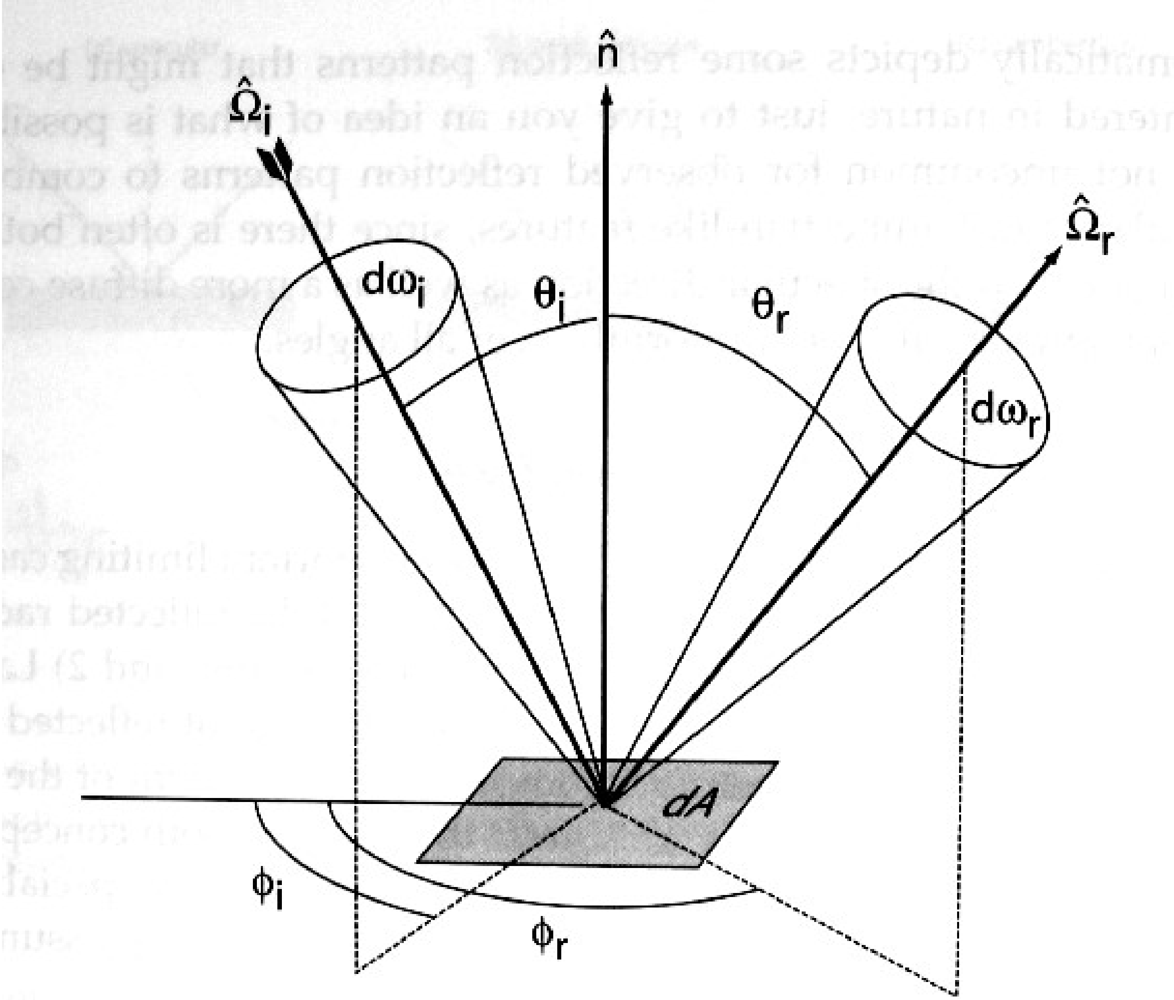


From Petty

Fig. 5.3: Examples of various types of surface reflection, presented in the form of polar plots in which the distance of the curve to the reflection point on the surface represents the relative intensity of reflected radiation in that direction.

Types of reflection: Can also think of the reflected light as emitted light from different types of surfaces.

Geometry for the BDRF (bidirectional reflection function)



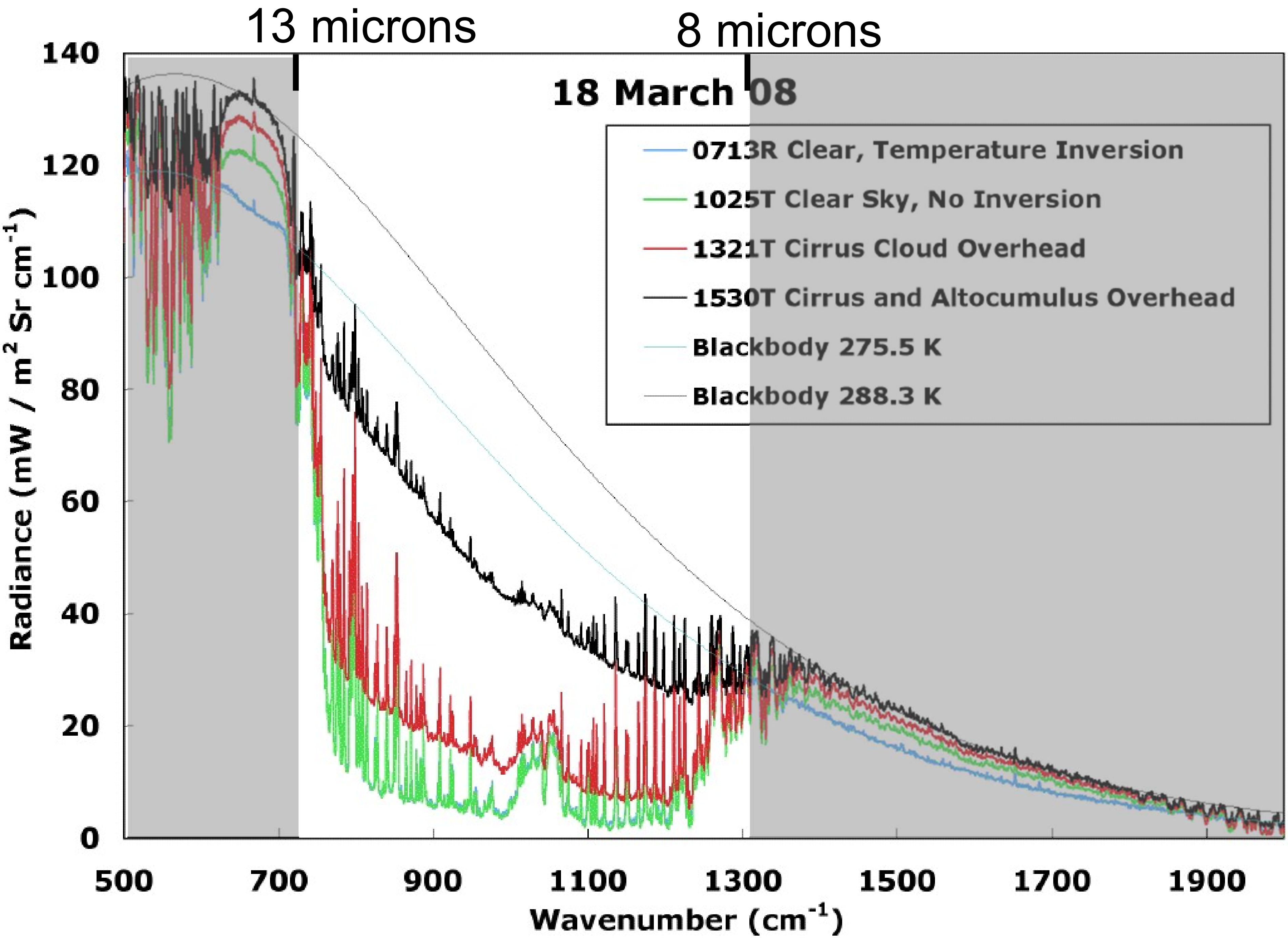
S is solar irradiance coming in.

I is the reflected radiance.

$$\rho(\theta_i,\phi_i;\theta_f,\phi_f) \equiv BDRF = \frac{S_0 \cos(\theta_i)}{I^{\uparrow}(\theta_r,\phi_r)} \; for\ a\ clear\ day.$$

$$I^{\uparrow}(\overrightarrow{\Omega_r}) = \int_{2\pi} \rho(\overrightarrow{\Omega_i},\overrightarrow{\Omega_r}) I^{\downarrow}(\overrightarrow{\Omega_r}) \widehat{n \bullet \Omega_i} \, d\omega_i \; for\ a\ cloudy\ day.$$

FTIR Radiance: Atmospheric IR Window



DEFINITION OF THE BRIGHTNESS TEMPERATURE
T_B

$$Radiance(\nu) = B(T_B, \nu)$$

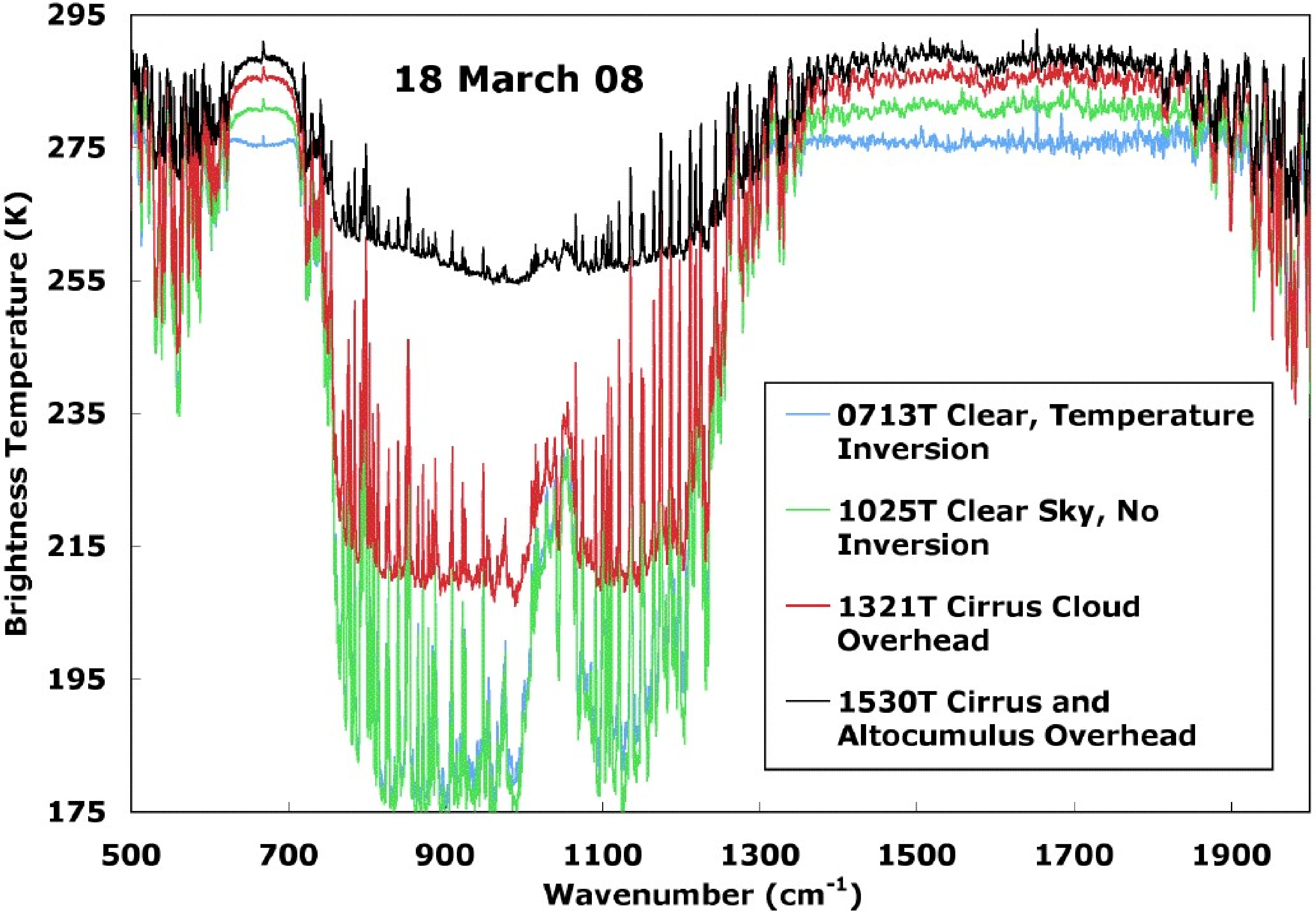
Measured Radiance at wavenumber v

=

Theoretical Radiance of a Black Body at temperature T_B

$$B(T_B, \nu) = 2x10^{11}\, hc^2\nu^3 \frac{1}{e^{100\, hc\nu/kT} - 1} \quad \frac{mW}{m^2\ Sr\ cm^{-1}}$$

FTIR Brightness Temperatures



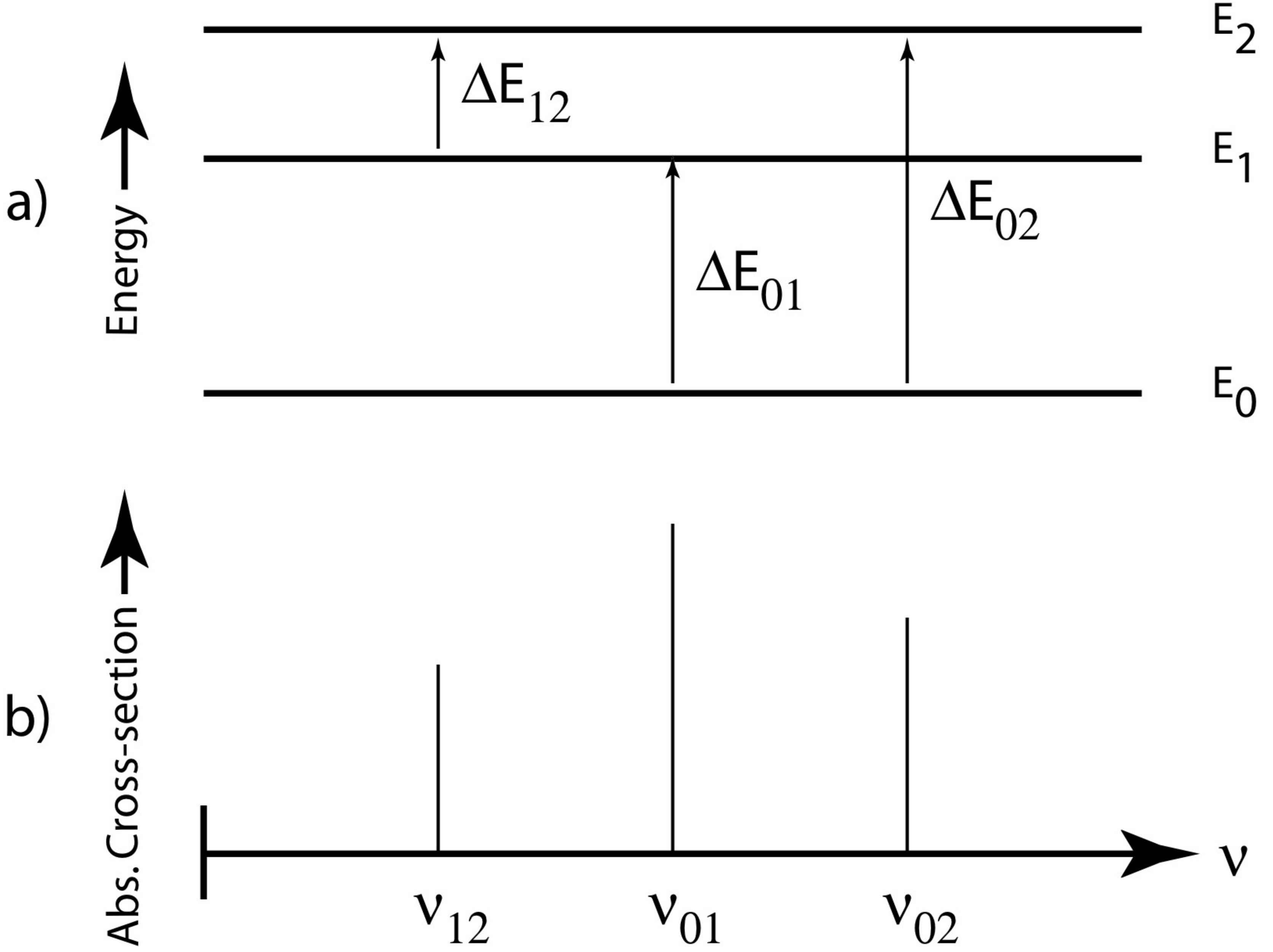
KEY POINTS

Absorption by Atmospheric Gases

- Visible and UV Absorption: due to electronic transitions. Monatomic - polyatomic.
- IR Absorption: due to vibration and rotation transitions. Polyatomic.
- Microwave Absorption: due to rotation transitions. Polyatomic.
- **Absorption cross sections depend on temperature and pressure**.
- Population of energy levels depends on temperature (thermal energy, kT). Transitions between levels therefore depend on temperature.
- Temperature (Doppler) broadening of absorption lines in the mesosphere.
- Pressure broadening of absorption lines (due to molecular collisions) in the troposphere.

Absorption and Emission Lines: Three level molecule.

$\nu_{ij}=\Delta E_{ij}/h$.



Dances of the Molecules in the Atmosphere: Which dance? Depends on temperature, available IR photons.

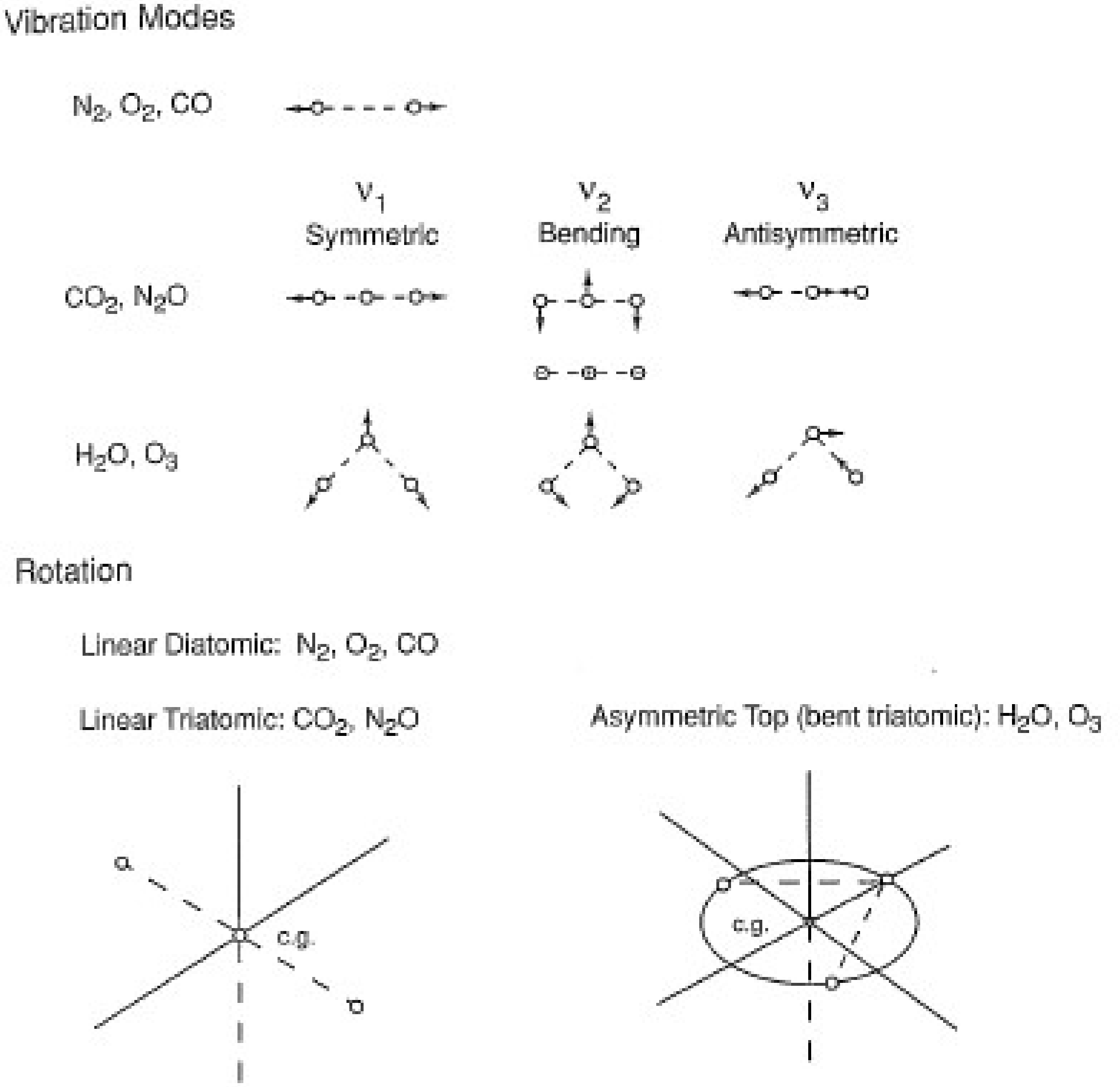


Figure 3.3 Vibrational modes of diatomic and triatomic atmospheric molecules and the axes of rotational freedom for linear and asymmetric top molecules.

From Liou

Atmospheric Temperature Profile: US "Standard" Atmosphere.

66 **3 Absorption and Scattering of Solar Radiation in the Atmosphere**

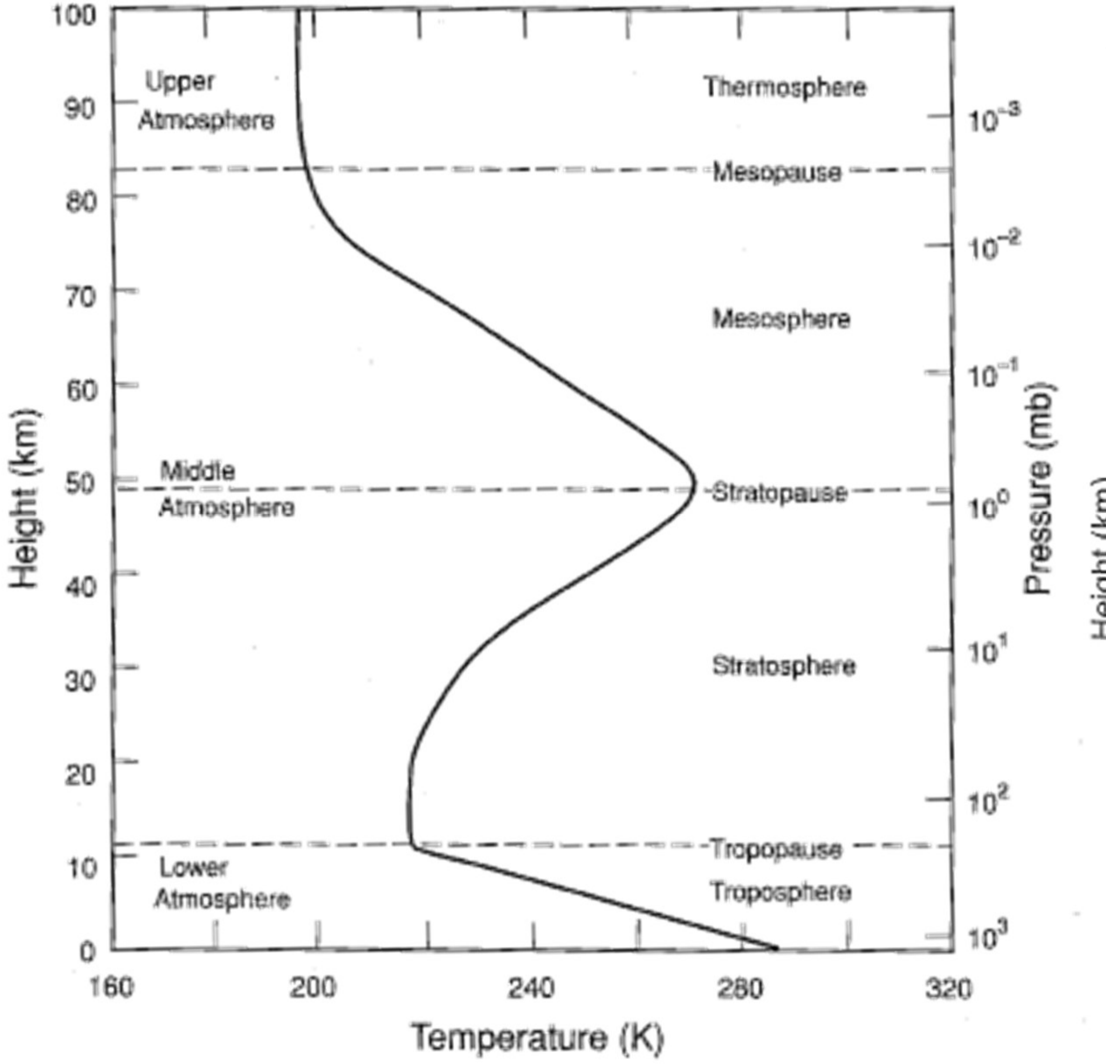
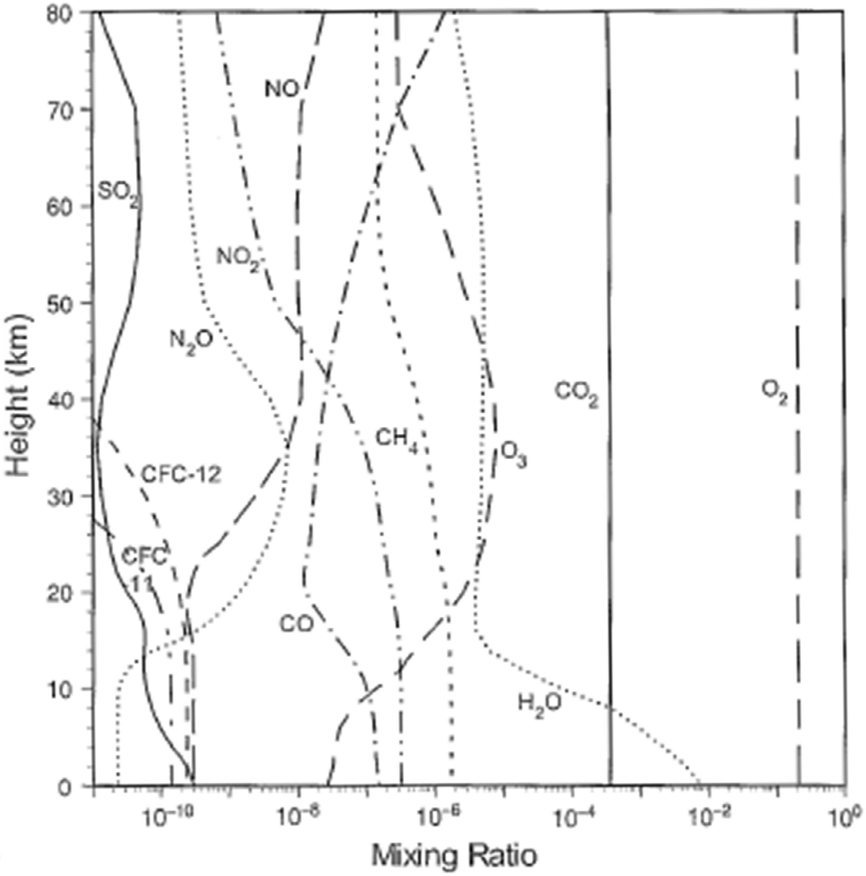


Figure 3.1 Vertical temperature profile after the U.S. Standard Atmosphere and definitions of atmospheric nomenclature.

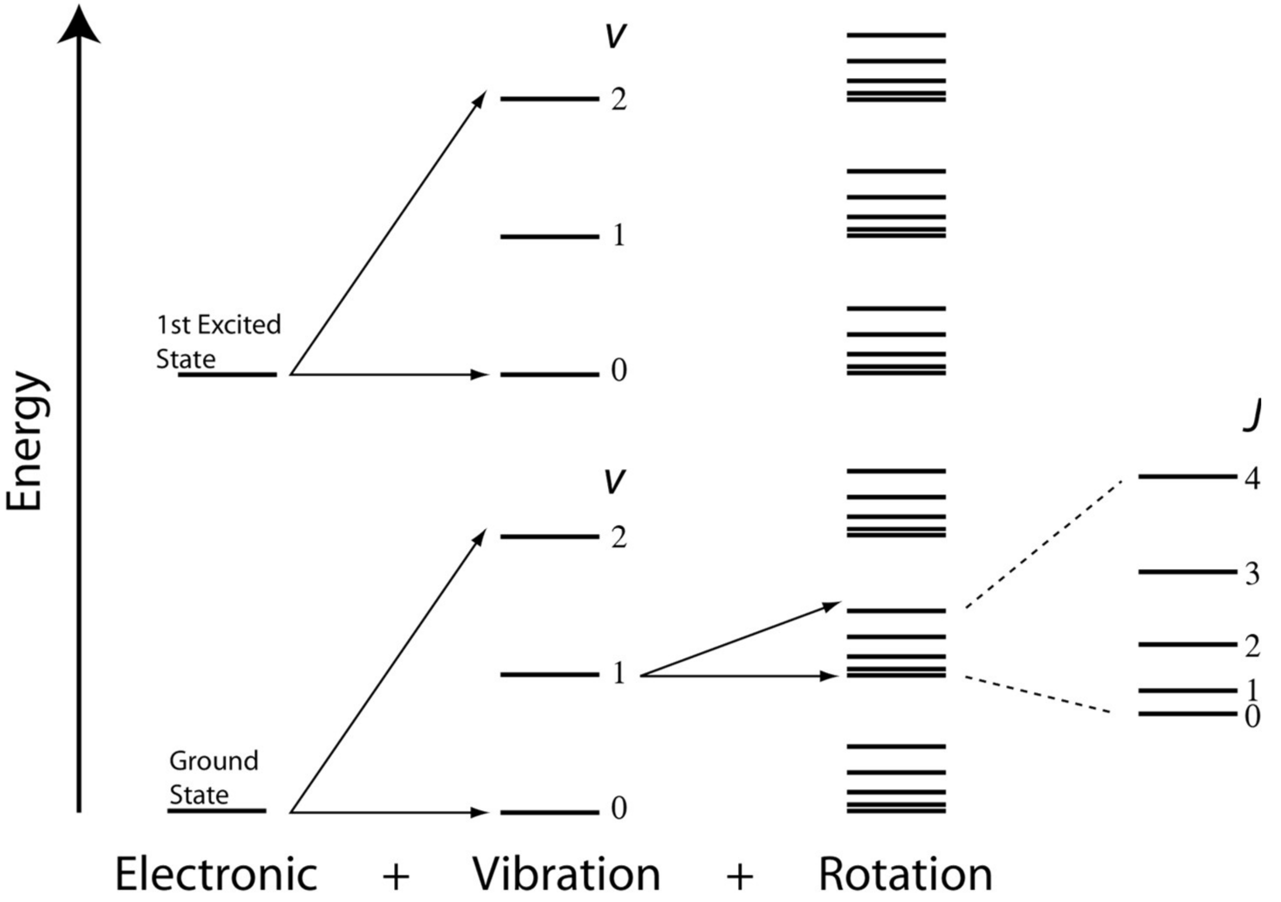
Dances of the Molecules in the Atmosphere: Which dance? Depends on temperature, available IR photons.



$$P(E_{ij}) \approx \exp(-h\nu_{ij} / kT)$$

From Liou

Transitions



$$E_{rot} = \frac{l(l+1)\hbar^2}{2\mu r_0^2} \qquad l = 0, 1, 2, ...$$

$$E_{vib} = \left(n + \frac{1}{2}\right) hf \qquad n = 0, 1, 2, ...$$

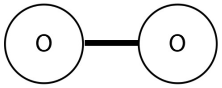
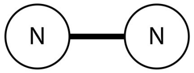
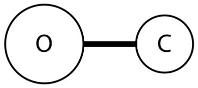
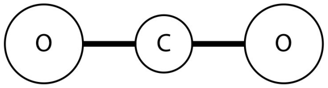
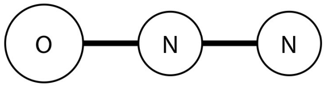
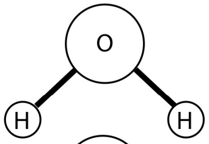
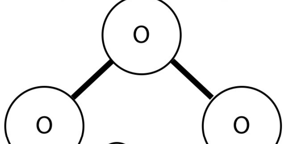
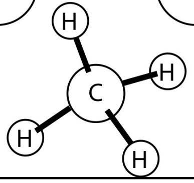
Dominant Transitions

Wavelengths	**Band**	**Dominant Transition**
< 1 µm, (200 nm - 1000 nm)	Near IR, Visible, UV	Electronic
1 µm - 20 µm	Near IR, Thermal IR	Vibration
> 20 µm	Far IR, Microwave	Rotation

$$E_{rot} = \frac{l(l+1)\hbar^2}{2\mu r_0^2} \qquad l = 0, 1, 2, ...$$

$$E_{vib} = \left(n + \frac{1}{2}\right) hf \qquad n = 0, 1, 2, ...$$

Rotations

Molecule	*Structure*		*Permanent Electirc Dipole Moment?*
Oxygen		linear	No (magnetic dipole)
Nitrogen		linear	No
Carbon Monoxide		linear	Yes
Carbon Dioxide		linear	No
Nitrous Oxide		linear	Yes
Water		asymmetric top	Yes
Ozone		asymmetric top	Yes
Methane		spherical top	Yes

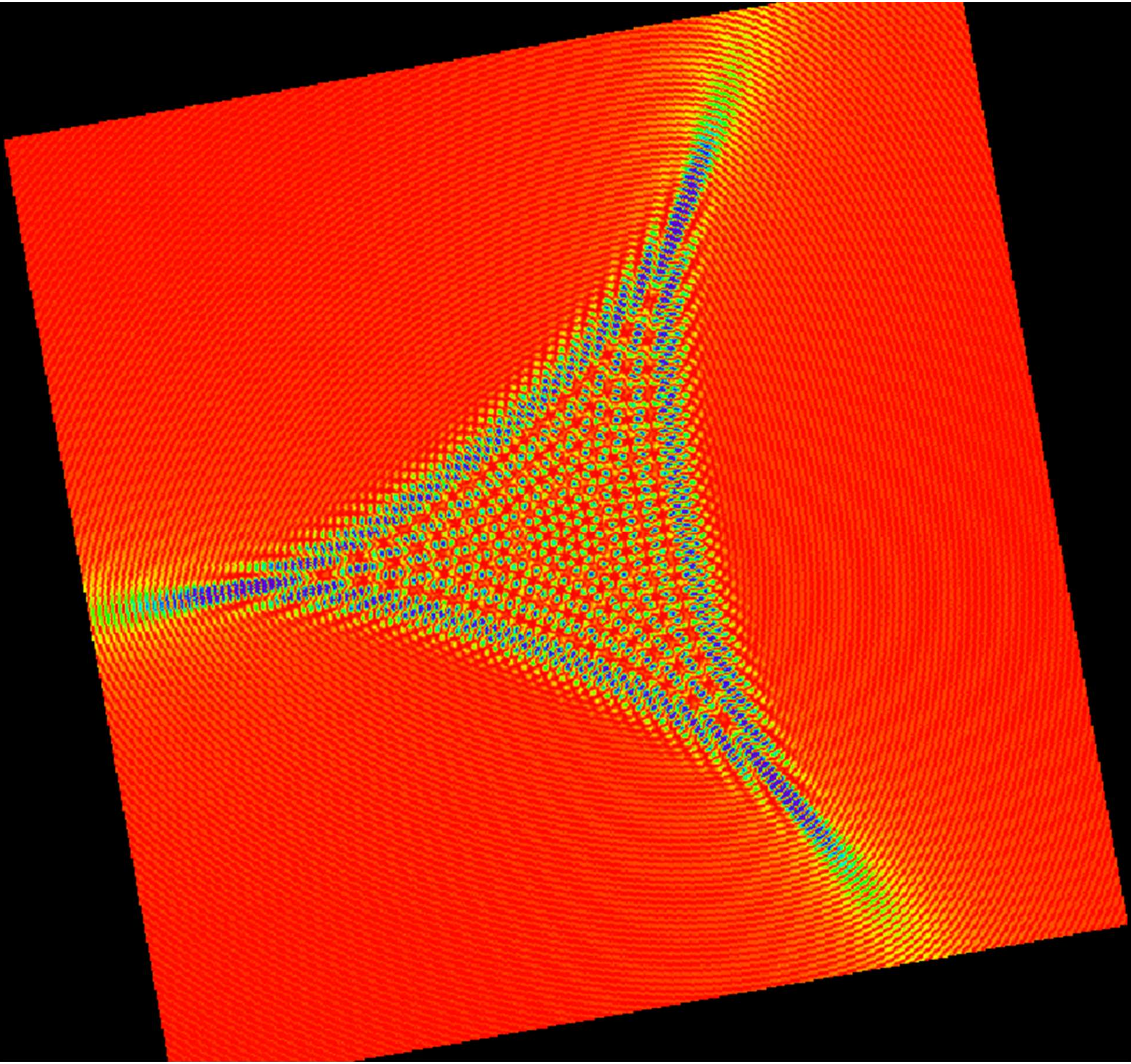
Description	**Moments of Inertia**	**Examples**
Monatomic	$I_1=I_2=I_3=0$	Argon, He, Xe
Linear	$I_1=0,$ $I_2=I_3>0$	N_2, O_2, CO_2, N_2O
Spherical Top	$I_1=I_2=I_3 > 0$	CH_4
Symmetric Top	$I_1\neq 0,\ I_2=I_3 > 0$	NH_3. CH_3Cl, CF_3Cl
Asymme		

$$E_{rot} = \frac{1}{2}I\omega^2, \quad L = I\omega, \quad I = \sum_i r_i^2 \delta m_i, \quad I = m' r^2 \ (diatomic)$$

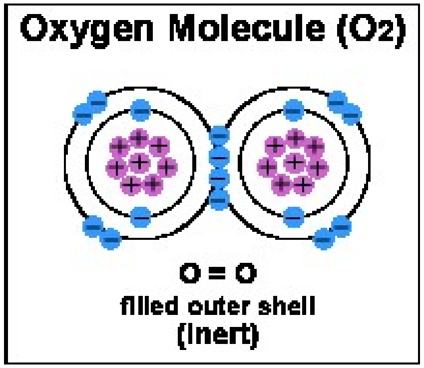
$$m' \equiv \frac{m_1 m_2}{m_1 + m_2}, \ L = \frac{h}{2\pi}\sqrt{J(J+1)}, \quad E_J = \frac{1}{2}I\omega^2 = \frac{J(J+1)h^2}{8\pi^2 I}$$

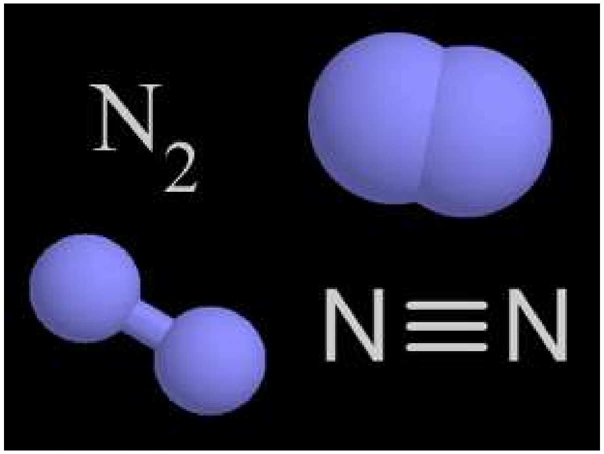
$$\Delta E = E_{J+1} - E_J = \frac{h^2}{8h^2 I}[(J+1)(J+2) - J(J+1)]$$

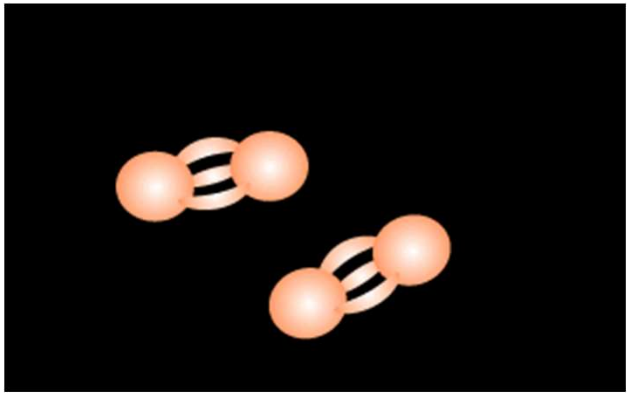
$$\Delta E = \frac{h^2}{4h^2 I}(J+1), \ \ \nu = \Delta E / h = \frac{h}{4\pi^2 I}(J+1), \ J = 0,1,2,...$$

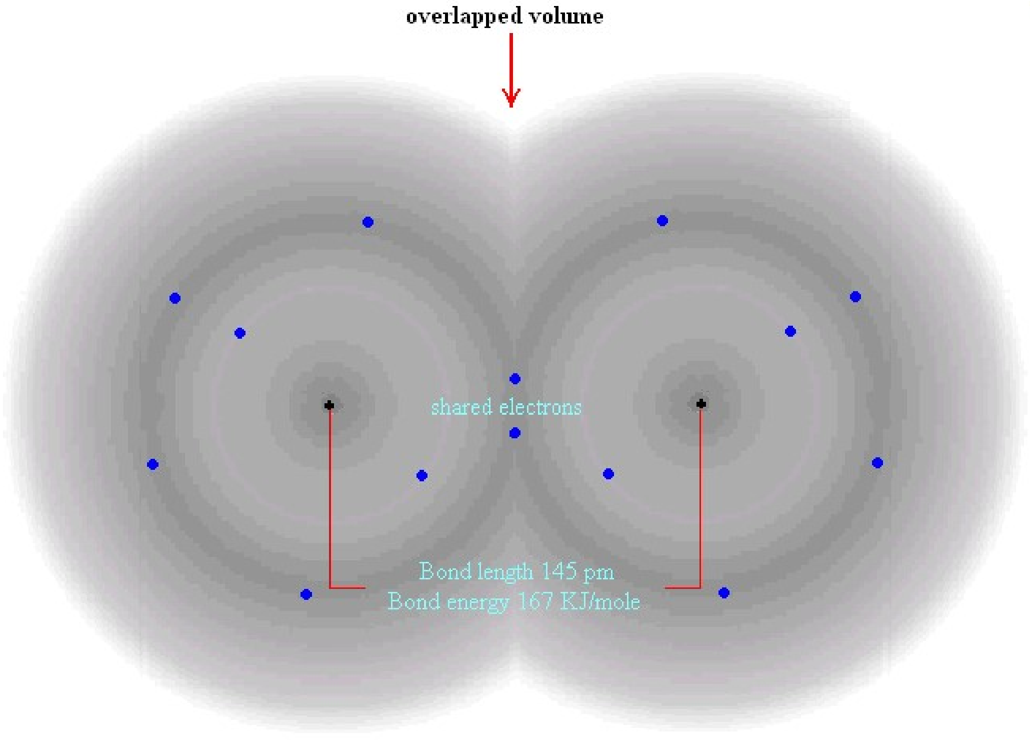


Why Don’t We Worry About Rotational and Vibrational Transitions for N_2, and worry only a little about O_2?





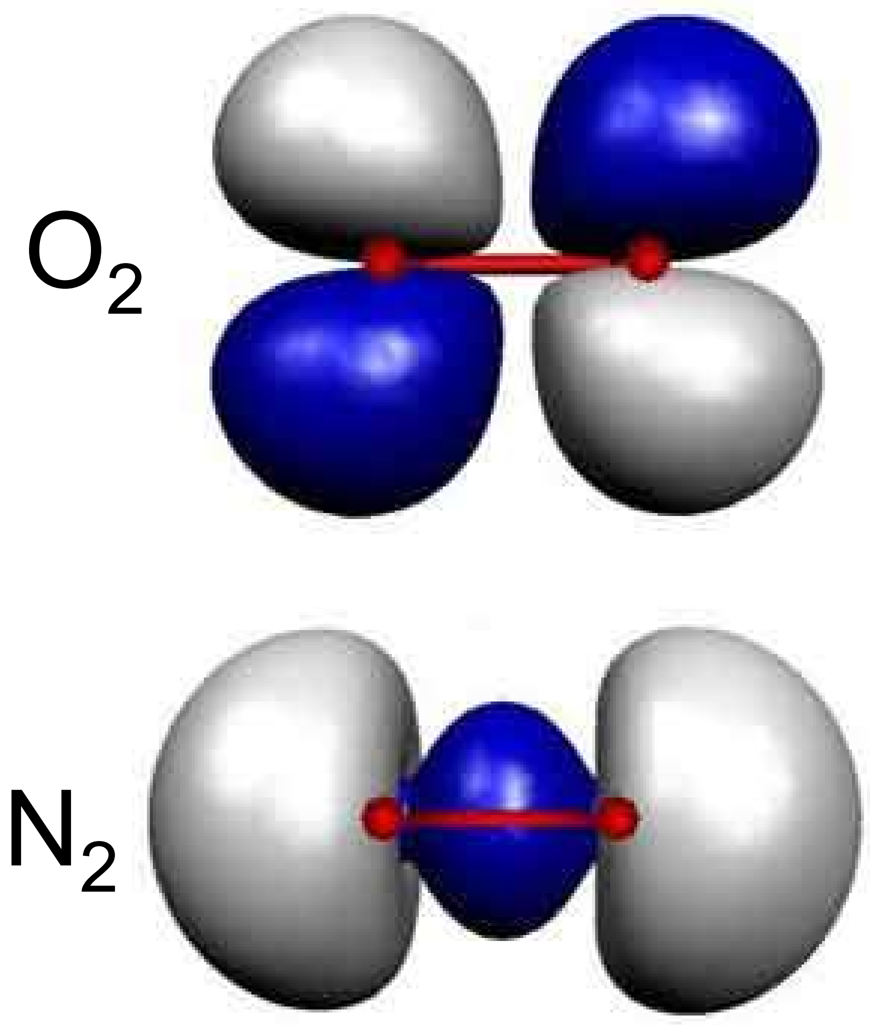




N-N (single bond nitrogen molecule)
cross section
Please note: Electrons in an electron shell will be configured in spherically.

Homonuclear Diatomic Molecules: *N_2 has no permanent electric or magnetic dipole moment due to the symmetry of positive and negative charge within the molecules.* (O_2 has a permanent magnetic dipole moment, rotation bands at 60 and 118 GHz.)

Why Don’t We Worry About Rotational and Vibrational Transitions for N_2, and worry only a little about O_2?

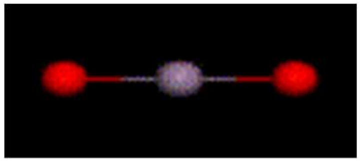


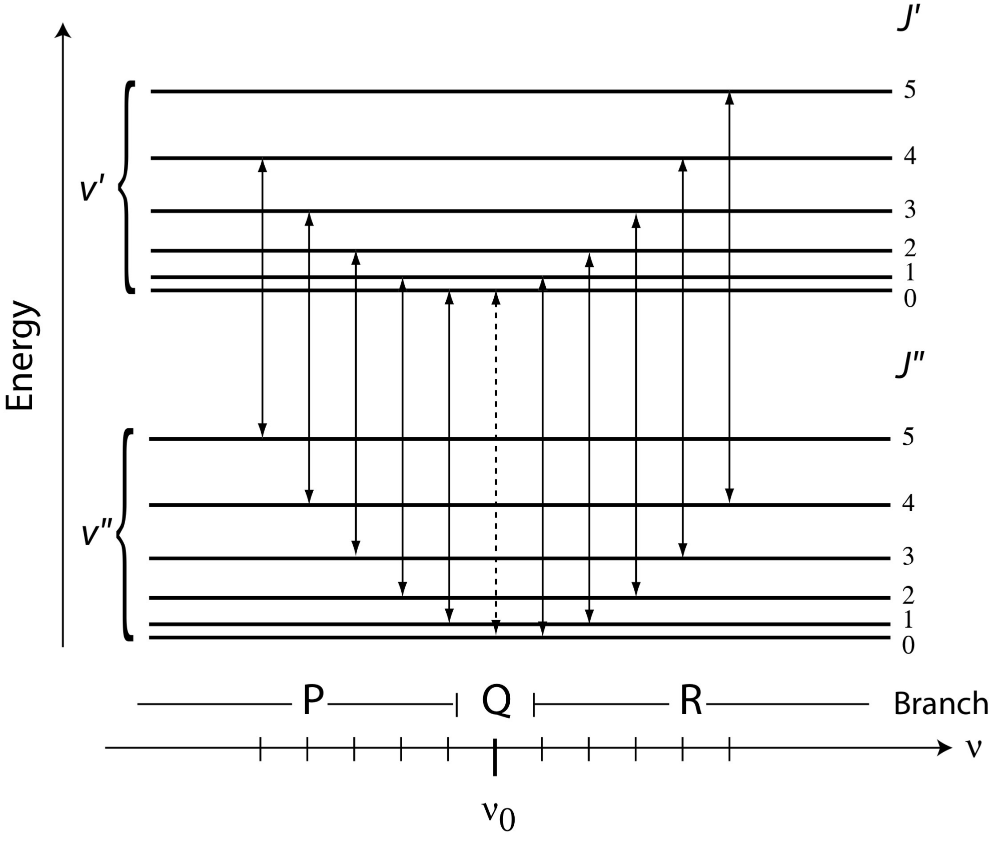
Bonding electron ‘clouds’ (orbitals) for O_2 and N_2 (bottom).

Homonuclear Diatomic Molecules: *N_2 has no permanent electric or magnetic dipole moment due to the symmetry of positive and negative charge within the molecules.* (O_2 has a permanent magnetic dipole moment, rotation bands at 60 and 118 GHz.)

Common Triatomic Molecules CO_2 and CH_4.

CO_2 and CH_4 (carbon dioxide and methane) have no permanent electric or magnetic dipole moment and don’t have pure rotational transitions. However, bending modes associated with vibrational energy levels can induce dipole moments that couple vibrational and rotational transitions in the thermal IR.



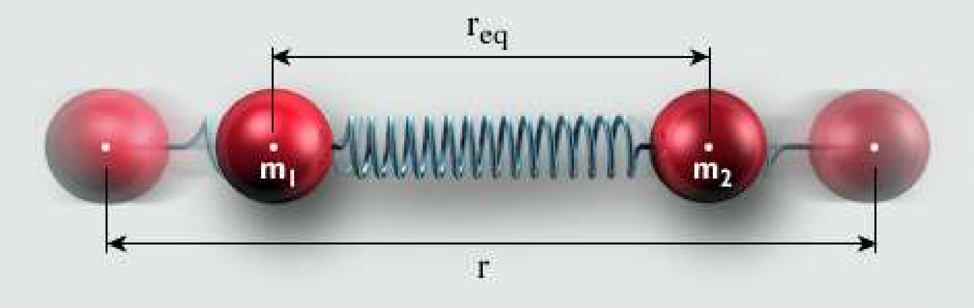


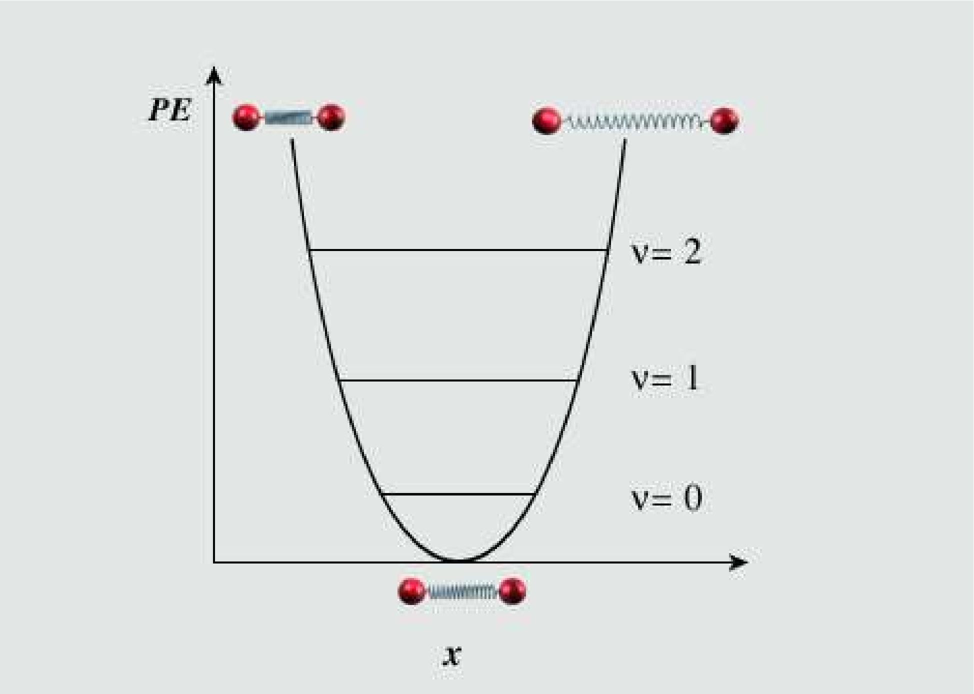
rotation

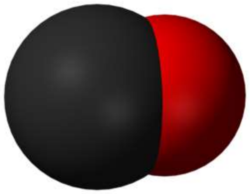
vibration {

rotation

Vibrational Transitions for Diatomic Molecules: CO







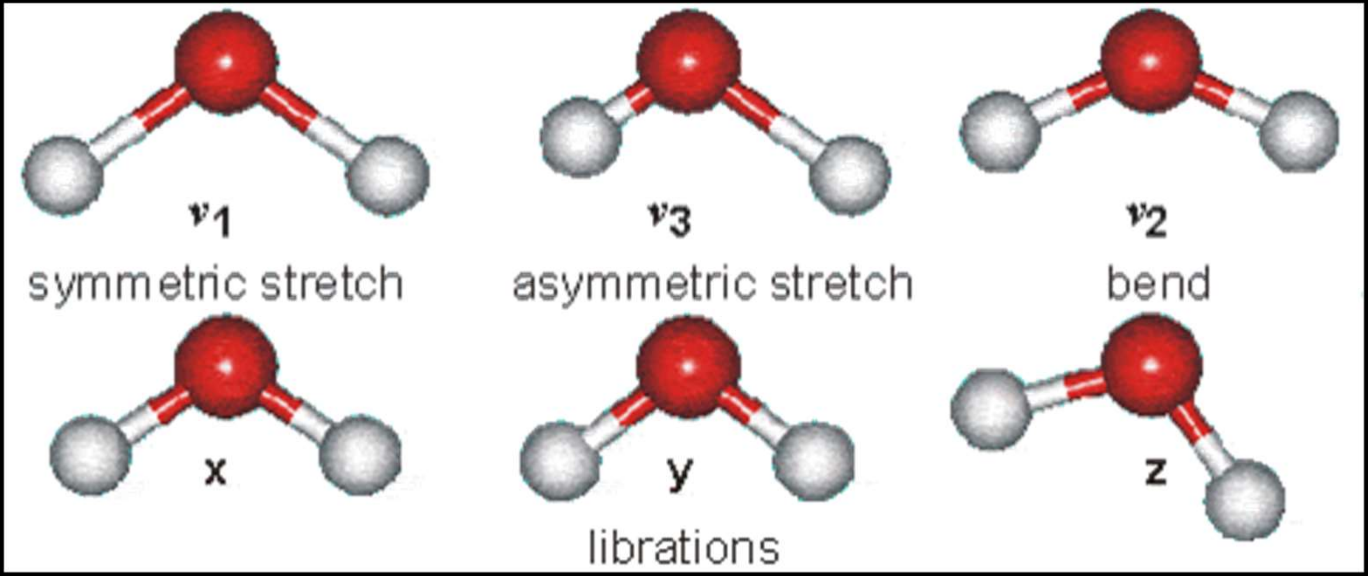
$$F = -k(r' - r)$$ {Newton's Second Law}

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m'}},\ m' \equiv \frac{m_1 m_2}{m_1 + m_2}$$ {Resonance Frequency, Reduced mass}

$$\nu = (n + \frac{1}{2})f$$ {Allowed frequencies for a quantum oscillator}

$$E_n = h\nu = (n + \frac{1}{2})hf$$ {Allowed energy levels}

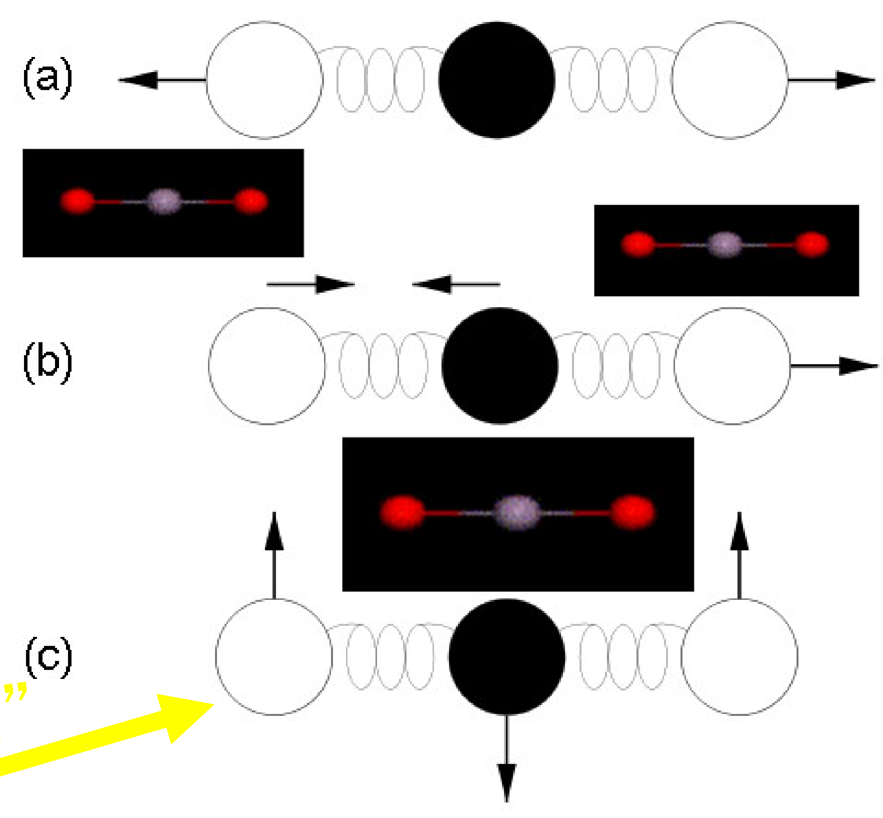
$$\Delta n = \pm N$$ {Vibrational transistions} $\left|\Delta E_v\right| = Nhf$ {Photon Energies}



http://www.lsbu.ac.uk/water/vibrat.html

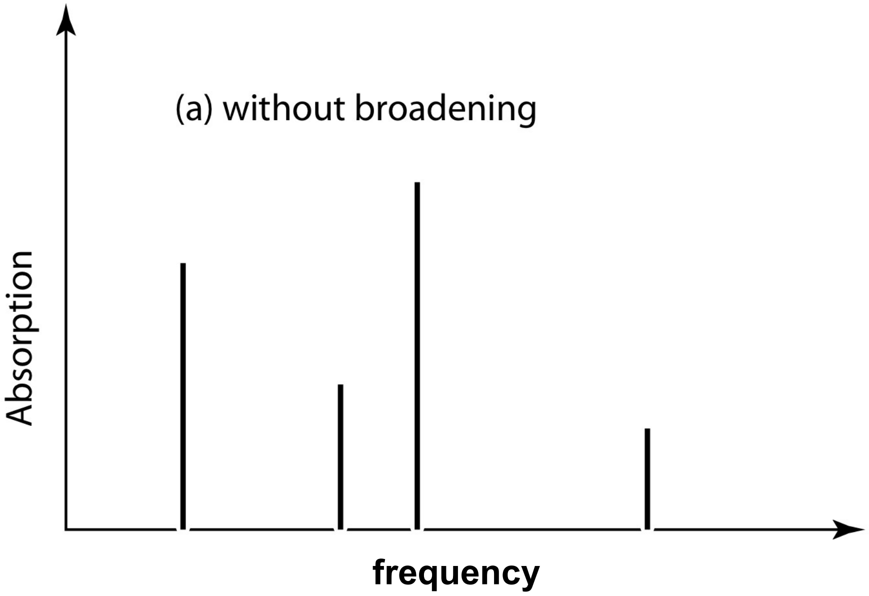
... of Carbon Dioxide Molecules

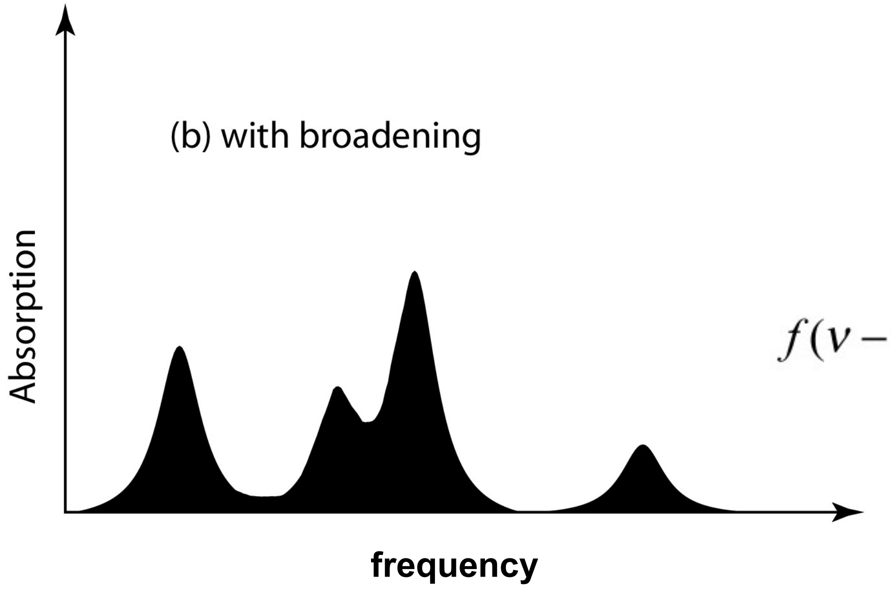
Vibration modes of carbon dioxide. Mode (a) is symmetric and results in no net displacement of the molecule's "center of charge", and is therefore not associated with the absorption of IR radiation. Modes (b) and (c) do displace the "center of charge", creating a "dipole moment", and therefore are modes that result from EM radiation absorption, and are thus responsible for making CO2 a greenhouse gas.



"15 um motion"

Line Broadening





Natural Broadening:
Finite time, finite widths (Heisenberg is uncertain about widths, certain they are not infinitely narrow!)

Doppler Broadening:

$$f_D(v-v_0)=\frac{1}{\alpha_D\sqrt{\pi}}\exp\left[-\frac{(v-v_0)^2}{\alpha_D^2}\right],\quad \alpha_D=v_0\sqrt{\frac{2k_BT}{mc^2}}$$

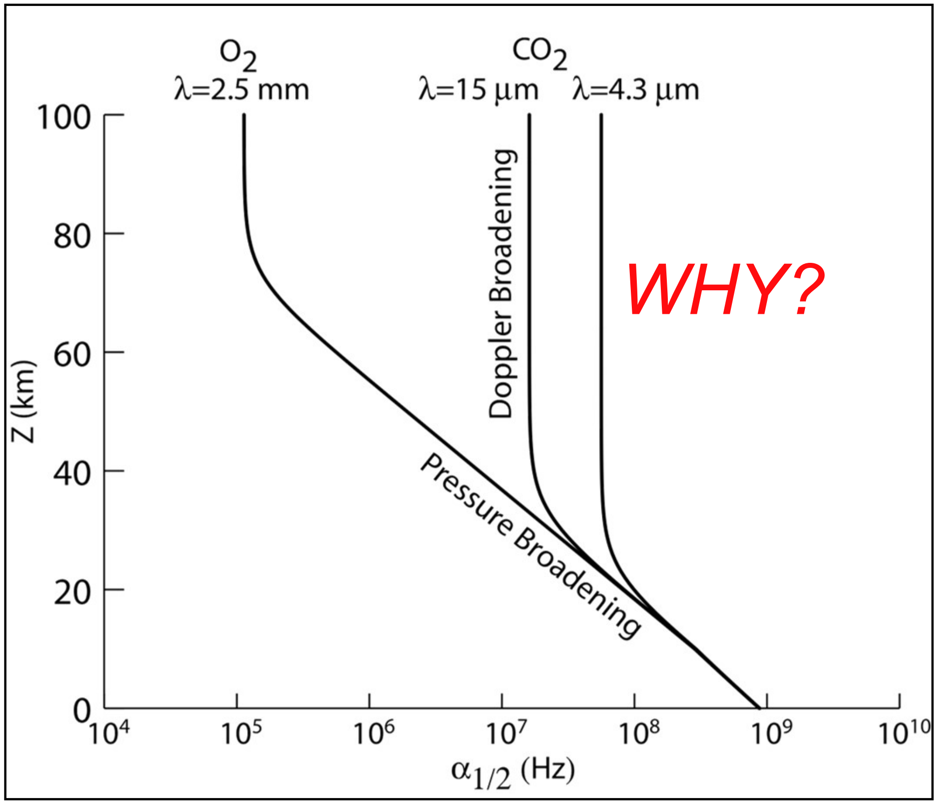
Molecules with relative motions due to thermal energy ‘see’ doppler shifts of the light. Important in the mesosphere.

Pressure Broadening: Lorentz line shape

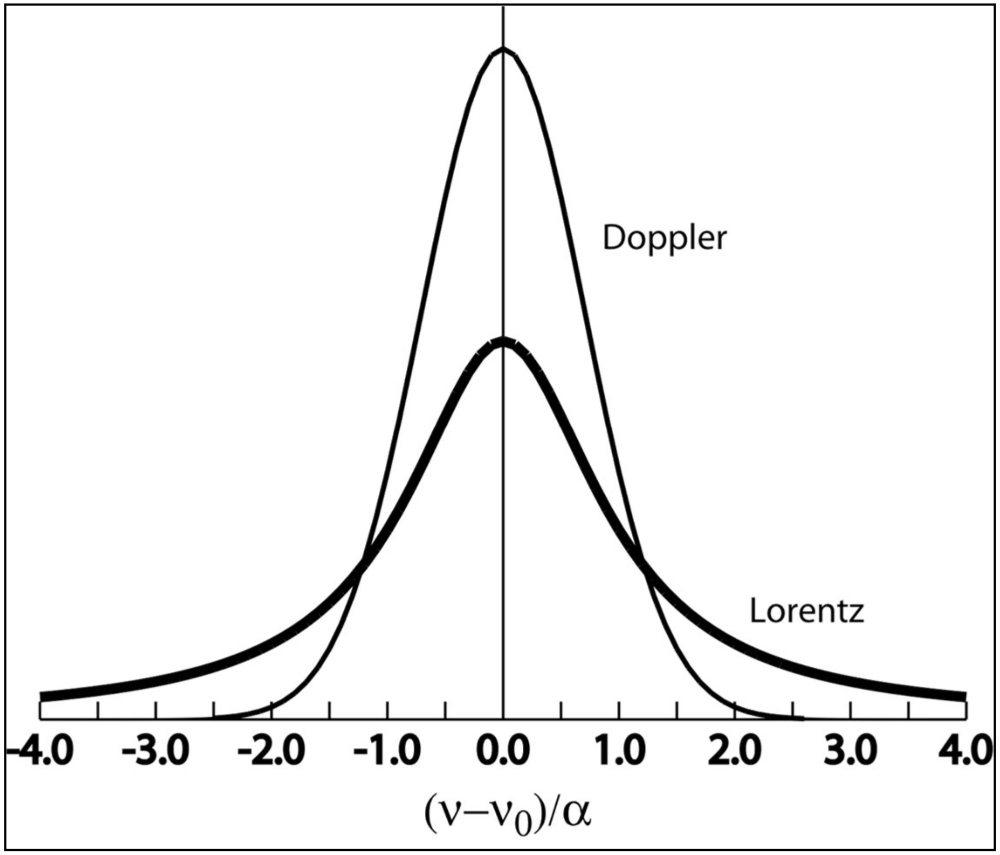
$$f(v-v_0)=\frac{\alpha_L/\pi}{(v-v_0)^2+\alpha_L^2},\quad \alpha_L\propto pT^{-1/2},\quad \alpha_L=\alpha_\infty\left(\frac{p}{p_0}\right)\left(\frac{T_0}{T}\right)^n$$

Molecular collisions distort energy levels for absorption and emission. Emperically determined (by measurement). Very important for the troposphere and lower stratosphere.

Absorption Cross Section per Molecule for a Single Transition



WHY?



$$\sigma_\nu(\nu) = S\ f(\nu - \nu_0)$$

$$\int_0^\infty f(\nu - \nu_0) d\nu = 1$$

$$\int_0^\infty \sigma_\nu d\nu = \int_0^\infty S f(\nu - \nu_0) d\nu = S$$

$$f_D(\nu - \nu_0) = \frac{1}{\alpha_D \sqrt{\pi}} \exp\left[-\frac{(\nu - \nu_0)^2}{\alpha_D^2} \right], \quad \alpha_D = \nu_0 \sqrt{\frac{2k_B T}{mc^2}}$$

$$f(\nu - \nu_0) = \frac{\alpha_L / \pi}{(\nu - \nu_0)^2 + \alpha_L^2}, \quad \alpha_L \propto pT^{-1/2}, \quad \alpha_L = \alpha_\infty \left(\frac{p}{p_0}\right)\left(\frac{T_0}{T}\right)^n$$

Hitran04
Database:
S, n, α_∞

$$\frac{f_D(\alpha_{1/2})}{f_D(0)} = \frac{1}{2}, \qquad \alpha_{1/2} = \alpha_D \sqrt{\ln 2}$$

Electronic, Vibrational, energy levels and the big break up (dissociation level)

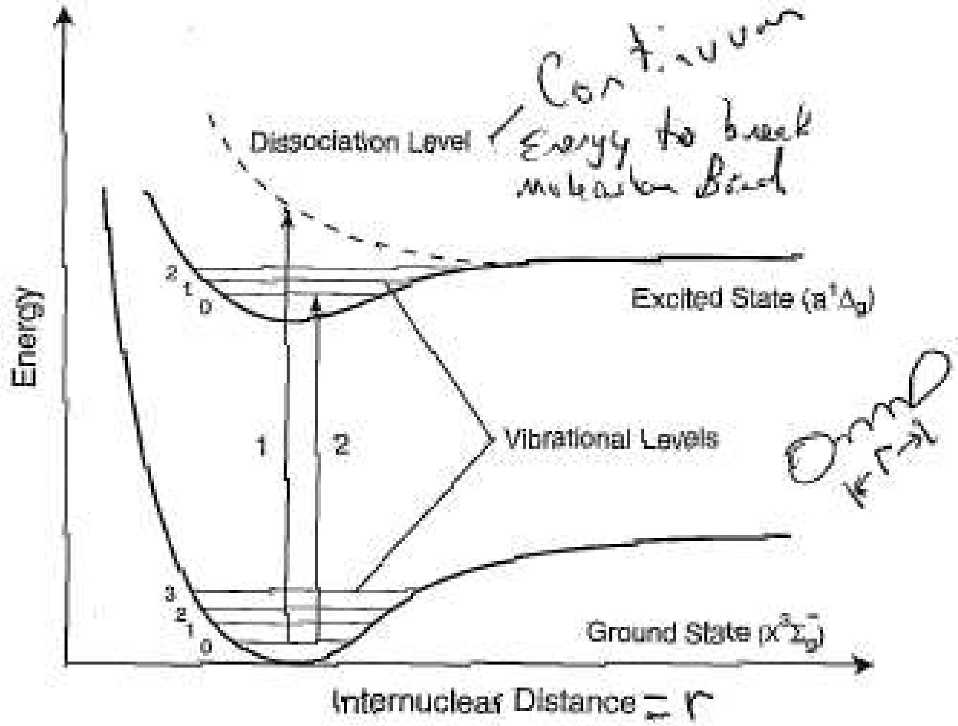


Figure 3.4 Illustrative potential energy curves for two electronic states of a diatomic molecule. The horizontal lines in the potential well represent vibrational energy levels.

From Liou

The Radiative Transfer Equation with Scattering

Key Points:

- Single and multiple scattering definitions. Scatter once, single, scatter several times, multiple.

- When does single scattering matter? Always. It may be small relative to absorption, but generally should consider it.

- When is multiple scattering important? When the optical depth for scattering is ‘large enough’, greater than say 0.5.

- General form of the multiple scattering equation.

- Single scattering approximation.

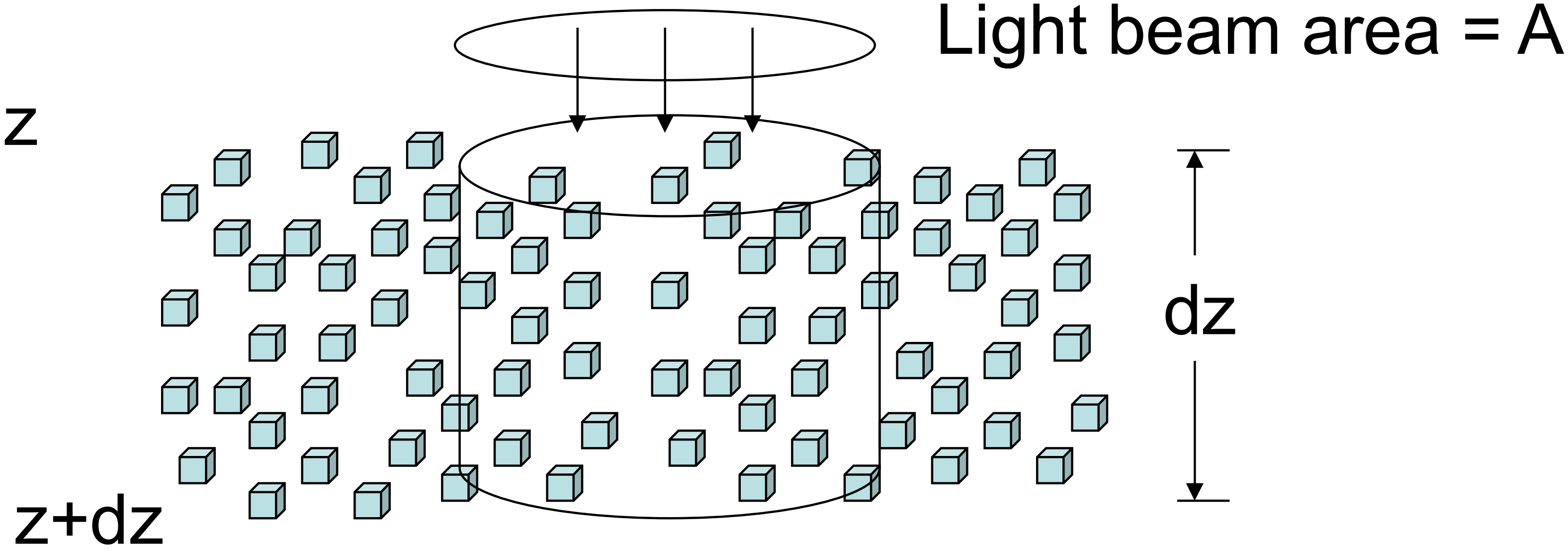
- Review of the phase function, and asymmetry parameter.

Demonstrations

Diffraction grating from a DVD. Grazing incidence, first order (over head) and second order (near backscattering).

Milk, Clouds, and multiple scattering: Rayleigh scattering by dilute milk, and polarization state.

Review: Optics of N identical (particles / volume)



Power removed in dz: = I(z) N A dz σ_{ext}

Bouger-Beer "law" (direct beam only!)

$$\left(I(z)-I(z+dz)\right)A = I(z)\ N\ \ A\ dz\ \boldsymbol{\sigma}_{ext}$$

$$-dI = I(z)\ N\ \boldsymbol{\sigma}_{ext}\ \ dz$$

$$\int_{I_0}^{I(z)} \frac{dI}{I} = -\int_0^z N\ \boldsymbol{\sigma}_{ext}\ \ dz'\ ,\ \ln\left(\frac{I(z)}{I_0}\right) = -N\ \boldsymbol{\sigma}_{ext}\ \ z$$

$$I(z) = I_0 \exp(-N\ \ \boldsymbol{\sigma}_{ext}\ \ z) = I_0 \exp(-\boldsymbol{\beta}_{ext}\ \ z)$$

ATMOSPHERIC EMISSION: PRACTICAL CONSEQUENCES OF THE **SCHWARZSCHILD EQUATION** FOR RADIATION TRANSFER WHEN SCATTERING IS NEGLIGIBLE

What process subtracts radiation?

What process adds radiation?

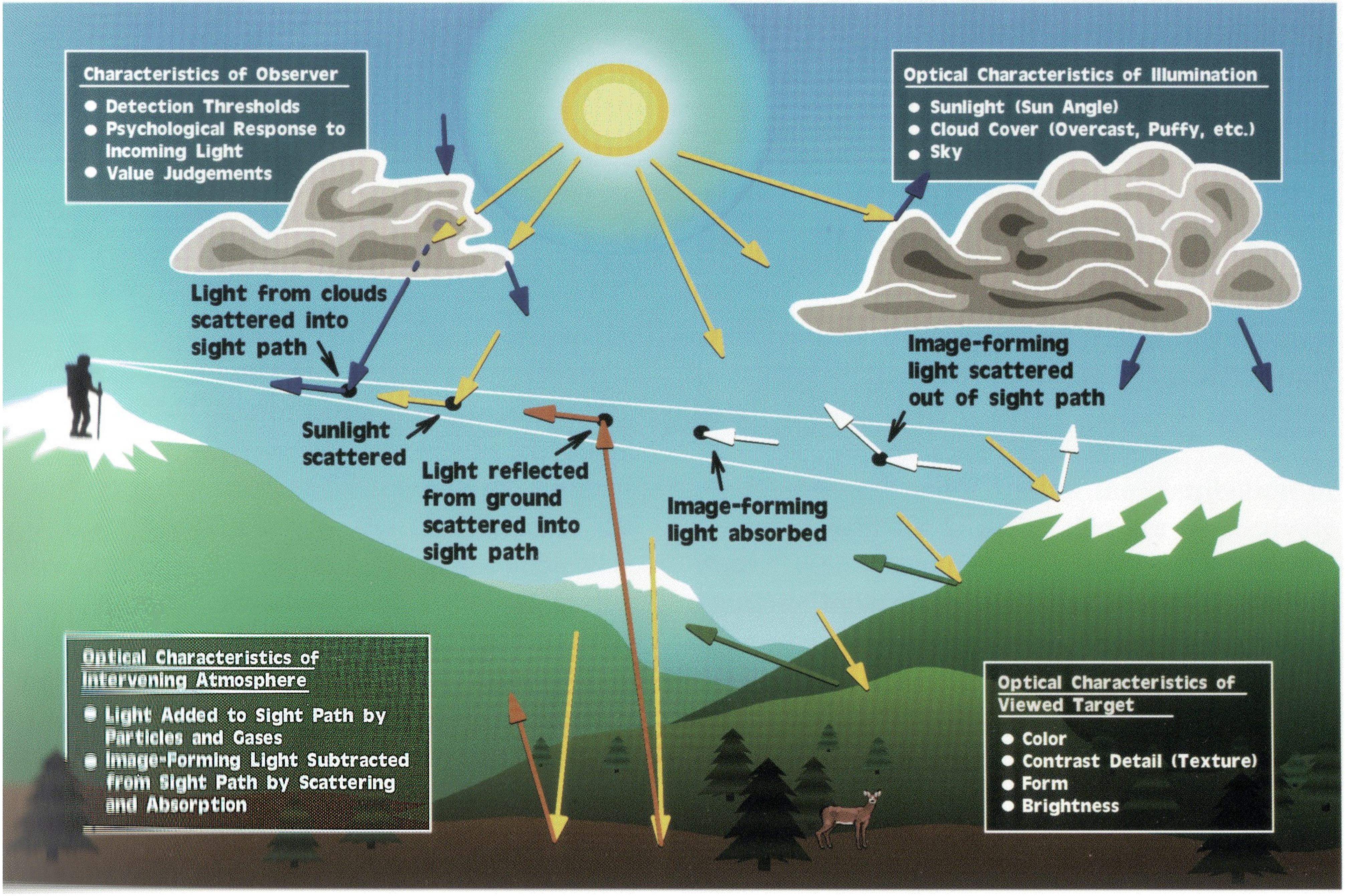
$$\boxed{\frac{dI}{ds} = \beta_a (B - I)\,.} \qquad (8.4)$$

What equation is used to calculate optical depth for a gaseous atmosphere?

$$\boxed{I(0) = I(\tau')e^{-\tau'} + \int_0^{\tau'} Be^{-\tau}\, d\tau\,.} \qquad (8.13)$$

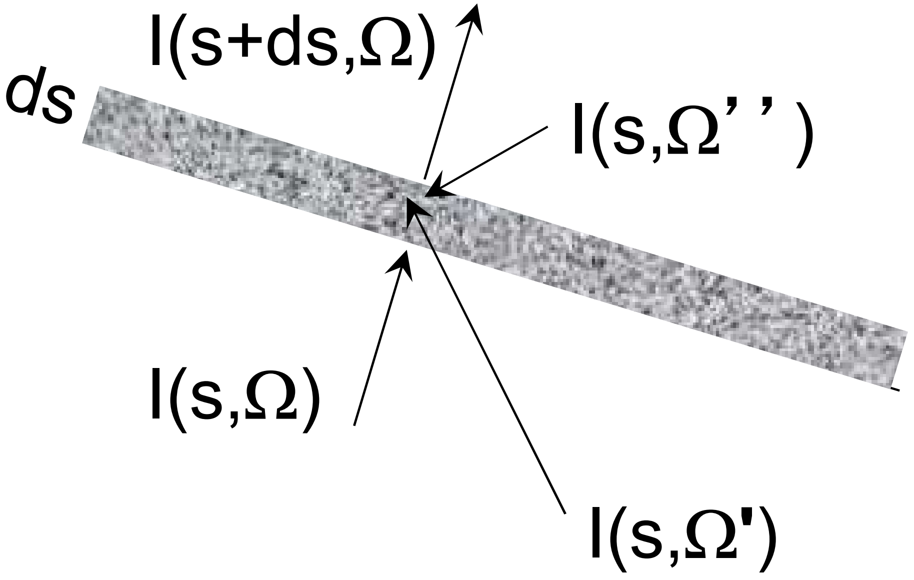
Key Point: *Almost all common radiative transfer problems involving emission and absorption in the atmosphere (without scattering) can be understood in terms of (8.13)!*[2]

Scattering of object rays from the sight path, and scattering of stray light into the sight path, both by gases and aerosols.



Visibility Overview: From Bill Malm, NPS Fort Collins

Putting it all together... General Radiative Transfer Equation



Phase function describes the fraction of light that is scattered into Ω from other directions.

$$I(s+ds)-I(s)=dI(s)=dI_{ext}+dI_{emit}+dI_{sca}$$

$$dI_{ext}=-\beta_{ext}ds\ I(s)=-(\beta_{abs}+\beta_{sca})ds\ I(s)$$

$$dI_{emit}=\beta_{abs}ds\ B[T(s)]$$

$$dI_{sca}=\frac{\beta_{sca}ds}{4\pi}\int_{4\pi}p(\hat{\Omega}',\hat{\Omega})I(\hat{\Omega}',s)d\omega'$$

$$p(\hat{\Omega}',\hat{\Omega})\equiv Phase\ \ Function,\ \int_{4\pi}p(\hat{\Omega}',\hat{\Omega})\ d\omega'=1$$

gains

losses

lost only from the forward direction Ω!!

new term

General Equation for Radiative Transfer

$$d\tau \equiv -\beta_{ext} ds \ , \varpi \equiv \frac{\beta_{sca}}{\beta_{ext}} \text{ (single scattering albedo)}, \ \omega \equiv Solid \ \ Angle$$

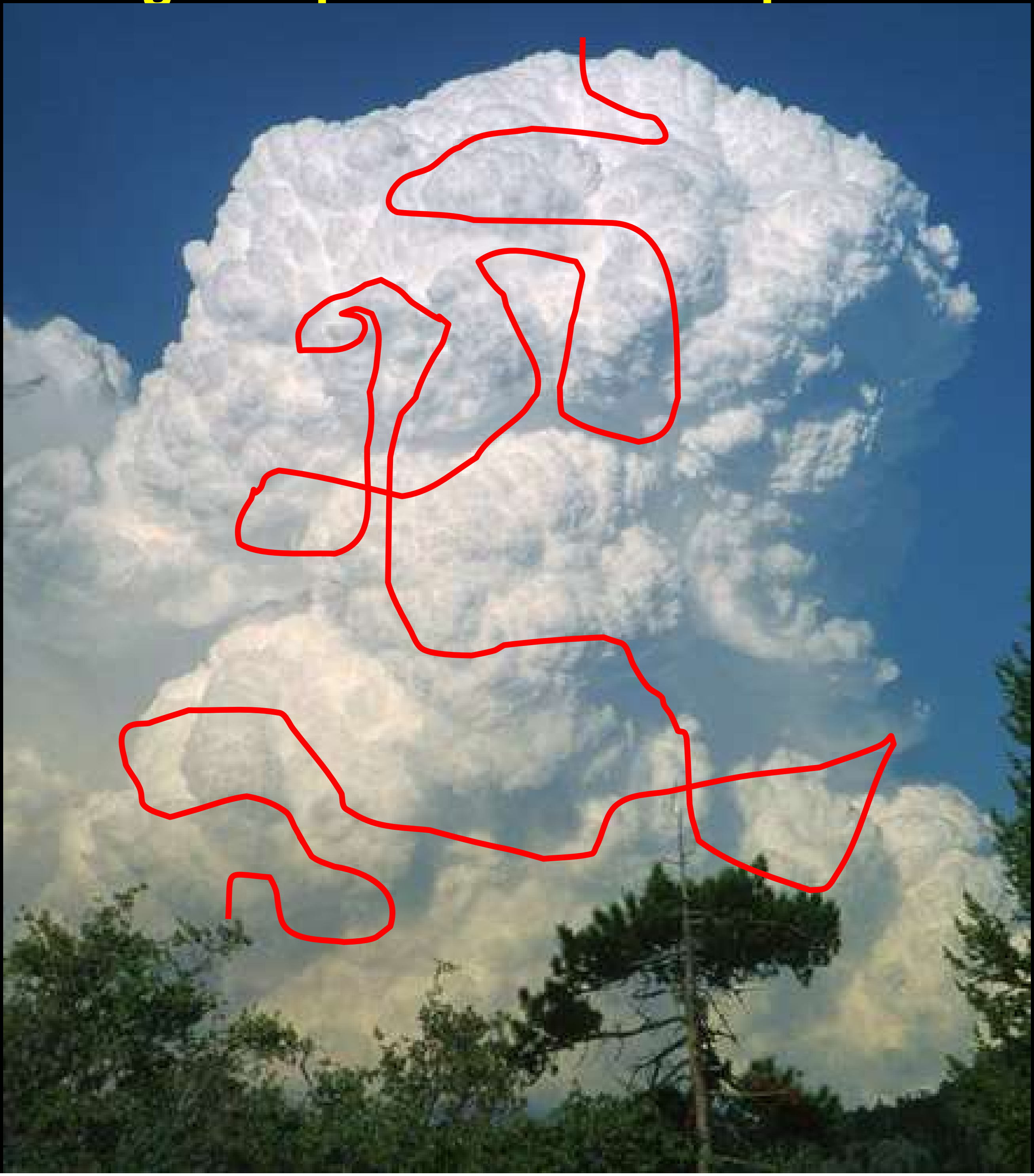
$$\frac{dI(\overset{\wedge}{\Omega})}{d\tau} = I(\overset{\wedge}{\Omega}) - (1-\varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} p(\overset{\wedge}{\Omega}',\overset{\wedge}{\Omega}) I(\overset{\wedge}{\Omega}')d\omega'$$

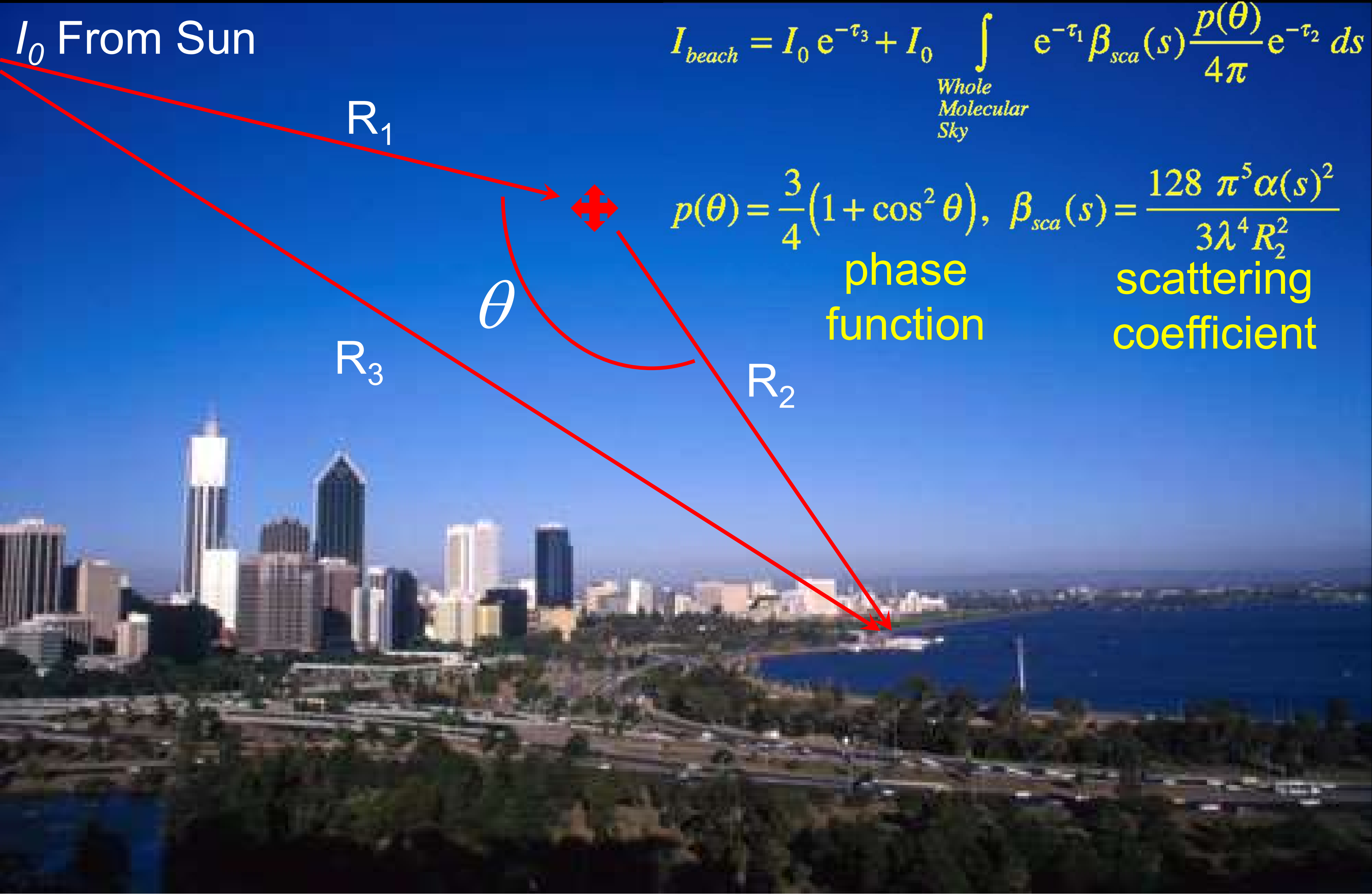
$$Plane \ \ Parallel \ \ Atmosphere: \ \mu \equiv \cos(\theta), \ \theta \equiv Zenith \ \ Angle, \ \phi \equiv Azimuth \ \ Angle$$

$$\mu \frac{dI(\mu,\phi)}{d\tau} = I(\mu,\phi) - (1-\varpi)B - \frac{\varpi}{4\pi}\int_0^{2\pi} d\phi' \int_{-1}^{1} p(\mu,\phi;\mu',\phi')I(\mu',\phi') \ d\mu'$$

Multiple Scattering Complex Due to Multiple Scatterings

Hypothetical
Photon
Path





$$I_{beach} = I_0 e^{-\tau_3} + I_0 \int_{\substack{\text{Whole}\\ \text{Molecular}\\ \text{Sky}}} e^{-\tau_1} \beta_{sca}(s) \frac{p(\theta)}{4\pi} e^{-\tau_2}\, ds$$

$$p(\theta) = \frac{3}{4}\left(1+\cos^2\theta\right),\ \ \beta_{sca}(s) = \frac{128\ \pi^5 \alpha(s)^2}{3\lambda^4 R_2^2}$$

phase function

scattering coefficient

Rayleigh and Raman Scattering: Quantum Perspective

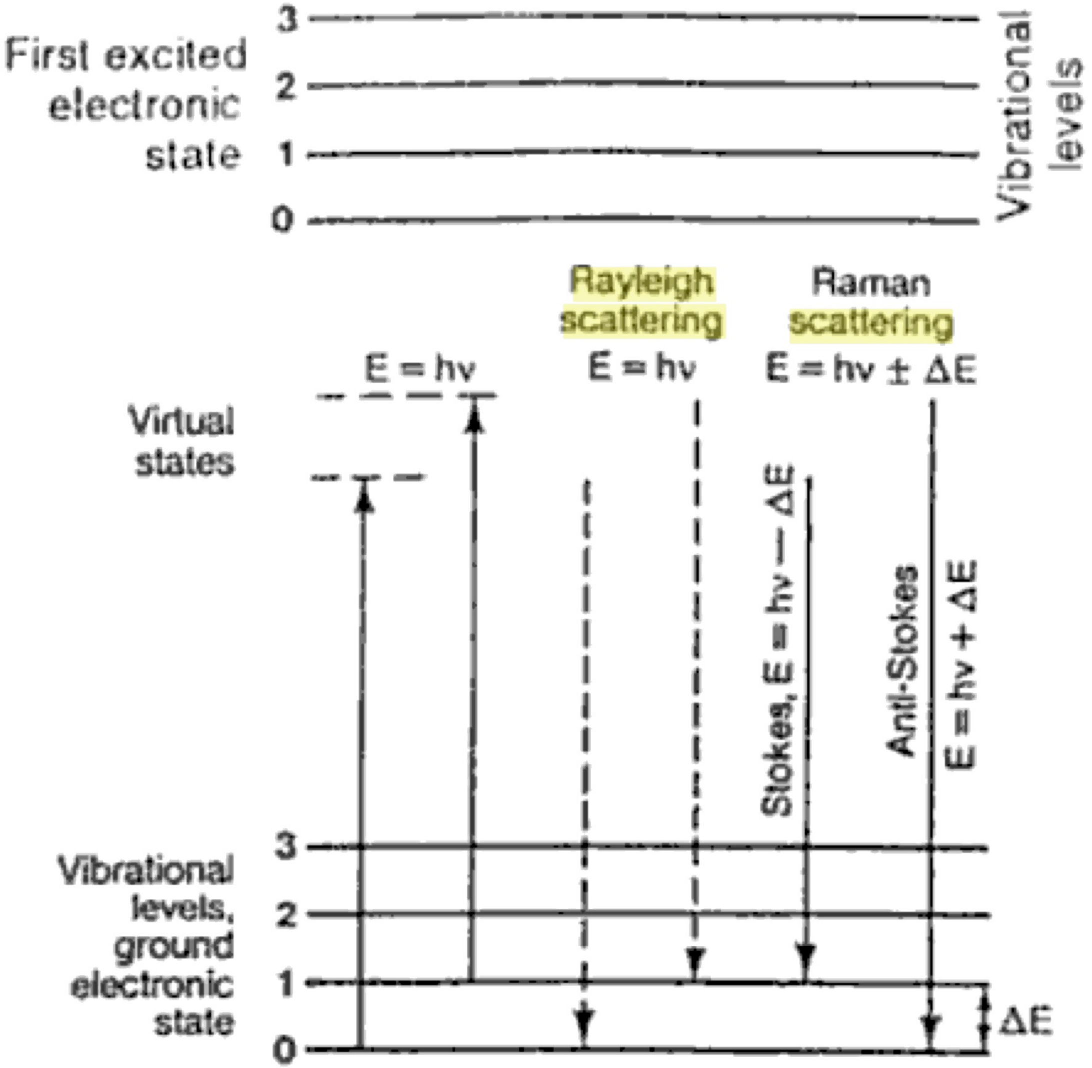


Figure 4.61 The process of Rayleigh and Raman scattering. Two virtual states are shown, one of higher energy. Rayleigh and Raman scattering are shown from each state. Normal IR absorption is shown by the small arrow on the far right marked ΔE, indicating a transition from the ground state vibrational level to the first excited vibrational level within the ground electronic state.

One in a million photons are Raman scattered.

Rayleigh scattering merely disperses radiation in space.

Stokes heats,
Anti Stokes cools.

Rayleigh Scattering (light scattering by air as dipole radiation)

$$\sigma_s = \frac{8\pi^3(m_r^2-1)^2}{3\lambda^4 N^2}\frac{6+3\delta}{6-7\delta} \qquad \beta_{sca}\left(\frac{1}{Mm}\right) = \sigma_s\left(\frac{cm^2}{molecule}\right) N\left(\frac{molecules}{cm^3}\right) 10^8\left(\frac{cm}{Mm}\right)$$

From Liou pg 93. δ is the molecular anisotropy parameter as the polarizability is really a tensor. The refractive index relationship is in relation to the polarizability of air. $\delta = 0.035$ for air.

$$(\mathrm{m_r} - 1)\, x\ 10^8 = 8{,}342.54 + \frac{2{,}406{,}147}{(130 - 1/\lambda^2)} + \frac{15{,}998}{(38.9 - 1/\lambda^2)}$$

Dry air, 15 C, 101325 Pa, 0.045% CO_2 by volume, vacuum λ in microns, (Birch, Metrologia, 1994, **31**, 315).

From http://www.kayelaby.npl.co.uk/general_physics/2_5/2_5_7.html.

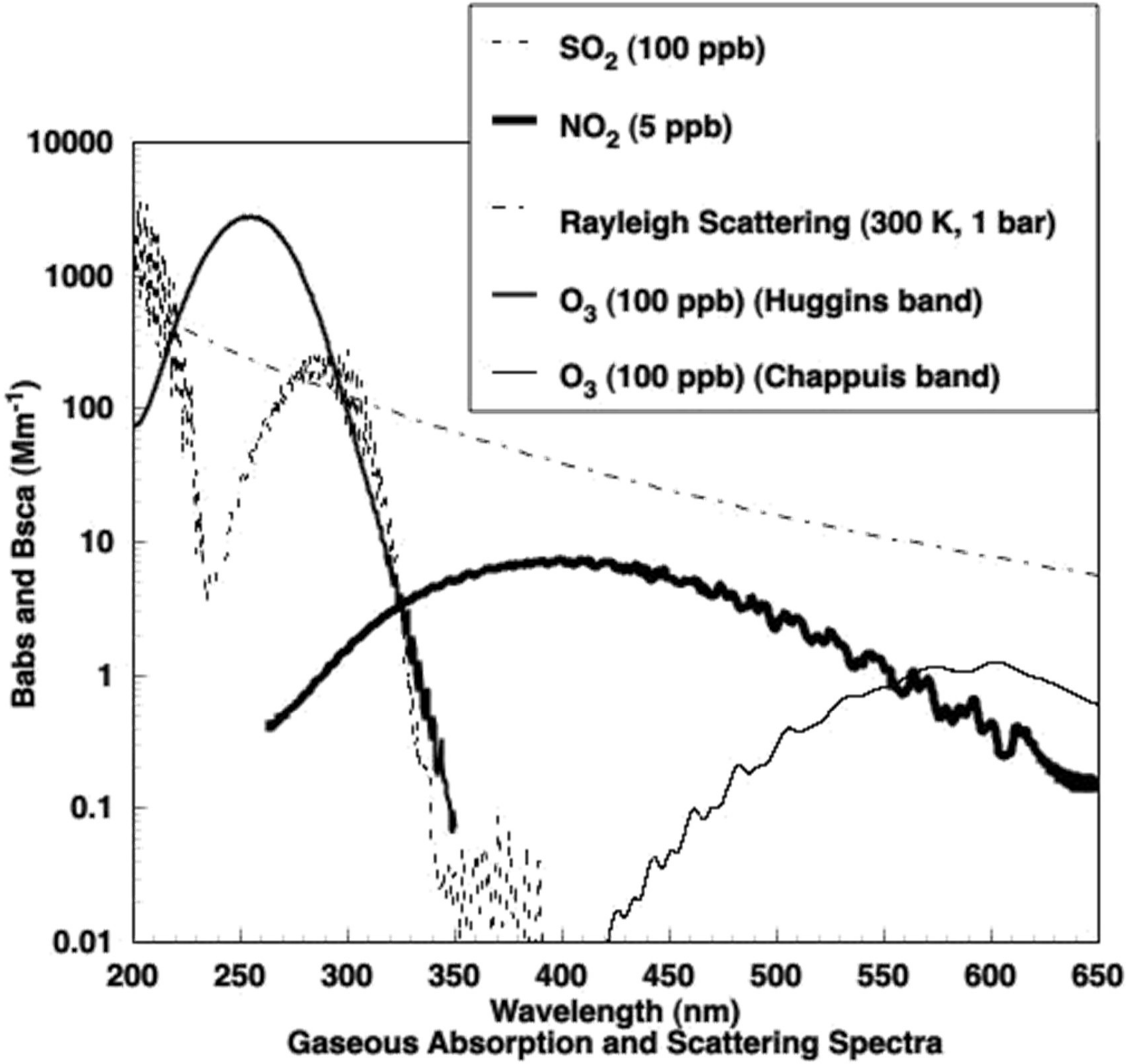
$$\mathrm{m_r(P,t)} - 1 = (\mathrm{m_r} - 1)\, x\ \frac{\mathrm{P[1 + P(60.1 - 0.972t)\ x\ 10^{-10}]}}{\mathrm{96\ 095.43(1 + 0.003\ 661t)}}$$

Dry air, t in Celcius, P in Pascal, 0.045% CO_2 by volume, Birch, Metrologia, 1994, **31**, 315).

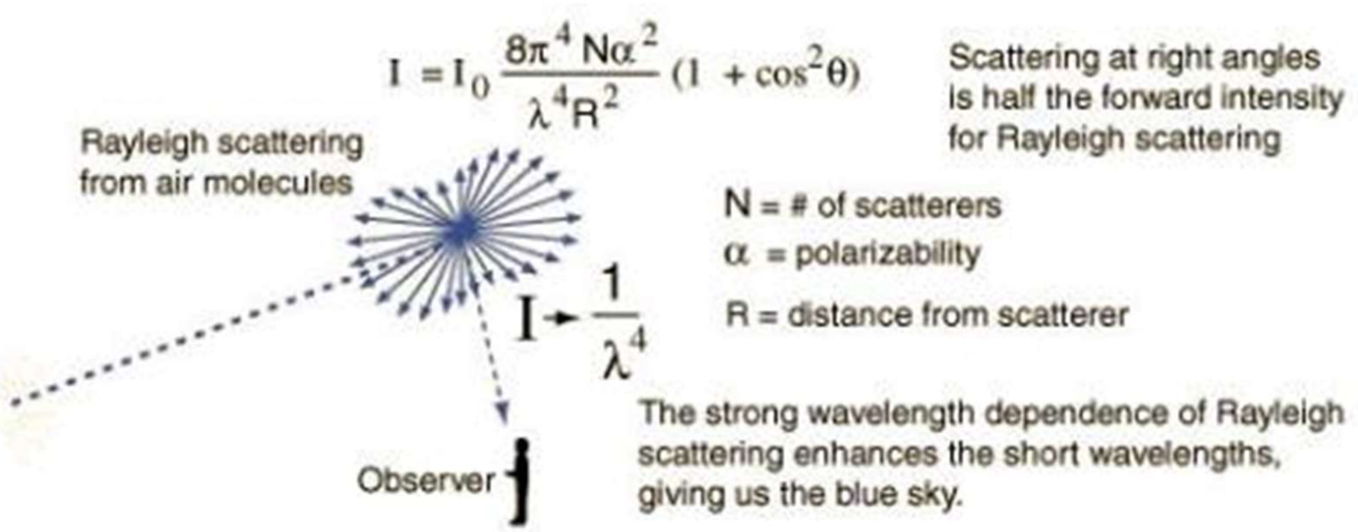
From http://www.kayelaby.npl.co.uk/general_physics/2_5/2_5_7.html.

$$N = \frac{P}{k_b T}$$ Number concentration of air molecules.

Rayleigh Scattering In Perspective Relative to Absorption



Rayleigh Scattering Intensity as a function of Scattering Angle.



$$I = I_0\frac{8\pi^4 N\alpha^2}{\lambda^4 R^2}(1 + \cos^2\theta)$$

Scattering at right angles is half the forward intensity for Rayleigh scattering

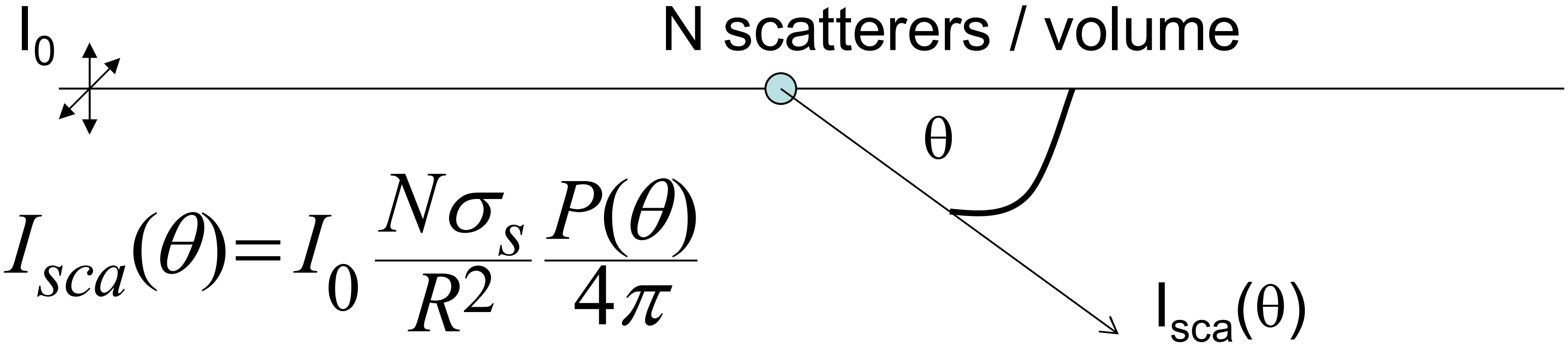
N = # of scatterers

α = polarizability

R = distance from scatterer

$I \rightarrow \frac{1}{\lambda^4}$

The strong wavelength dependence of Rayleigh scattering enhances the short wavelengths, giving us the blue sky.



$$I_{sca}(\theta)=I_0\frac{N\sigma_s}{R^2}\frac{P(\theta)}{4\pi}$$

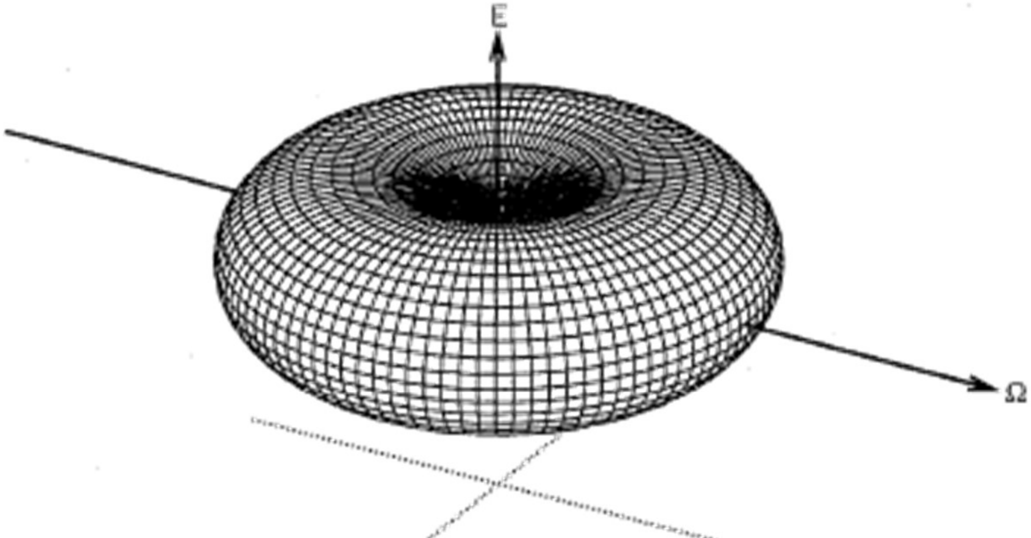
$$P(\theta)=\frac{3}{4}\left(1+\cos^2(\theta)\right)$$

Random E-field incident, random scatterer orientation.

From: http://www.jenkinsdisplays.com/led_bulbs/rayleigh_scattering.html

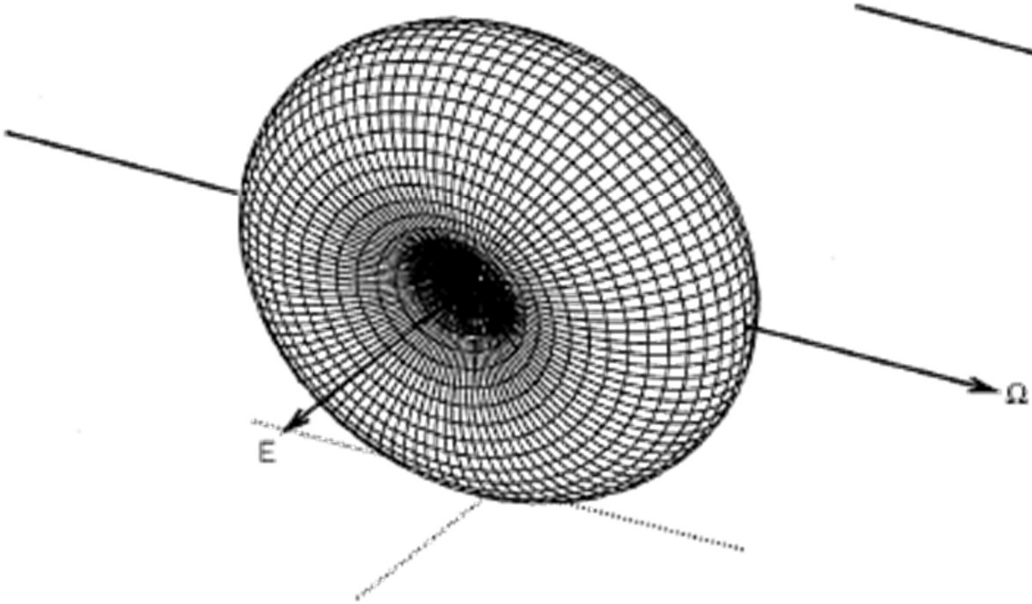
Dipole Radiation Pattern: (Petty, Ch12).

Incident E-field vertical: Dipole charge oscillation vertical.

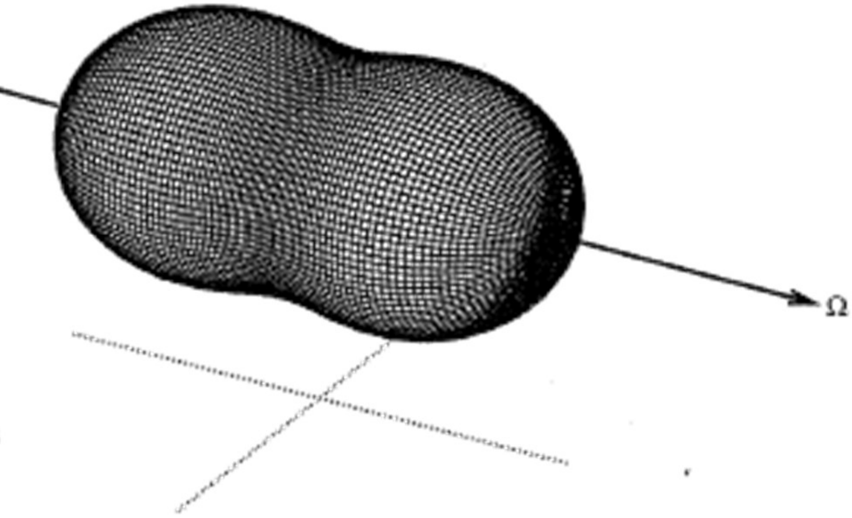


Ω = incident direction

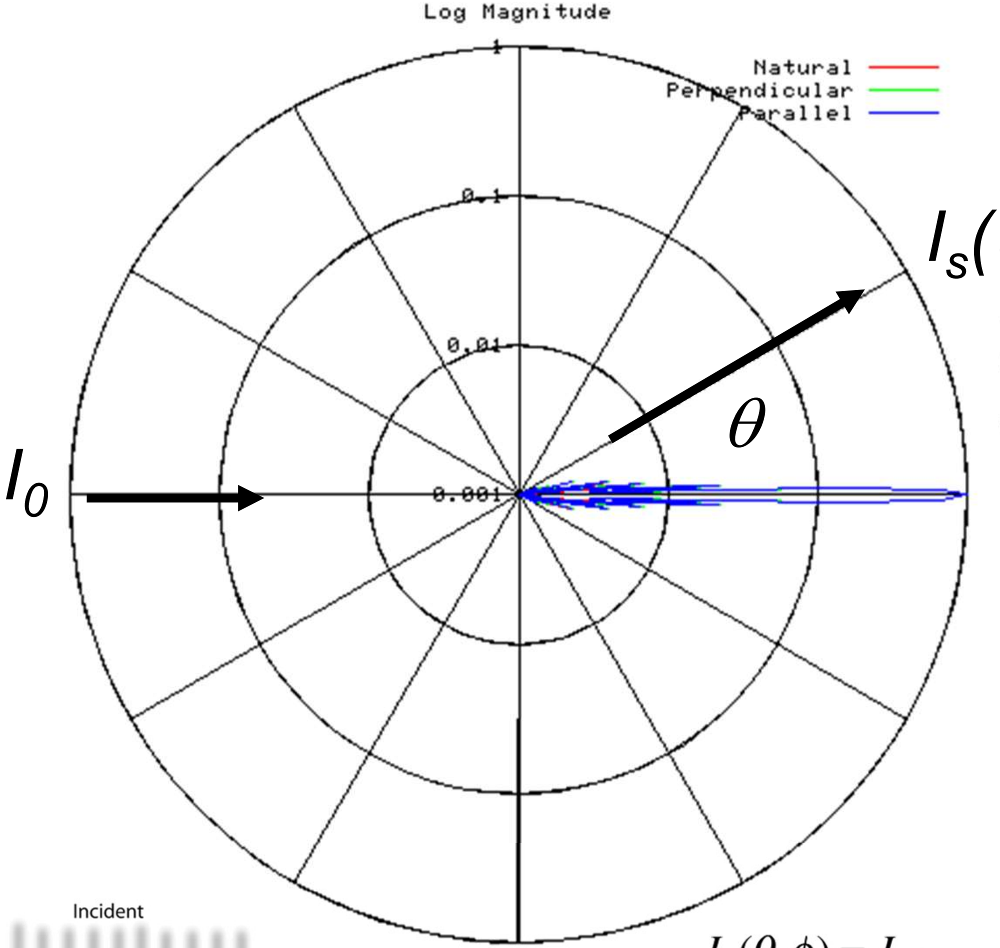
Incident E-field Horizontal

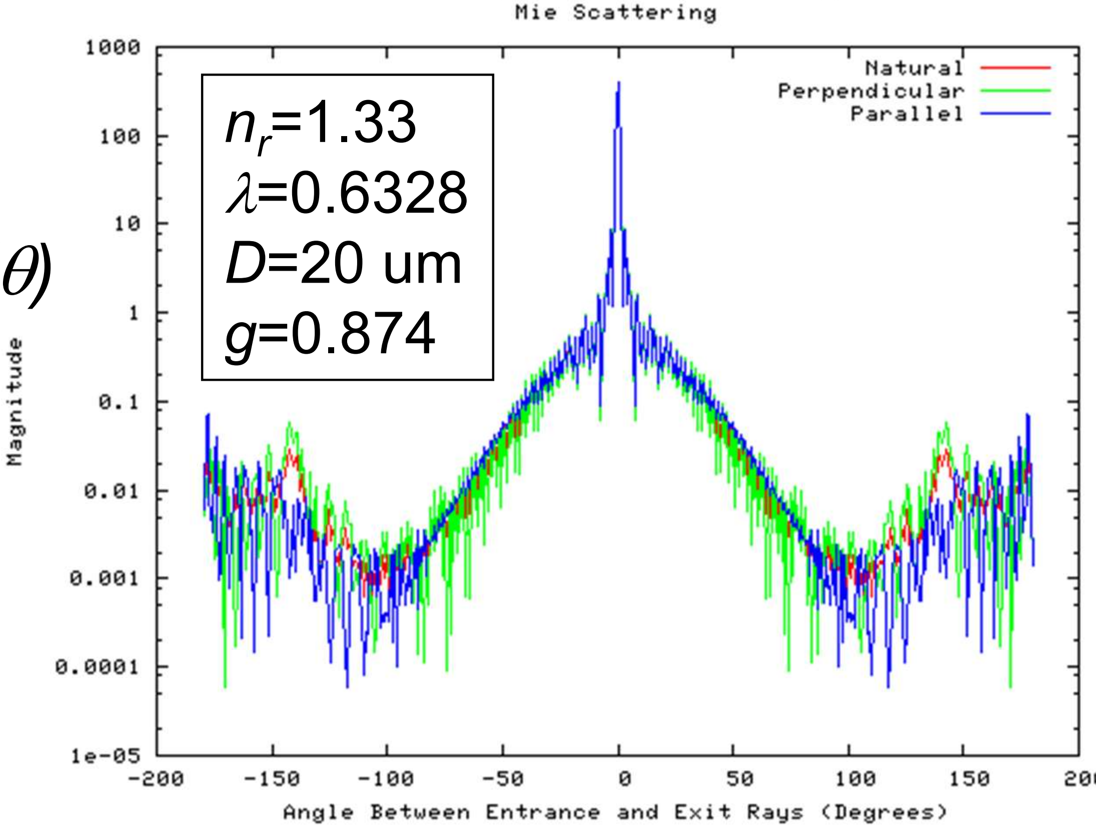


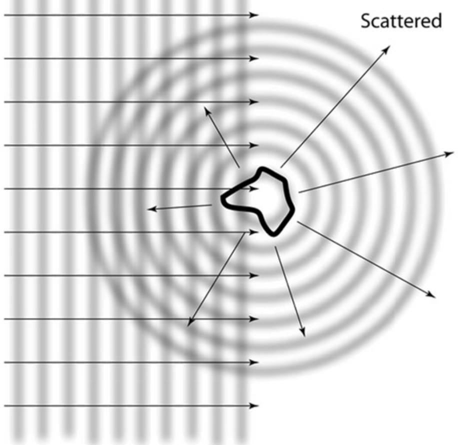
Irradiance Average for Random E-field: sum of the polarized patterns / 2.



Aside: Asymmetry Parameter of Scattering, g. -1<g<1







$$I_s(\theta,\phi)=I_0\,\frac{\dfrac{d\sigma_{sca}(\theta,\phi)}{d\Omega}}{\sigma_{sca}=\int\left(\dfrac{d\sigma_{sca}}{d\Omega}\right)\,d\Omega}$$

$$g = asymmetry\ \ parameter = \frac{\int\left(\dfrac{d\sigma_{sca}}{d\Omega}\right)\cos(\theta)\ d\Omega}{\int\left(\dfrac{d\sigma_{sca}}{d\Omega}\right)\,d\Omega},$$

$$-1 \le g \le 1$$

$$P_{\downarrow\downarrow}=P_{\uparrow\uparrow}=\frac{1+g}{2}$$

$$P_{\downarrow\uparrow}=P_{\uparrow\downarrow}=\frac{1-g}{2}$$

‘Typical’ Water Droplet Cloud Optical Properties

2688 JOURNAL OF CLIMATE VOLUME 16

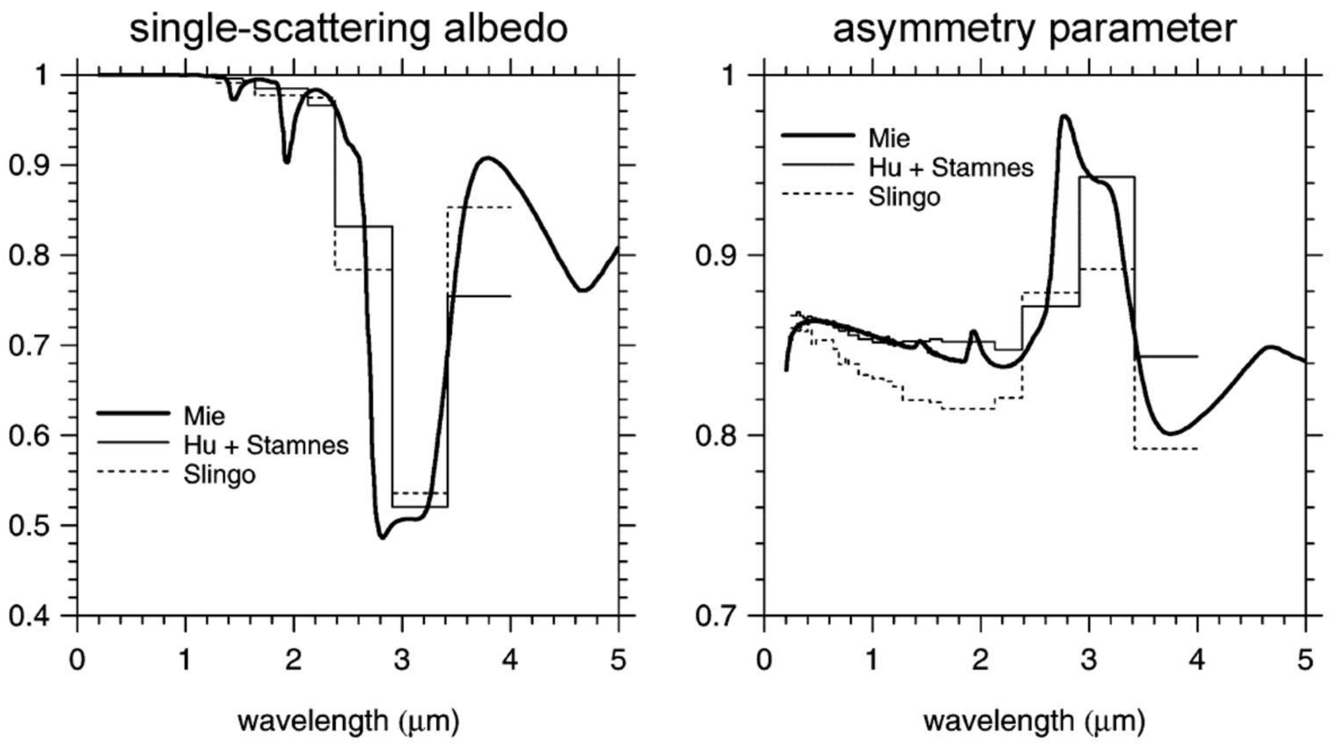
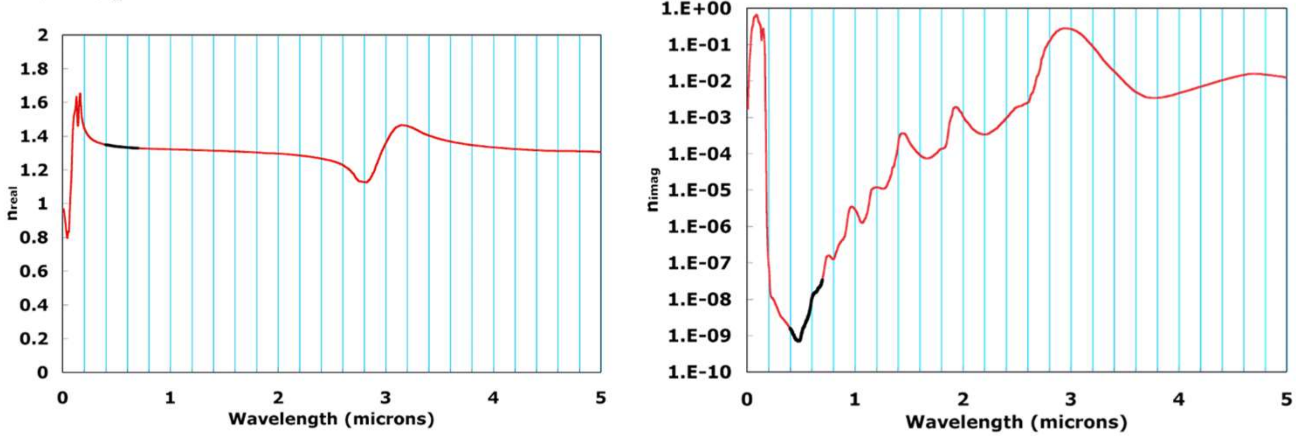


FIG. 7. Cloud droplet single-scattering albedo and asymmetry parameter as a function of wavelength based on Mie calculations for a gamma droplet size distribution with r_e = 10 μm and effective variance of 0.1. Also shown are values from Slingo's (1989) and Hu and Stamnes's (1993) parameterizations.

D_{eff} = 20 um
Variance = 0.1

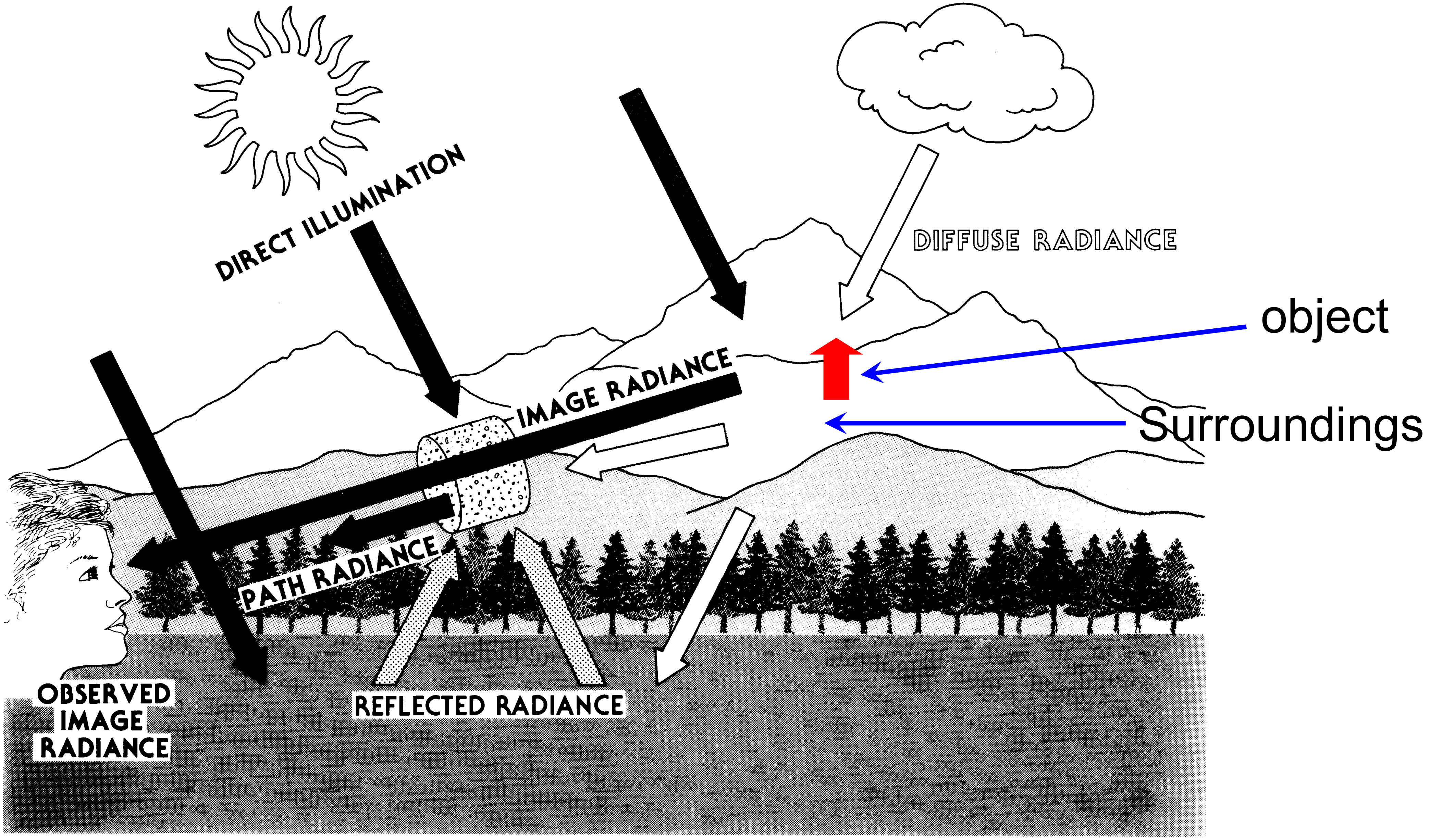
Why does the single scatter albedo go so low at around 3 microns?

Why does the asymmetry parameter go so large at around 3 microns?



COMPLEX REFRACTIVE INDEX OF WATER: Visible in black.

Optics of Visibility: I=Radiance, Radiant Intensity
C= Visual Contrast = $(I_{surroundings} - I_{object}) / I_{surroundings}$

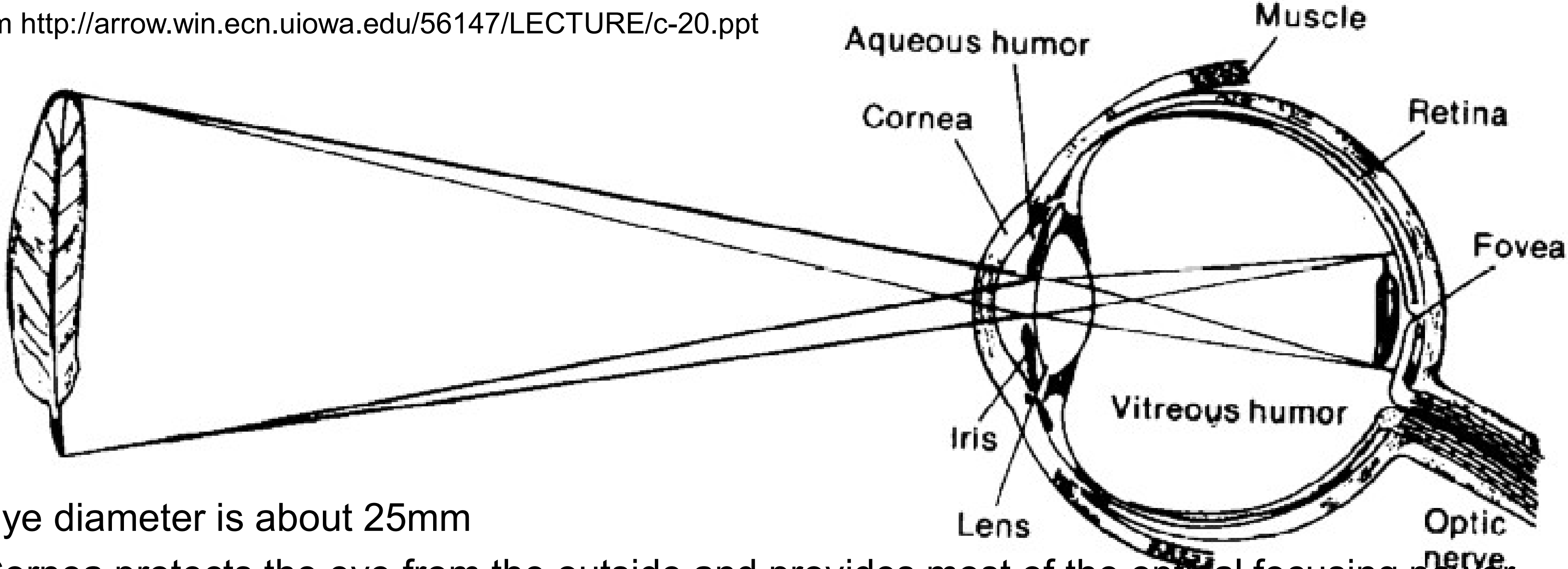


from http://vista.cira.colostate.edu/improve/Education/Workshops/WESTAR/Malm/partiii_malm.ppt

The Anatomy of the Human Visual System

- The human eye

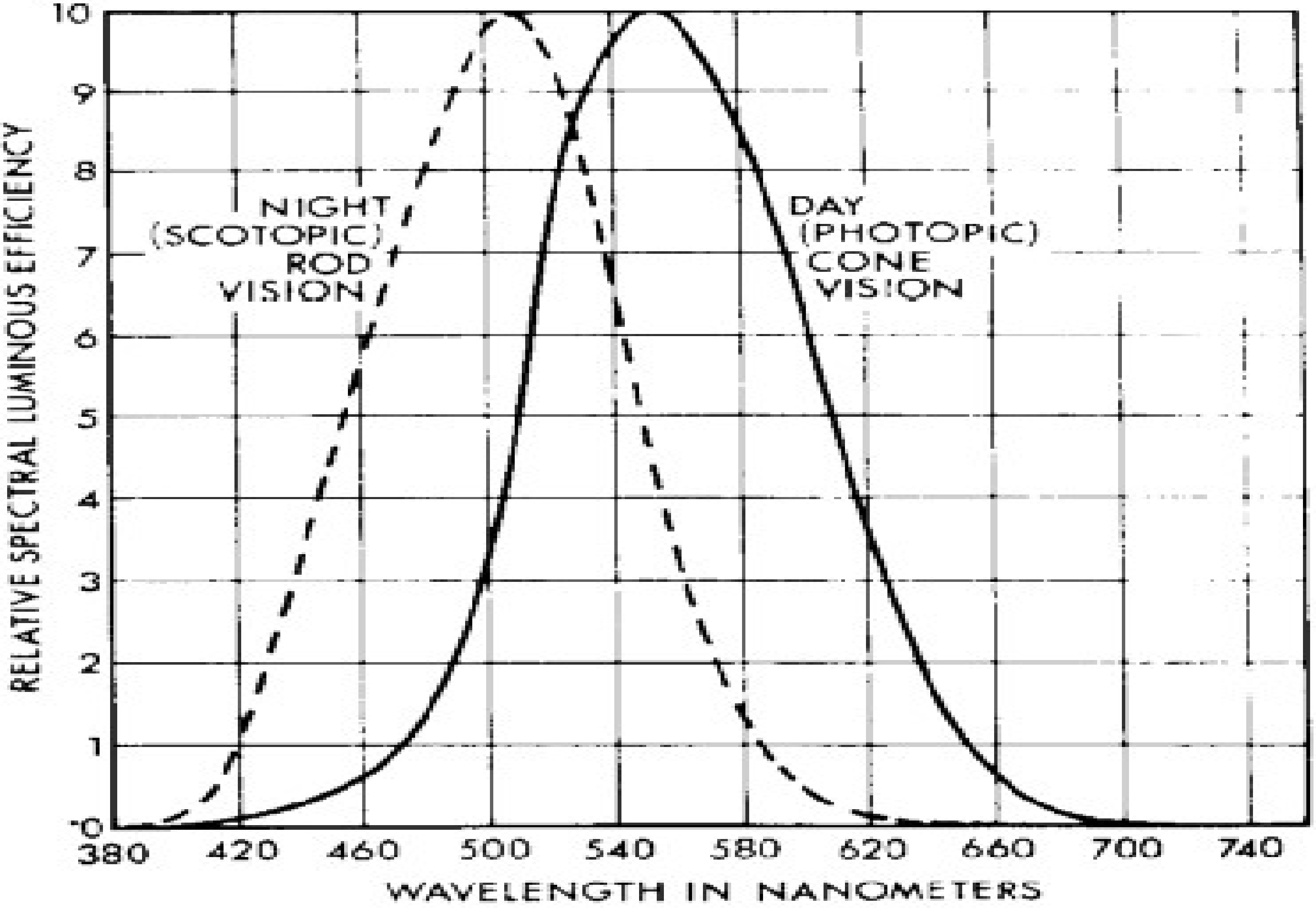
from http://arrow.win.ecn.uiowa.edu/56147/LECTURE/c-20.ppt



- Eye diameter is about 25mm
- Cornea protects the eye from the outside and provides most of the optical focusing power
- Aqueous humor provides nourishment to cornea and lens
- Iris regulates the amount of light that is allowed into the eye
  - Pupil is the aperture left open by the iris
  - Small pupil under high luminance conditions provides sharp image (pinhole camera effect)
  - Adjustment of pupil gets more difficult with age
- Focal length of lens is adjusted by ciliary muscle
  - focal distance, diopters (1/focal length)
  - lens gets harder with age and accommodation becomes more difficult

From Proctor and VanZandt (1993)

Daytime and Nightime Sensitivity of the Eye



from http://arrow.win.ecn.uiowa.edu/56147/LECTURE/c-20.ppt