Chapter 3: Atmospheric Thermodynamics

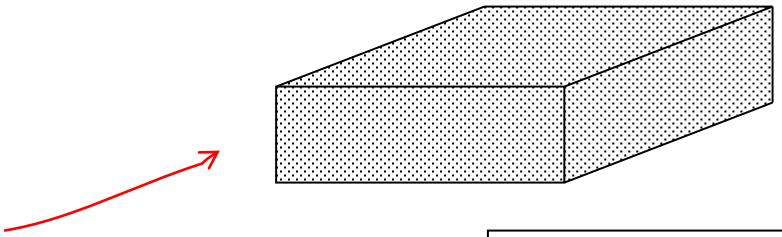
Objectives:

1. Demonstrate quantities used by Atmospheric Scientists to relate properties of air parcels aloft with those at the surface.
2. Develop increasingly more accurate models for the temperature, pressure, and density of dry and moist air in the atmosphere.
3. Lapse rate and the stability of dry and moist air parcels.
4. Energy and enthalpy applied to the atmosphere.

Should be familiar with these topics as we cover this chapter:

a. Ideal gas equation applied to dry and moist air.
b. Virtual temperature.
c. Potential temperature.
d. Hydrostatic equation.
e. Increasingly detailed description of the temperature and pressure distribution in the atmosphere.
f. SkewT logP diagrams.
z. Relative humidity, absolute humidity.
g. Dew point temperature.
h. Wet bulb temperature.
i. Equivalent potential temperature.
j. Latent heat release and absorption in condensation and evaporation of water.
k. Stability of air parcels.
l. Indices on soundings. Lapse rate, adiabatic lapse rate, deviations from adiabatic lapse rate, pseudoadiabats.

Equation of State for an Ideal Gas: Air



1. molecule size is ignorable.
2. molecules don’t interact (attract or repel each other).
3. molecular collisions are like hard point like spheres.

Most primitive, intuitive form of the I.G.L. (ideal gas law):
$PV = NkT$

V = volume

P = pressure
N = # molecules
T = absolute temperature (Kelvin)
k = Boltzmann’s constant = 1.38 x 10^{-23} Joules / (molecule K)

Now we manipulate to find a satisfying form of the I.G.L for analysis:

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Various Equivalent Forms of the I.G.L.

Start with the primitive form of the I.G.L.: $PV = NkT$

$n \equiv \frac{N}{N_a}$ $N_a = 6.022x10^{23}$ (molecules per mole) or $N_a = 6.022x10^{26}$ (molecules per kmole)

$m =$ mass of gas in volume V (kg units). M_w = molecular mass (kg/kmole)

$n = \frac{m}{M_w}$. $R^* = N_a k = 8314.3 \frac{Joules}{kmole\ K}$ (universal gas constant)

Then

$PV = nR^*T$. Use n, $PV = m \frac{R^*}{M_w} T$. $Motivates\ R \equiv \frac{R^*}{M_w}$ = gas constant.

Let $\alpha \equiv \frac{V}{m}$ = specific volume. Let $\rho \equiv \frac{m}{V}$ = gas density.

So ... $P\alpha = RT$ $P = \rho RT$.

Note the useful bottom line form $P\alpha=RT$: We will use this most often.

Partial Pressure and Ideal Gas Mixtures

$$PV = nR^*T \quad , \quad n = n_{N_2} + n_{O_2} + n_{Ar} + n_{CO_2} + n_{H_2O} + \ldots = \sum_{i=1}^{All\ Gases} n_i$$

$$P = \sum_{i=1}^{All\ Gases} P_i \quad , \quad P_i = \text{Partial Pressure of ith gas} = \frac{n_i\ R^*T}{V} \ .$$

Each gas separately obeys the ideal gas law (*I.G.L.*)

$$n_i = \text{\# kmoles of component i} = \frac{m_i}{M_{w_i}} \quad .$$

$$P_i = \frac{m_i}{V}\,\frac{R^*}{M_{w_i}}\,T = \rho_i\ R_i\ T\ .$$

$$\alpha_i \equiv \frac{1}{\rho_i} \ .$$

So, $P_i\ \alpha_i = R_i\ T$.

EACH GAS SEPARATELY OBEYS THE IDEAL GAS LAW.

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Applications of Dalton’s Law of Partial Pressures…

What is the total pressure in the room?

What is the partial pressure due to nitrogen molecules N_2*?*

What is the partial pressure due to oxygen molecules, O_2*?*

What is the partial pressure due to carbon dioxide molecules, CO_2*?*

Applications of Dalton's Law of Partial Pressures…

What is the total pressure in the room? *860 mb ish.*

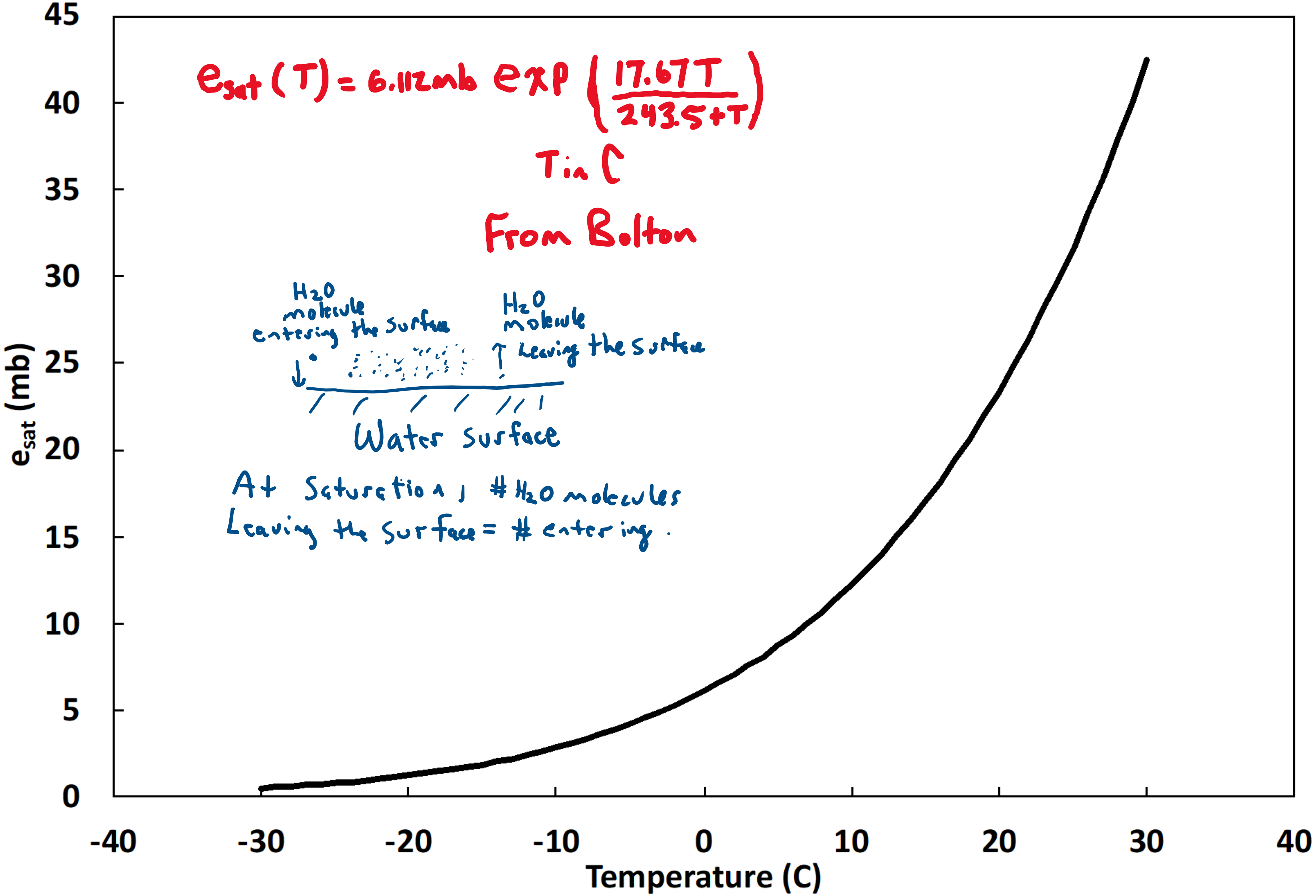
What is the partial pressure due to nitrogen molecules N_2*?*
*860 mb * 0.78 = 670 mb. Air is composed of 78%* N_2 *molecules.*

What is the partial pressure due to oxygen molecules, O_2*?*
*860 mb * 0.21 = 180 mb.*

What is the partial pressure due to carbon dioxide molecules, CO_2*?*
*860 mb * 0.000385 = 0.34 mb.*

For 10 mb water vapor partial pressure, air is about 1% water vapor.

Water Vapor Saturation Vapor Pressure



$$e_{sat}(T) = 6.112\text{mb}\ \exp\left(\frac{17.67T}{243.5+T}\right)$$

T in C

From Bolton

At Saturation, # H_2O molecules Leaving the surface = # entering.

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Applications of Dalton's Law of Partial Pressures…

Wait a minute… how can it be that these molecules apply pressure according to their number concentration? They don't all have the same mass… What is going on?

The fine print from Wikipedia…

Dalton's law is not exactly followed by real gases. Those deviations are considerably large at high pressures. In such conditions, the volume occupied by the molecules can become significant compared to the free space between them. Moreover, the short average distances between molecules raises the intensity of intermolecular forces between gas molecules enough to substantially change the pressure exerted by them. Neither of those effects are considered by the ideal gas model.

The Earth's atmosphere is a dilute gas, so the ideal gas equation is a reasonable approximation.

Kinetic Theory of Pressure (Wikipedia…)

Pressure [edit]

Pressure is explained by kinetic theory as arising from the force exerted by liquid molecules impacting on the walls of the container, which shows that the molecules of liquid would need less energy at the surface of the liquid to leave. Consider a gas of *N* molecules, each of mass *m*, enclosed in a cuboidal container of volume $V=L^3$. When a gas molecule collides with the wall of the container perpendicular to the *x* coordinate axis and bounces off in the opposite direction with the same speed (an elastic collision), then the momentum lost by the particle and gained by the wall is:

$$\Delta p = p_{i,x} - p_{f,x} = 2mv_x$$

where v_x is the *x*-component of the initial velocity of the particle.

The particle impacts one specific side wall once every

$$\Delta t = \frac{2L}{v_x}$$

(where *L* is the distance between opposite walls).

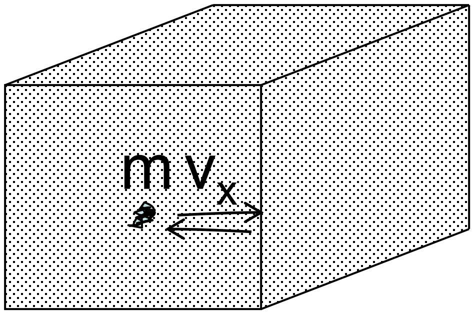
The force due to this particle is:

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_x^2}{L}.$$

The total force on the wall is

$$F = \frac{Nm\overline{v_x^2}}{L}$$

Box of sides L



Nature is fair …
On average, molecules share the burden of random kinetic energy, also known as heat. K.E.=$mv^2/2$. On average, molecules with smaller m move faster than large m molecules.

where the bar denotes an average over the *N* particles. Since the assumption of molecular chaos imposes $\overline{v_x^2} = \overline{v^2}/3$, we can rewrite the force as

$$F = \frac{Nm\overline{v^2}}{3L}.$$

This force is exerted on an area L^2. Therefore the pressure of the gas is

$$P = \frac{F}{L^2} = \frac{Nm\overline{v^2}}{3V}$$

where $V=L^3$ is the volume of the box. The fraction $n=N/V$ is the number density of the gas (the mass density $\rho=nm$ is less convenient for theoretical derivations on atomic level). Using *n*, we can rewrite the pressure as

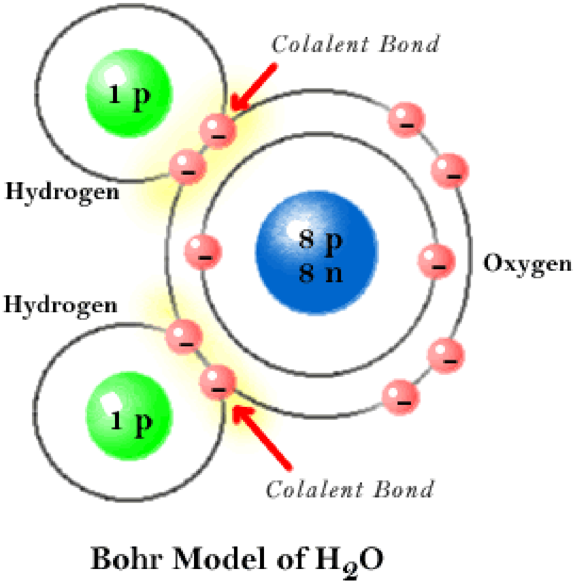
$$P = \frac{nm\overline{v^2}}{3}.$$

Pressure in the kinetic theory of gases …

This is a first non-trivial result of the kinetic theory because it relates pressure, a macroscopic property, to the average (translational) kinetic energy per molecule $\frac{1}{2}m\overline{v^2}$ which is a microscopic property.

Equipartition Theorem: $\frac{3}{2}kT = \frac{1}{2}m\bar{v}^2$

Special Case: Partial Pressure of Water Vapor, *e*



Bohr Model of H₂O

$$\boxed{\begin{array}{l} e\ \alpha_v = R_v\ T\ , \qquad \alpha_v = \dfrac{1}{\rho_v} \\ R_v = \dfrac{R^*}{M_{w_{H_2O}}} = \dfrac{8314.3\ J/kmole}{18\ kg/kmole} = 461\ \dfrac{J}{kg\ K} \end{array}}$$

Aside: Ratio of gas constants for dry air and water vapor.

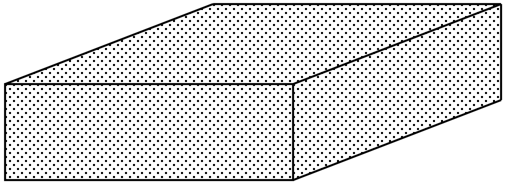
$$\frac{R_D}{R_v} = \frac{R^*/M_{w_{dry\,air}}}{R^*/M_{w_{H_2O}}} = \frac{M_{w_{H_2O}}}{M_{w_{dry\,air}}} = \frac{18}{29} \equiv \varepsilon\ .$$

$\varepsilon = 0.622$. We will use this often.

$$Note: R_D = 287\ \frac{J}{kg\ K}\ .$$

Virtual Temperature T_v.

dry air



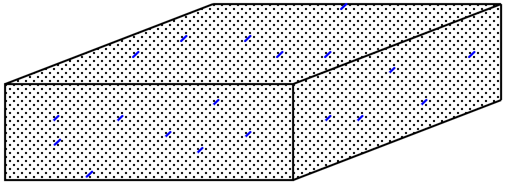
$P = P_D$

Total pressure=
partial pressure
due to dry air.

same for both

T = temperature
P = pressure
V = volume
N = # molecules

moist air



$P = P_D'+e$

Total pressure=
partial pressure
due to dry air + water vapor.

$$\rho_{\text{dry air}} > \rho_{\text{moist air}}$$

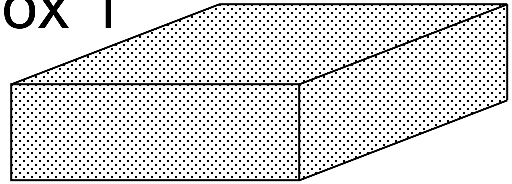
$$P = \rho RT$$

TWEAK …

Raise the temperature of the dry air on the left to lower its density so that it is the same as the density of the moist air on the right. We have to let some of the molecules out of the box.

This raised temperature is the virtual temperature by definition. It is a useful construct because the I.G.L. for dry or moist air is written $P\,\alpha = R_D\,T_v$.

Virtual Temperature T_v Calculation.



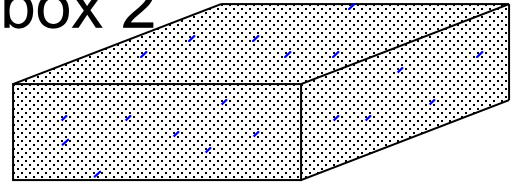
$P = P_D$ $\quad T_v$

Total pressure=
partial pressure
due to dry air.

same for both

T = temperature
P = pressure
V = volume

$$P = \rho RT$$



$P = P_D' + e$ $\quad T$

Total pressure=
partial pressure
due to dry air + water vapor.

Crank up the temperature of box 1, keeping pressure and volume constant (let some dry air molecules leak out), until the mass (density) of box 1 is the same as that of box 2.

From the I.G.L.

BOX 1 density = BOX 2 density

$$\rho_D = \frac{P}{R_D\,T_v} = \rho_D + \rho_v = \frac{P-e}{R_D\,T} + \frac{e}{R_v\,T}$$

Solve for T_v .

$$T_v = \frac{T}{1 - \frac{e}{P}(1-\varepsilon)}$$

General Note:
$T_v > T$.

Virtual Temperature Example

$$\boxed{T_v = \frac{T}{1-\frac{e}{P}(1-\varepsilon)}}$$

Let
e=10 mb
P=1000 mb
T=280 K
Remember
ε=0.622

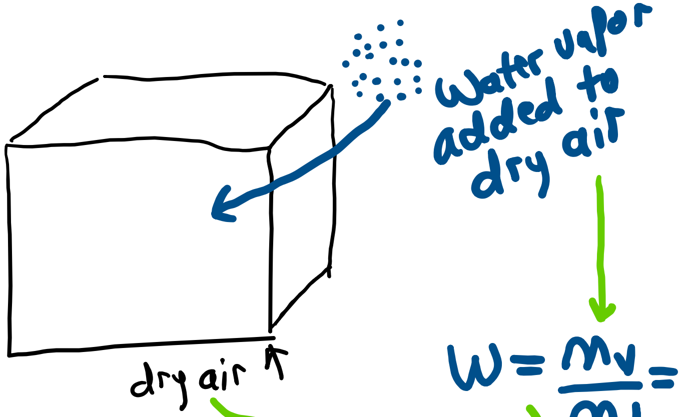
Then $T_v \approx T\,[1+e(1-\varepsilon)/P] = T(1+0.0038) \approx 281\text{ K}$

(binomial expansion was used to show an equivalent form)

This gives us a rough idea of the temperature increase needed to make dry air have the same density as the moist air described above.

Can use $T_v = T(1+0.61w)$ where
w = water vapor mixing ratio in $\frac{\text{Kg Water Vapor}}{\text{Kg Dry air}}$.

Water Vapor Mixing Ratio *w* [kg/kg] units



$$w = \frac{m_v}{m_d} = \frac{\rho_v}{\rho_d} = \frac{\frac{e}{R_v T}}{\frac{p-e}{R_d T}} = \epsilon \frac{e}{p-e} \approx \epsilon \frac{e}{p}$$

p = total Pressure

e = Partial Pressure due to Water Vapor

T = temperature

$$\epsilon = \frac{R_d}{R_v} = \frac{18\,\text{g/mole}}{28.97\,\text{g/mole}} = 0.622$$

Relative Humidity

$$RH = 100 \frac{w}{w_{sat}(T)}$$

$$RH = 100 \frac{w_{sat}(T_{dew})}{w_{sat}(T)}$$

Example:

$p = 1000$ mb

$T = 20C$

$w_{sat} = 14.54$ g/kg

$T_{dew} = -1C$

$w_{sat}(T_{dew}) = 3.53 \frac{g}{kg}$

$RH = 24.3\%$

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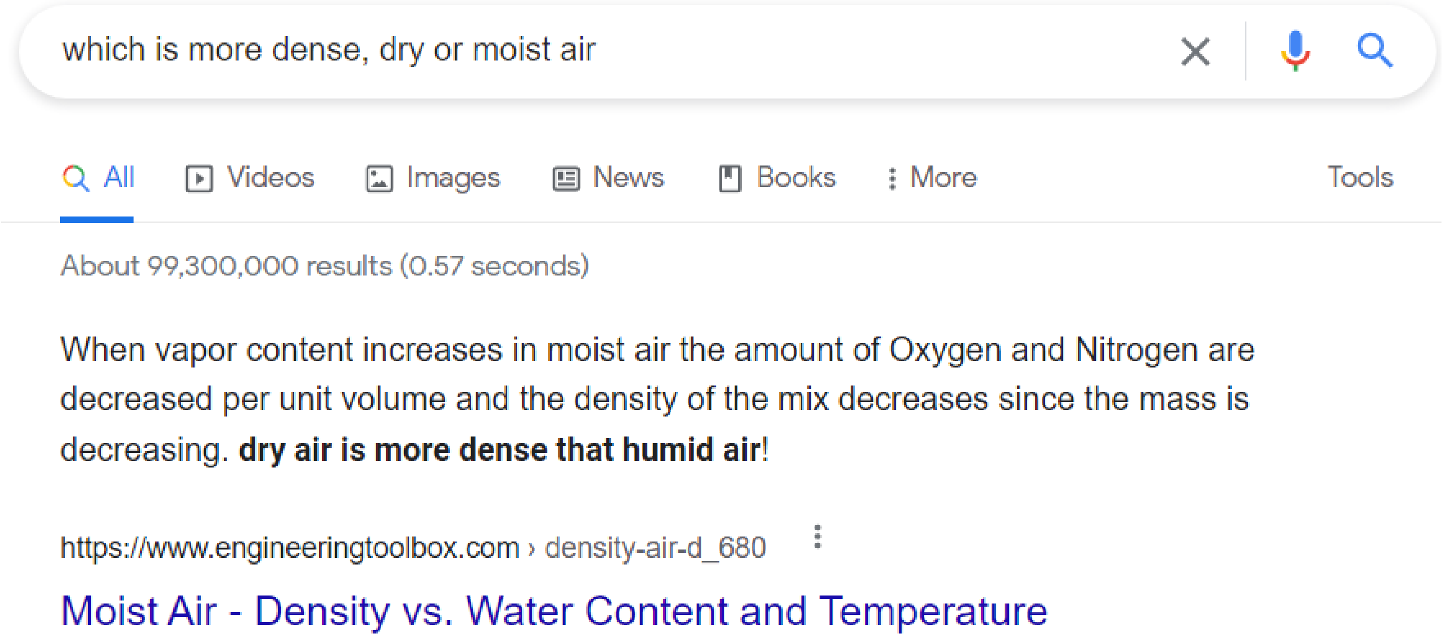
Which is more dense, dry or moist air, at the same temperature and pressure? Explain.

Which is more dense, dry or moist air, at the same temperature and pressure? Explain.



which is more dense, dry or moist air

About 99,300,000 results (0.57 seconds)

When vapor content increases in moist air the amount of Oxygen and Nitrogen are decreased per unit volume and the density of the mix decreases since the mass is decreasing. **dry air is more dense that humid air**!

https://www.engineeringtoolbox.com › density-air-d_680

Moist Air - Density vs. Water Content and Temperature

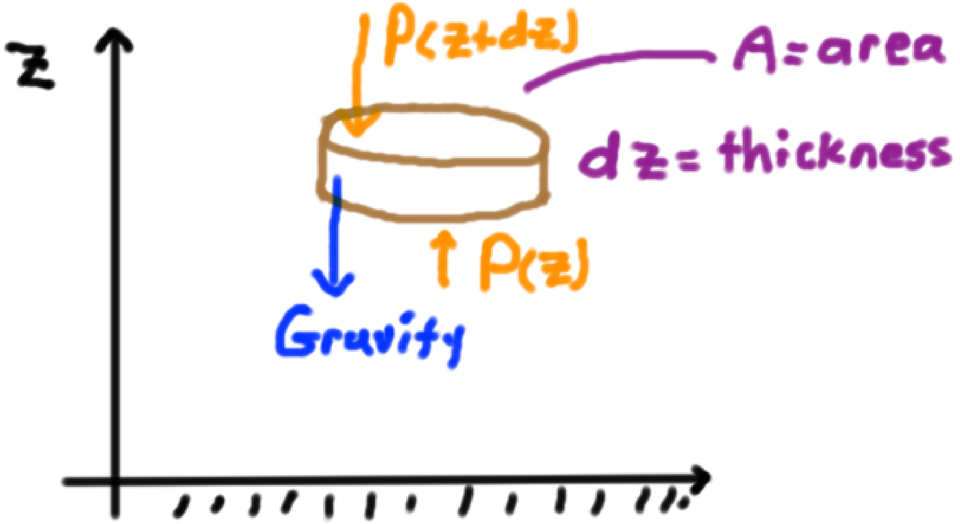
https://www.theweatherprediction.com/habyhints/260/

In short, the virtual temperature is greater than the air temperature.

Hydrostatic Equation

Fluid, gas or liquid

is not moving



$\underline{\underline{m}} = \rho(z) A dz$

= mass of slab

Newton's 2nd Law for the slab: $m\overset{\text{acceleration}}{a} = 0$

$0 = -\underline{\underline{m}}g + P(z)A - P(z+dz)A$

$P(z+dz) \approx P(z) + \frac{dP}{dz}dz$

So $\boxed{\frac{dP}{dz} = -\rho(z)g \quad \text{Hydrostatic Equation!}}$

Integrating, $\int_{P(z)}^{P(\infty)} dP(z) = -P(z) = -\int_{z}^{\infty} \rho(z) g dz$

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Hydrostatic Equation Ideas

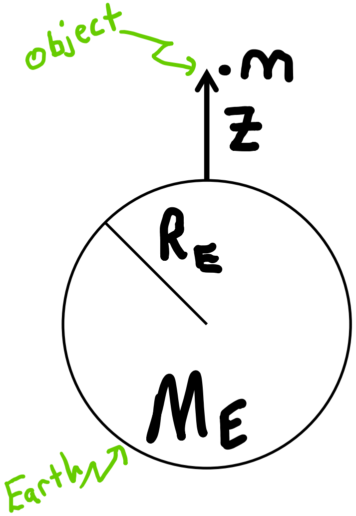
$$dP = -\rho g \, dz \qquad P_{(z)} = \int_{z}^{\infty} \rho g \, dz$$

It is the weight of the air above the person that gives rise to the observed pressure.

Pressure is less here ⇒ less atmosphere above.

Examples:

- Ocean
- Isothermal Atmosphere



Variation of g With Altitude

Let $r = R_E + z$, G = gravitation constant

$G = 6.67 \times 10^{-11}\ N\frac{m^2}{kg^2}$

Define: $mg(z) = \frac{GM_Em}{r^2}$

So letting $g_0 = \frac{GM_E}{R_E^2}$, $\boxed{g(z) = \frac{g_0}{(1+z/R_E)^2} \approx g_0\left(1 - \frac{2z}{R_E}\right)}$

Example:

$z = 10$ km ~ tropopause height

$R_E = 6370$ km

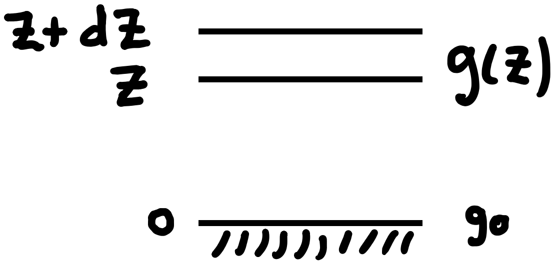
$g_0 = 9.8\ m/s^2$

Then:

$g(z = 10\,km) = 9.77\ m/s^2$

(only slightly less than g_0)

Geopotential (Potential Energy)



(Used to define layer thickness, 500mb height, etc)

$$d\Phi = \frac{\text{work}}{\text{mass}} \text{ to move air from } z \text{ to } z+dz$$

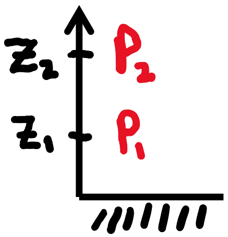
$$d\Phi = \underbrace{g(z)}_{\text{Force/mass}}\,\underbrace{dz}_{\text{distance}} = -\frac{dP}{\rho} \quad \text{from the hydrostatic Eq.}$$

$$\text{Geopotential} \equiv \Phi(z) = \int_0^z g(z')\,dz', \quad \Phi(0)=0$$

$$\boxed{\text{Geopotential Height: } \underbrace{Z}_{\text{No bar}} \equiv \frac{1}{g_0}\int_0^z g(z')\,dz' = \frac{1}{g_0}\Phi(z)}$$

Example: $z = 10\text{ km}, \quad Z = 9.986\text{ km}$

$Z \leq z$ in general



Thickness of Atmospheric Layers $Z_2 - Z_1$

$$d\Phi = -\frac{dP}{\rho} \quad , \quad \frac{1}{\rho} = \frac{R_D T_v}{P}$$

Virtual Temp.

Pressure

Dry air constant

Integrating,

$$Z_2 - Z_1 = \frac{1}{g_0}\int_{\Phi_1}^{\Phi_2} d\Phi = \frac{\Phi_2 - \Phi_1}{g_0} = -\frac{R_D}{g_0}\int_{P_1}^{P_2} T_v \underline{\underline{\frac{dP}{P}}} = \frac{R_D}{g_0}\int_{P_2}^{P_1} T_v \underline{\underline{d(\ln P)}}$$

Note: $\frac{dP}{P} = d(\ln P)$

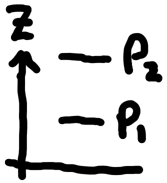
Summarizing,

The thickness of the atmospheric layers between P_1 and P_2 is...

$$\boxed{Z_2 - Z_1 = \frac{R_D}{g_0}\int_{P_2}^{P_1} T_v \, d(\ln P)}$$

Isothermal Atmosphere Approximation

(temperature the same throughout!)



$$Z_2 - Z_1 = \frac{R_0 T_v}{g_0}\int_{P_2}^{P_1} d(\ln P) = \frac{R_0}{g_0} T_v \underbrace{(\ln P_1 - \ln P_2)}_{\ln(P_1/P_2)}$$

$$P_2(z) = P_1(z)\, e^{-\frac{(Z_2-Z_1)}{H}}$$

H ≡ Scale height of atmosphere

$$H = \frac{R_0 T_v}{g_0} = 287 \frac{J}{kg\,K}\,\frac{1}{9.8}\,\frac{kg}{N}\,T_v$$

$$\boxed{H = 29.3\, T_v \ \text{meters} \quad \text{Use Kelvin units}}$$

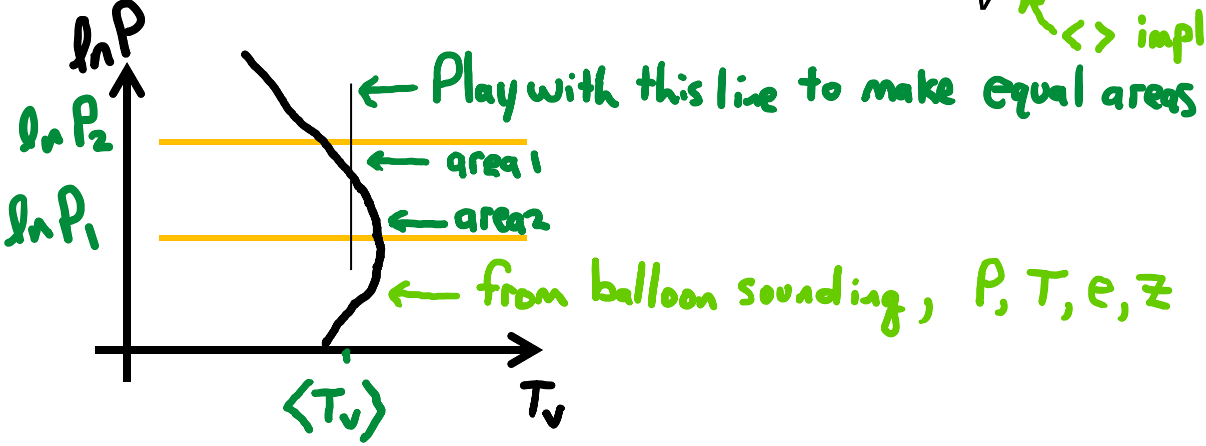
Example: $T_v = 288\,K = 15C$,

$H = 8.5$ km

Next up:
fix the isothermal assumption

Layer Thickness From H=29.3<T_v> Idea

< > implies mean value



Play with this line to make equal areas

area 1

area 2

from balloon sounding, P, T, e, Z

Define: $\langle T_v \rangle$ = mean virtual temperature of layer

$$\langle T_v \rangle = \int_{P_2}^{P_1} T_v \, d(\ln P) \Big/ \ln(P_1/P_2)$$

Then: $Z_2 - Z_1 = \langle H \rangle \ln\left(\frac{P_1}{P_2}\right)$, $\langle H \rangle = 29.3 \langle T_v \rangle$ meters

height → Hypsometric Equation ← measure

Conceptually: As $\langle T_v \rangle$ increases, the air between 2 pressure levels expands So the layer thickness $Z_2 - Z_1$ increases

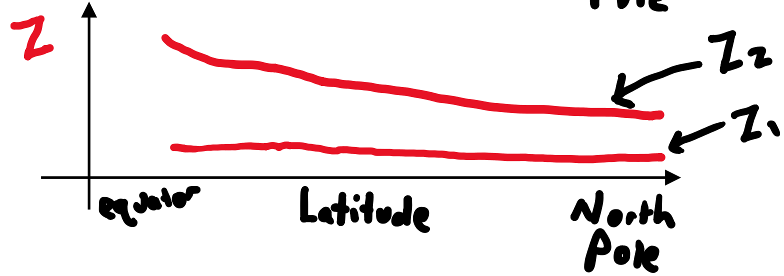
Conceptual Example

Determine the 500 mb geopotential height Z_2 for balloon soundings launched at sea level.

P_1 = Surface Pressure at Z_1=0 meters.

P_2 = 500 mb Pressure at Z_2= ?

$$\boxed{Z_2 = Z_1 + \frac{R_0 \langle T_v \rangle}{g_0} \ln\left(P_1/P_2\right)}$$



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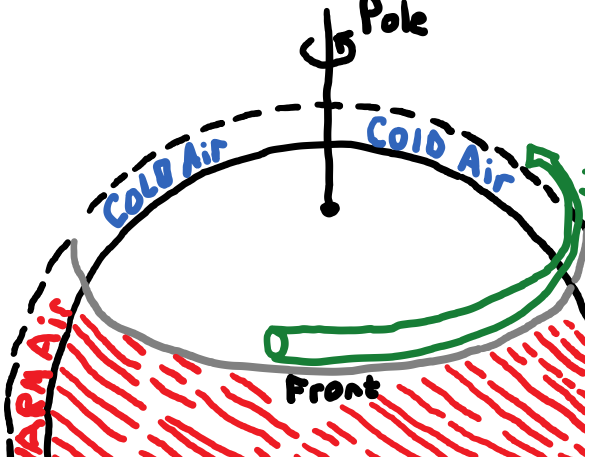
Clash of Warm and Cold Air on a Frontal Zone: Idealized example with zonal flow



Zonal flow:
No meander of the jet stream.

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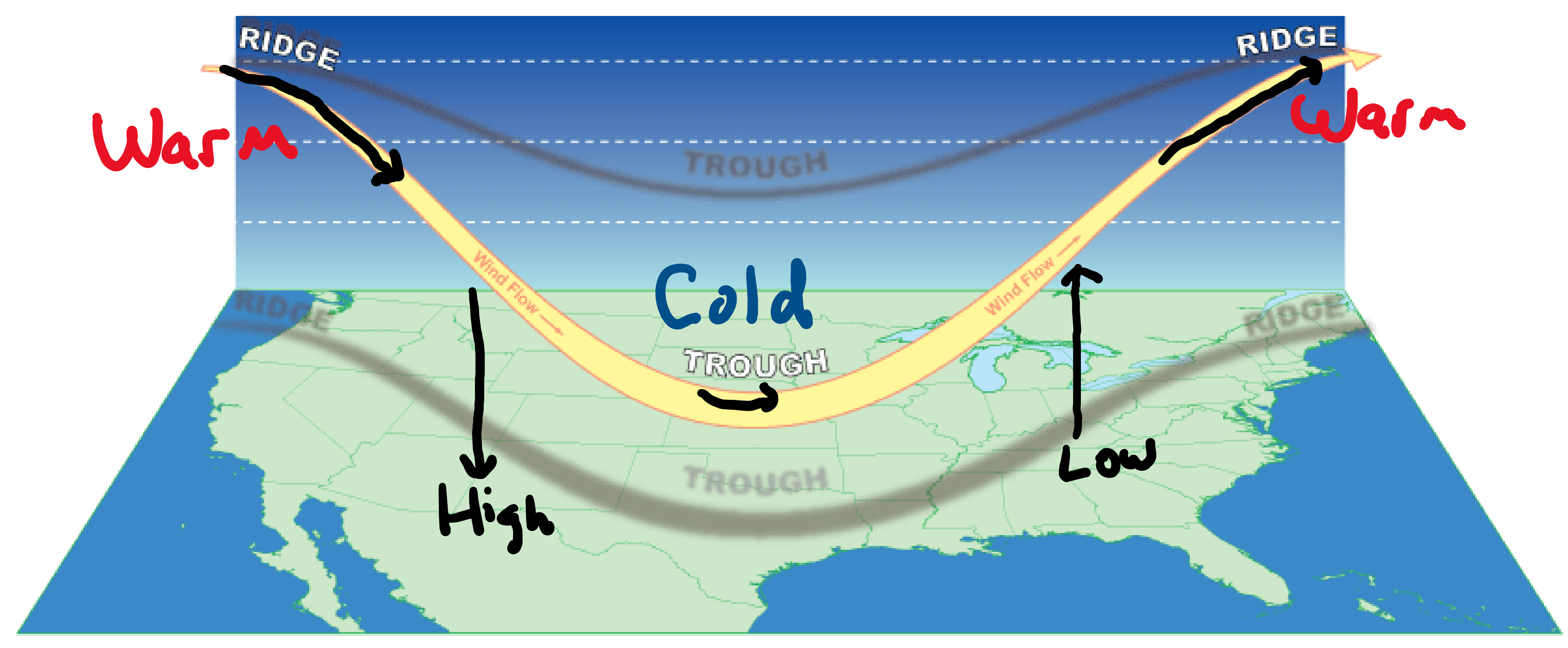
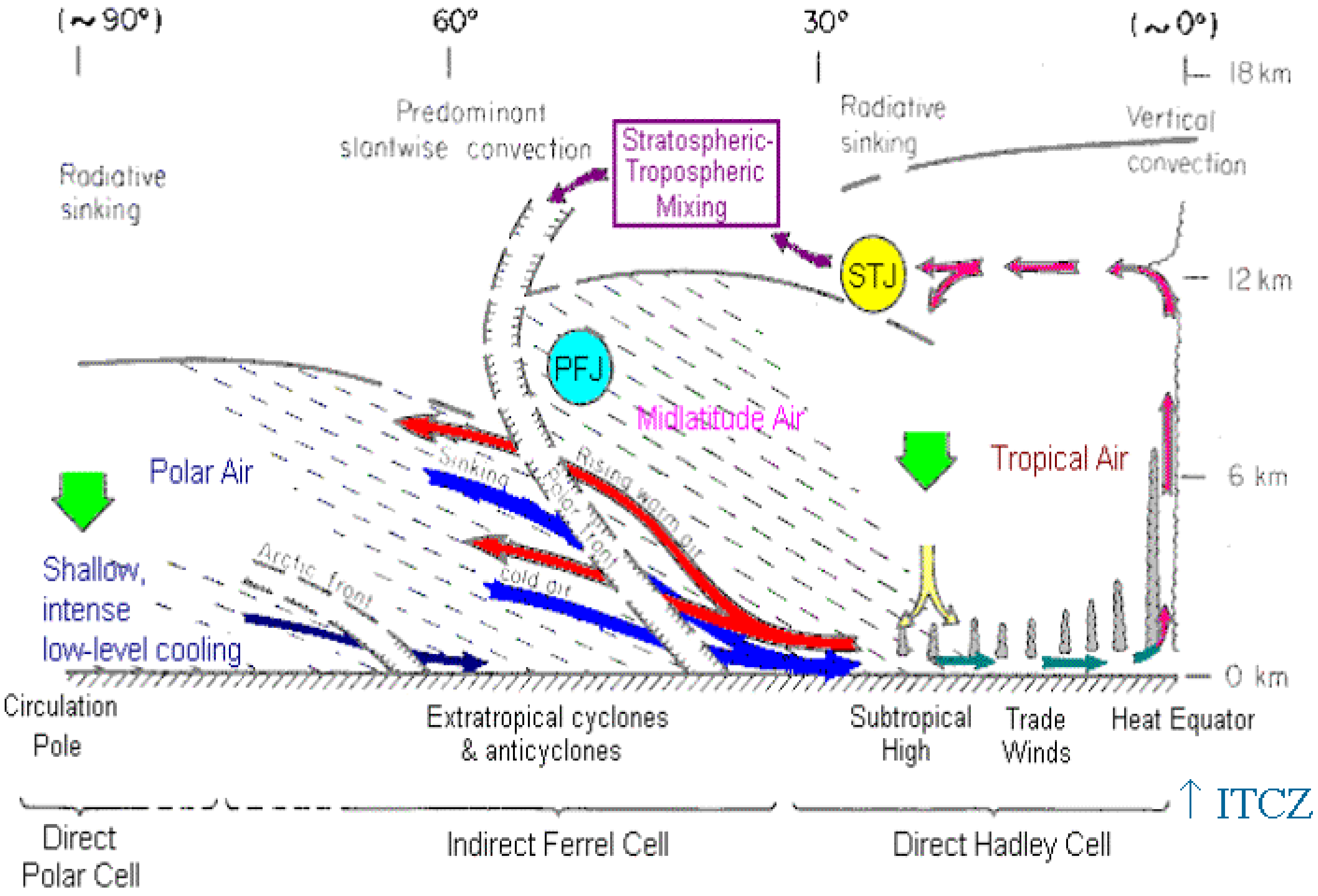
Adapted from Reiter, *Jetstreams*

Meander of the Jet Stream



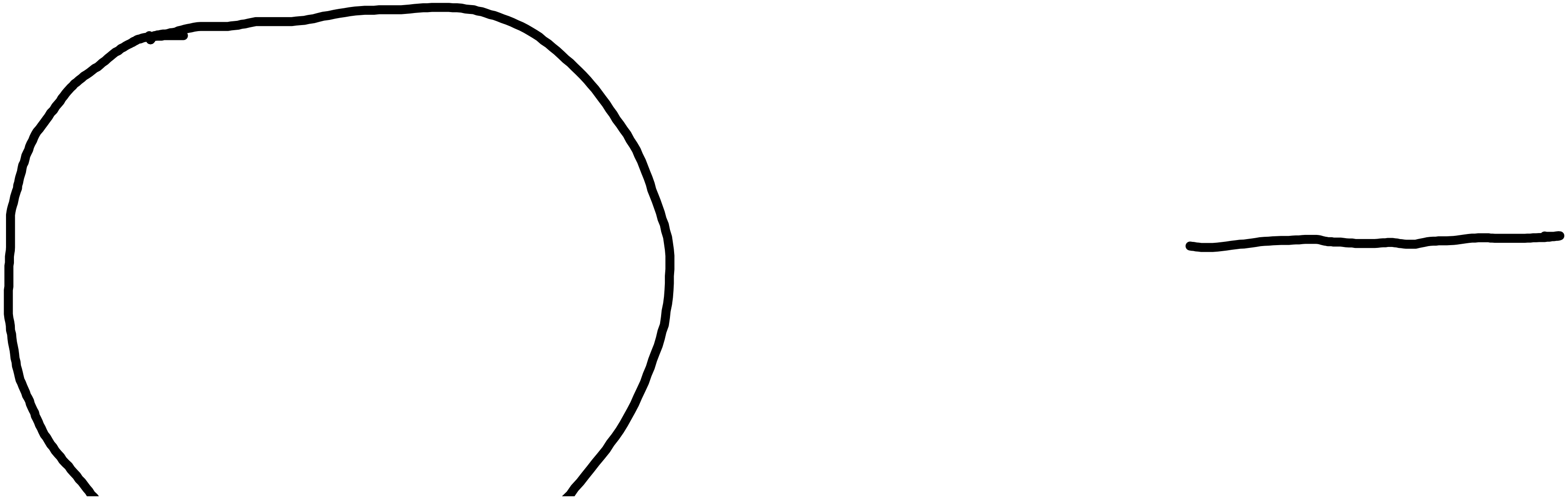
Wind flowing from a ridge toward a trough is decreasing in height above the surface. Conversely, wind flowing from a trough into a ridge is increasing in height. Where is it cold and warm?
From: https://www.weather.gov/jetstream/verses

More Comprehensive View



From: http://das.uwyo.edu/~geerts/cwx/notes/chap01/tropo.html

Geostrophic Wind: Balance between the pressure gradient force and the Coriolis force. Coordinate System and Definition of Ω.



Relation between geopotential height and Wind

Example: 500mb Surface ~ 5600 m above sea level

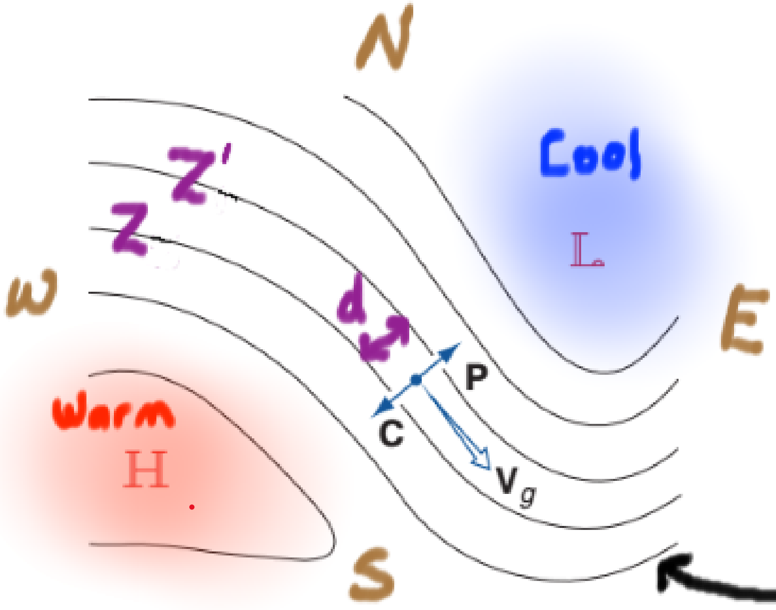


Fig. 7.9 The geostrophic wind V_g and its relationship to the horizontal pressure gradient force P and the Coriolis force C in the northern hemisphere.

$\vec{P}$ = Pressure Gradient Force

$\vec{C}$ = Coriolis Force

$\vec{V}_g$ = Geostrophic Wind

ϕ = Latitude

$\Omega = 2\pi/\text{day} \approx 7.3\times10^{-5} \frac{\text{Radians}}{\text{Second}}$

Geopotential Height Contours

$$\boxed{V_g = \frac{(Z - Z')}{d}\,\frac{g}{2\Omega\sin\phi}}$$

From $\frac{dP}{dx} = \frac{dP}{dZ}\frac{dZ}{dx} = -\rho g\frac{dZ}{dx} = 2\rho\Omega\sin\phi\, V_g$

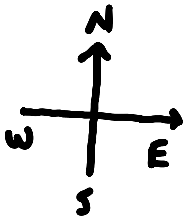
Wind is parallel to height contours.

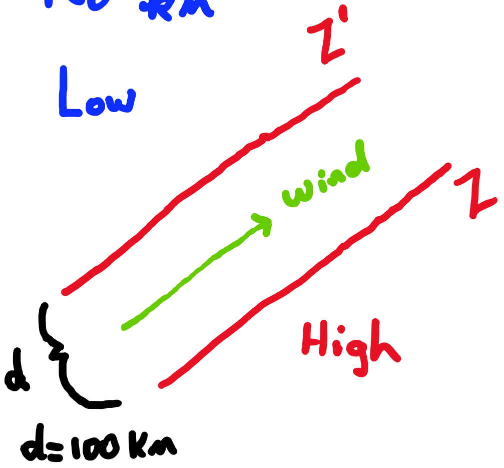
Low height corresponds to low pressure at this level of the atmosphere.

Example: 49 Knot winds are observed over Reno at the 500 mb level, Southwesterly flow at 220 degrees.

49 Knots $= 25 \frac{m}{s}$

Calculate the height difference for height contours spaced by 100 km





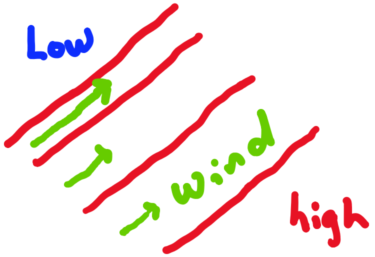
← Top View at 500 mb

$Z > Z'$ ← Geopotential heights

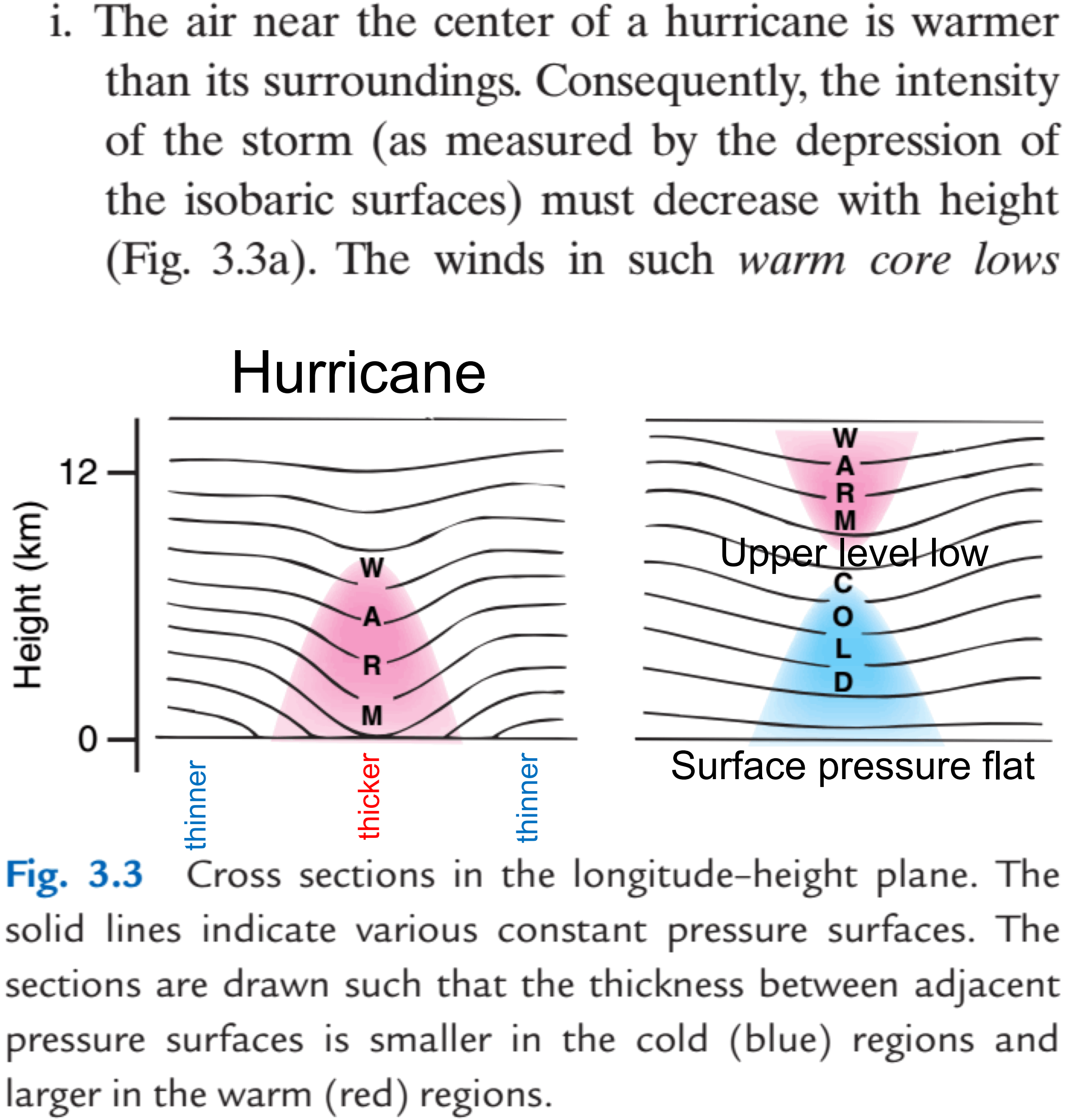
$$Z - Z' = \frac{d\, 2\Omega \sin\phi\, V}{g} = \frac{10^5 \text{meters} \cdot 2 \cdot 7.3 \times 10^{-5} \frac{1}{s}\ \sin(39.5^\circ)\ 25 \frac{m}{s}}{9.8\ m/s^2}$$

$$\boxed{Z - Z' = 23.6 \text{ meters}}$$

Usually $Z - Z' = 25$ meters for example, and the contour spacing determines the wind speed.



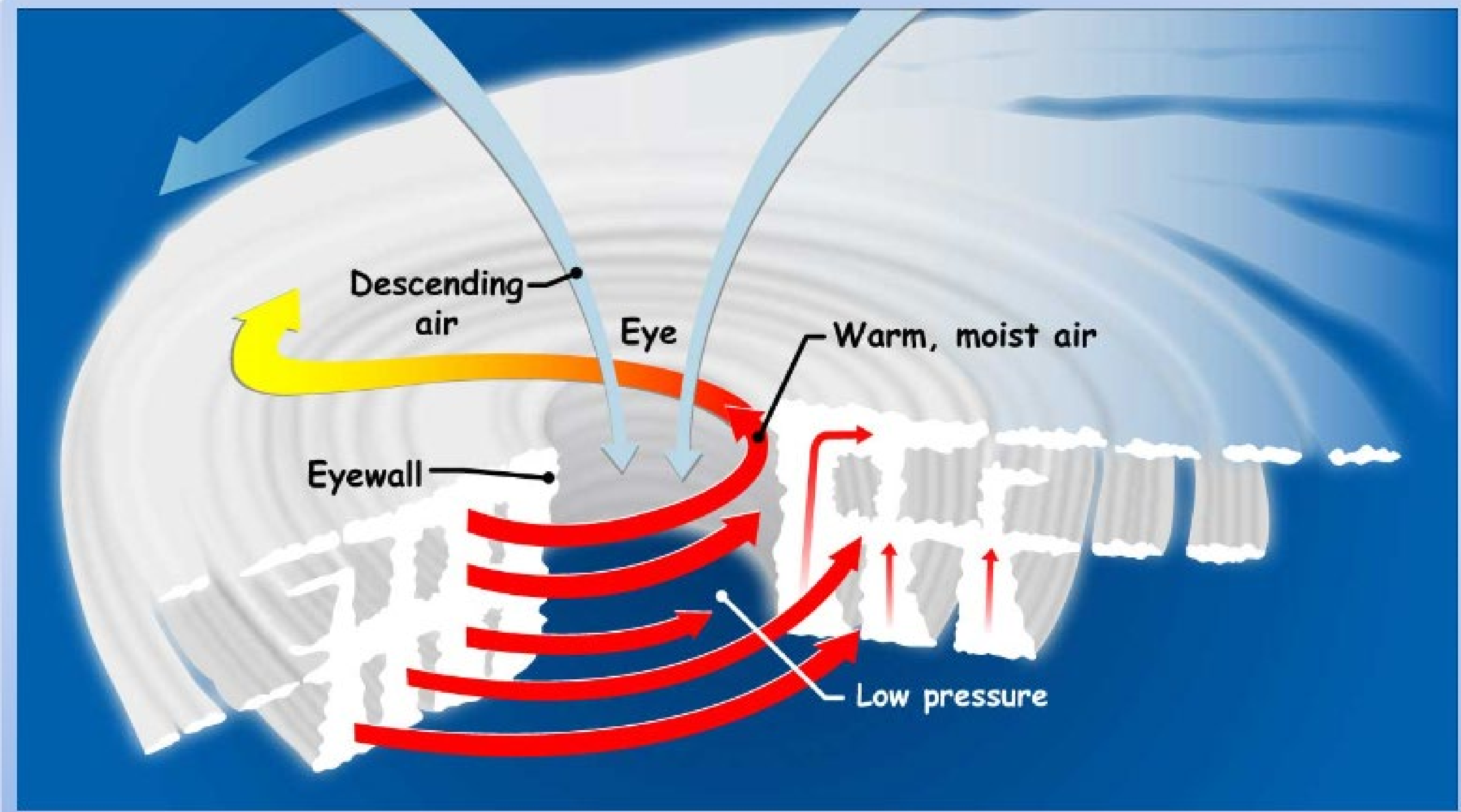
Layer Thickness and Constant Pressure Surfaces



i. The air near the center of a hurricane is warmer than its surroundings. Consequently, the intensity of the storm (as measured by the depression of the isobaric surfaces) must decrease with height (Fig. 3.3a). The winds in such *warm core lows*

Fig. 3.3 Cross sections in the longitude–height plane. The solid lines indicate various constant pressure surfaces. The sections are drawn such that the thickness between adjacent pressure surfaces is smaller in the cold (blue) regions and larger in the warm (red) regions.

Hurricanes: Problem 4 in the homework

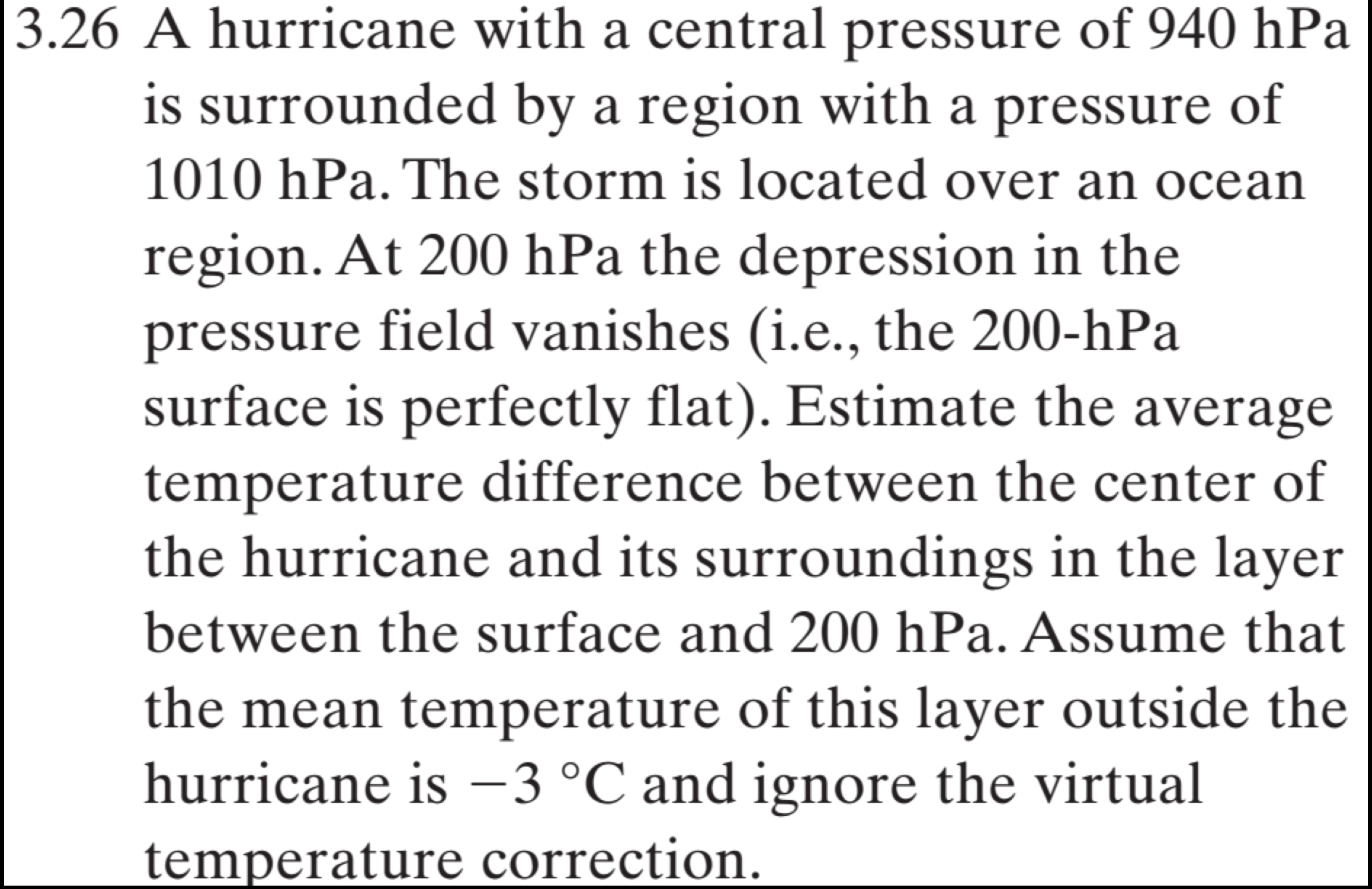


If you could slice into a tropical cyclone, it would look something like this. The small red arrows show warm, moist air rising from the ocean's surface, and forming clouds in bands around the eye. The blue arrows show how cool, dry air sinks in the eye and between the bands of clouds. The large red arrows show the rotation of the rising bands of clouds.

From: https://gpm.nasa.gov/education/articles/how-do-hurricanes-form

Pat Arnott

Let's Do Problem 3.26

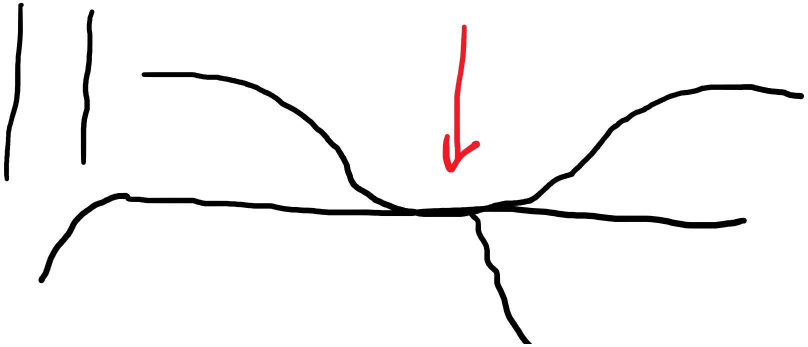


3.26 A hurricane with a central pressure of 940 hPa is surrounded by a region with a pressure of 1010 hPa. The storm is located over an ocean region. At 200 hPa the depression in the pressure field vanishes (i.e., the 200-hPa surface is perfectly flat). Estimate the average temperature difference between the center of the hurricane and its surroundings in the layer between the surface and 200 hPa. Assume that the mean temperature of this layer outside the hurricane is −3 °C and ignore the virtual temperature correction.

Calculate Z from the surroundings using the hypsometric equation.
Z remains the same for the hurricane. Calculate T for the hurricane from the pressure. (It may be useful to obtain a general equation for the temperature).

Problem 3.26: Average Temperature in the Eye

2



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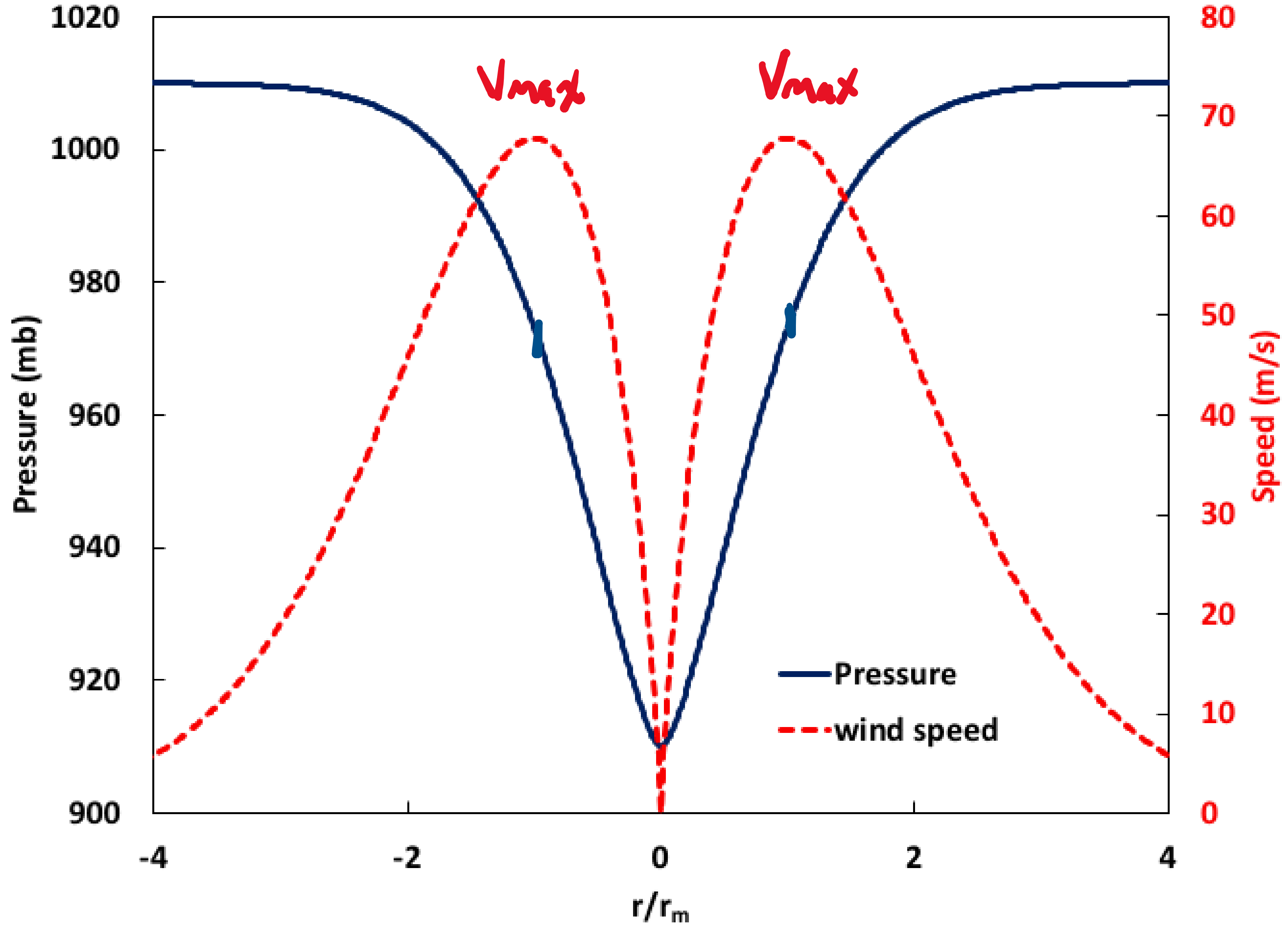
Hurricane Pressure and Wind Speed



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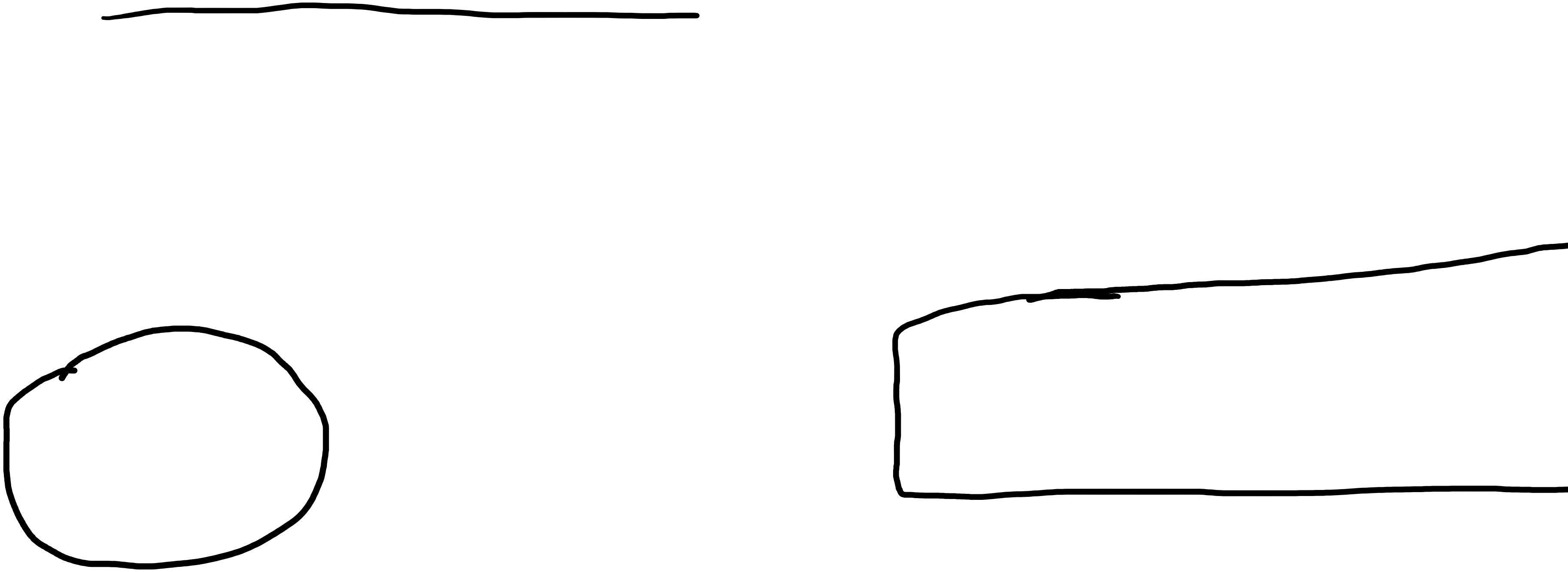
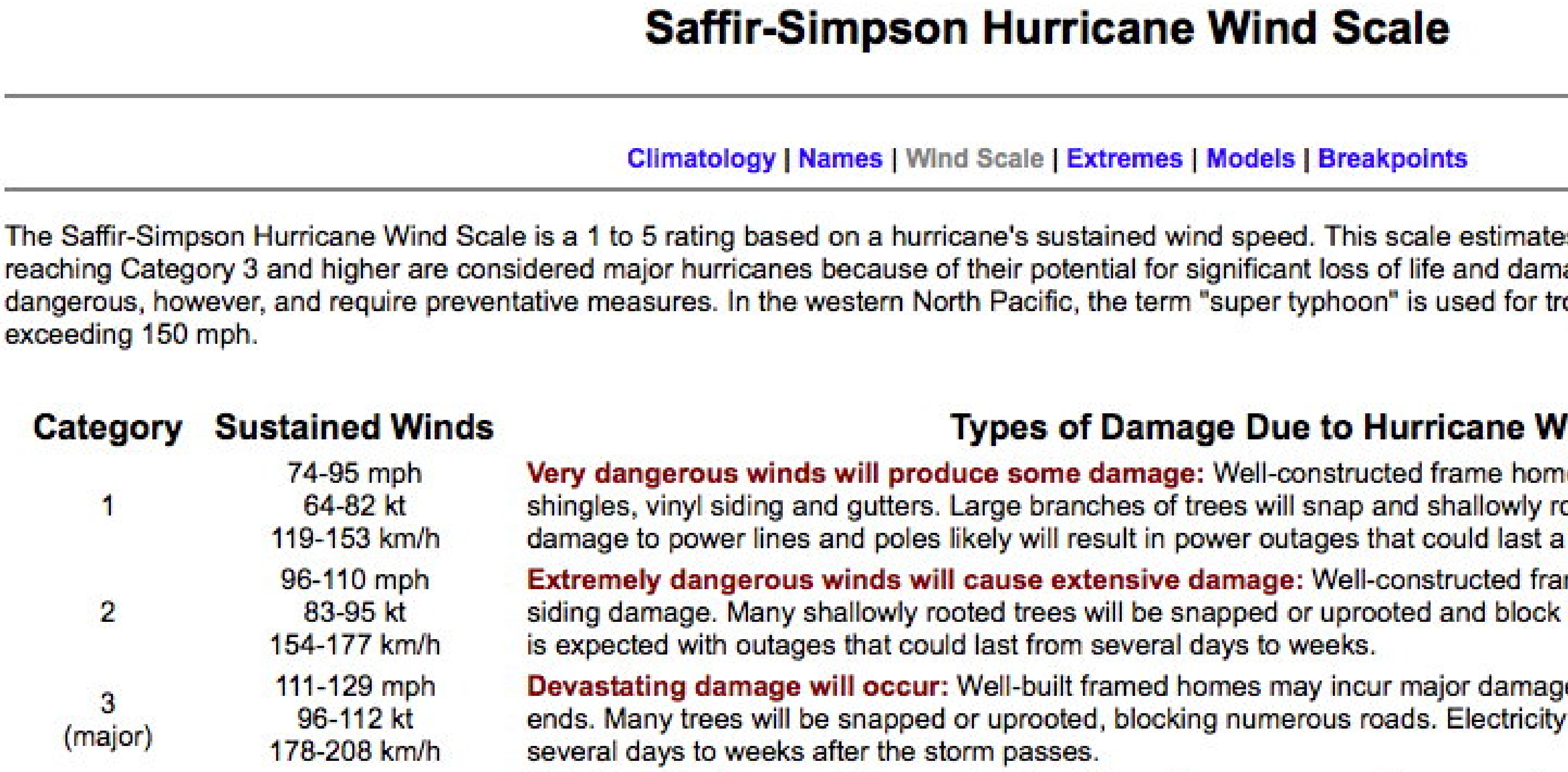
Hurricane Pressure and Wind Speed: Cyclostrophic Flow Approximation: Pressure gradient force balanced by centrifugal force



Problem 4 in Homework: Make a table with 4 columns "Category" "V_{max}" "ΔP" and "ΔT".



Saffir-Simpson Hurricane Wind Scale

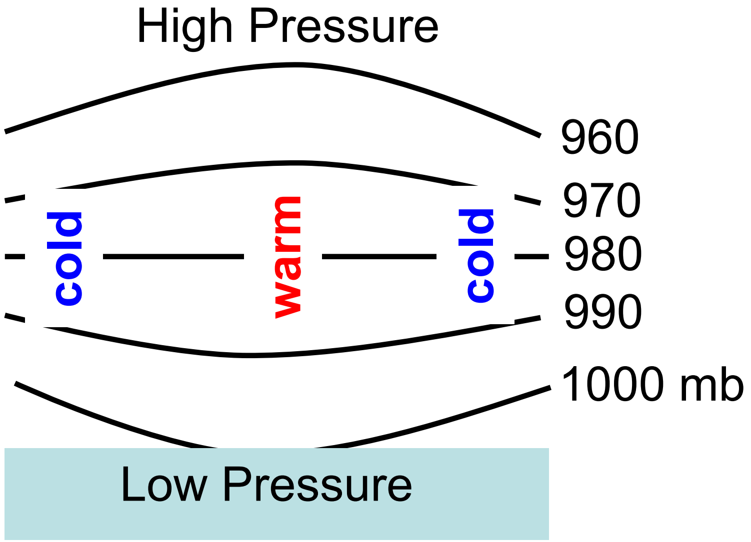
Climatology | Names | Wind Scale | Extremes | Models | Breakpoints

The Saffir-Simpson Hurricane Wind Scale is a 1 to 5 rating based on a hurricane's sustained wind speed. This scale estimate
reaching Category 3 and higher are considered major hurricanes because of their potential for significant loss of life and dama
dangerous, however, and require preventative measures. In the western North Pacific, the term "super typhoon" is used for tro
exceeding 150 mph.

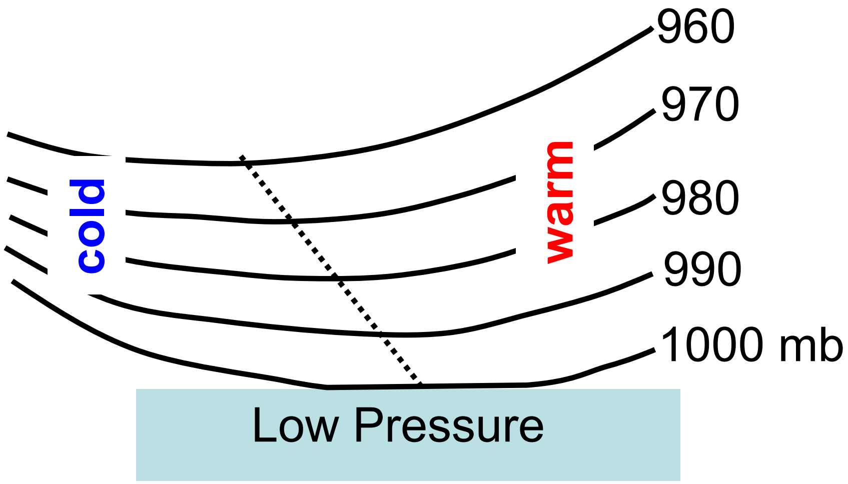
Category	Sustained Winds	Types of Damage Due to Hurricane W
1	74-95 mph 64-82 kt 119-153 km/h	**Very dangerous winds will produce some damage:** Well-constructed frame hom shingles, vinyl siding and gutters. Large branches of trees will snap and shallowly ro damage to power lines and poles likely will result in power outages that could last a
2	96-110 mph 83-95 kt 154-177 km/h	**Extremely dangerous winds will cause extensive damage:** Well-constructed fra siding damage. Many shallowly rooted trees will be snapped or uprooted and block is expected with outages that could last from several days to weeks.
3 (major)	111-129 mph 96-112 kt 178-208 km/h	**Devastating damage will occur:** Well-built framed homes may incur major damag ends. Many trees will be snapped or uprooted, blocking numerous roads. Electricity several days to weeks after the storm passes.

More On Atmospheric Layer Thickness

(adapted from Petty)



Surface warm core cyclone
becomes an anticyclone aloft



Tilt of pressure trough to cold air

Practice by making sketches of ...

1. A surface warm-core anticyclone strengthens with height
2. A surface cold-core cyclone becomes more intense with height

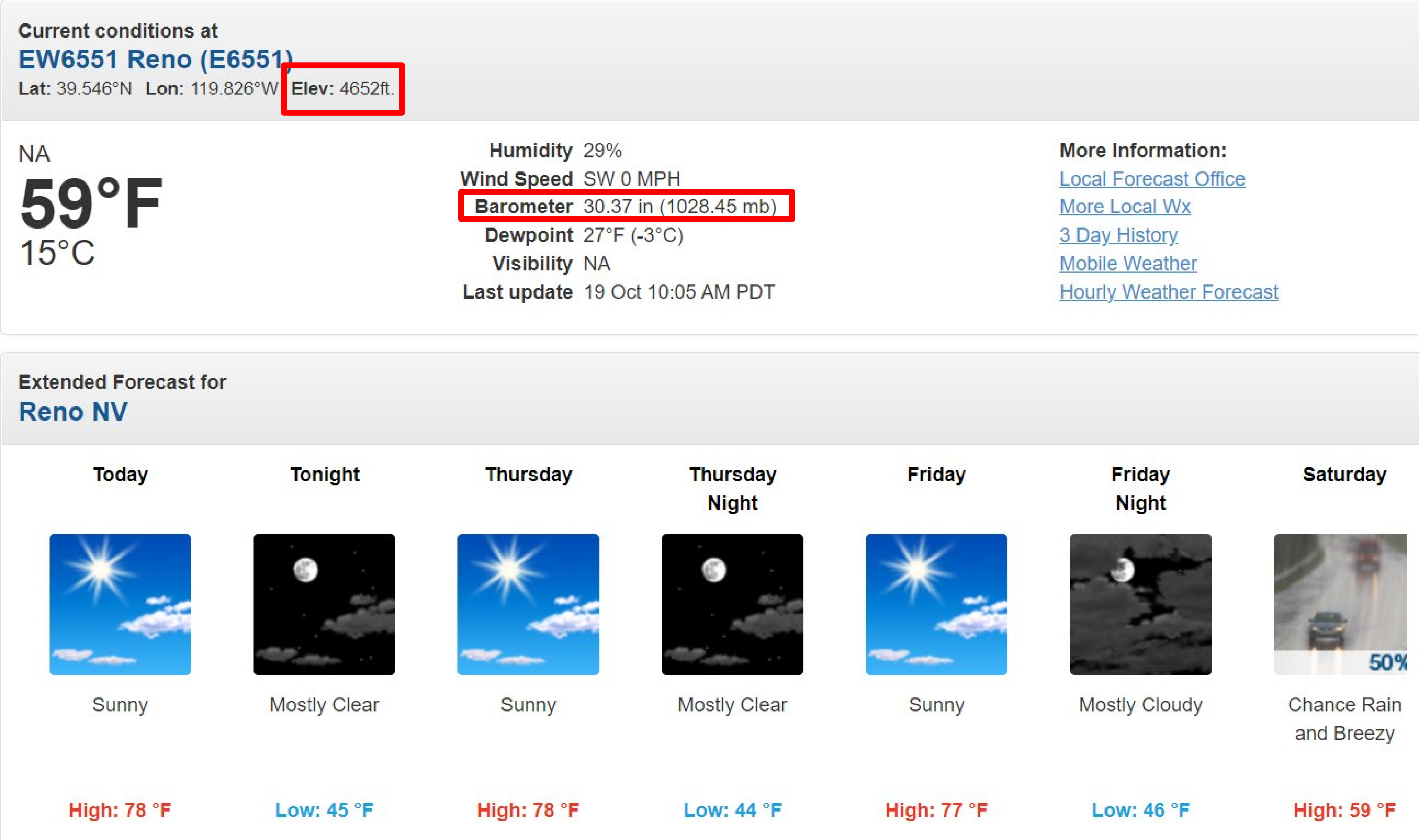
Pressure Definitions:
Station and Mean Sea Level Equivalent Pressure

STATION PRESSURE: This is the pressure that is observed at a specific elevation and is the true barometric pressure of a location.

MEAN SEA LEVEL EQUIVALENT PRESSURE: This is the pressure reading most commonly used by meteorologists to track weather systems at the surface. It is an attempt to remove the effects of elevation from pressure readings. This reduction estimates the pressure that would exist at sea level at a point directly below the station using a temperature profile based on temperatures at the station. In practice the temperature used in the reduction is an average temperature for the preceding twelve hours. Mean sea level pressure should be used with caution at high elevations as temperatures can have a very profound effect on the reduced pressures, sometimes giving rise to fictitious pressure patterns and anomalous mean sea level pressure values.

https://www.weather.gov/bou/pressure_definitions

A Weather Report for Reno: Which pressure is reported here, Station or Mean Sea Level Equivalent?



Current conditions at

EW6551 Reno (E6551)

Lat: 39.546°N **Lon:** 119.826°W **Elev:** 4652ft.

NA

59°F

15°C

Humidity	29%
Wind Speed	SW 0 MPH
Barometer	30.37 in (1028.45 mb)
Dewpoint	27°F (-3°C)
Visibility	NA
Last update	19 Oct 10:05 AM PDT

More Information:

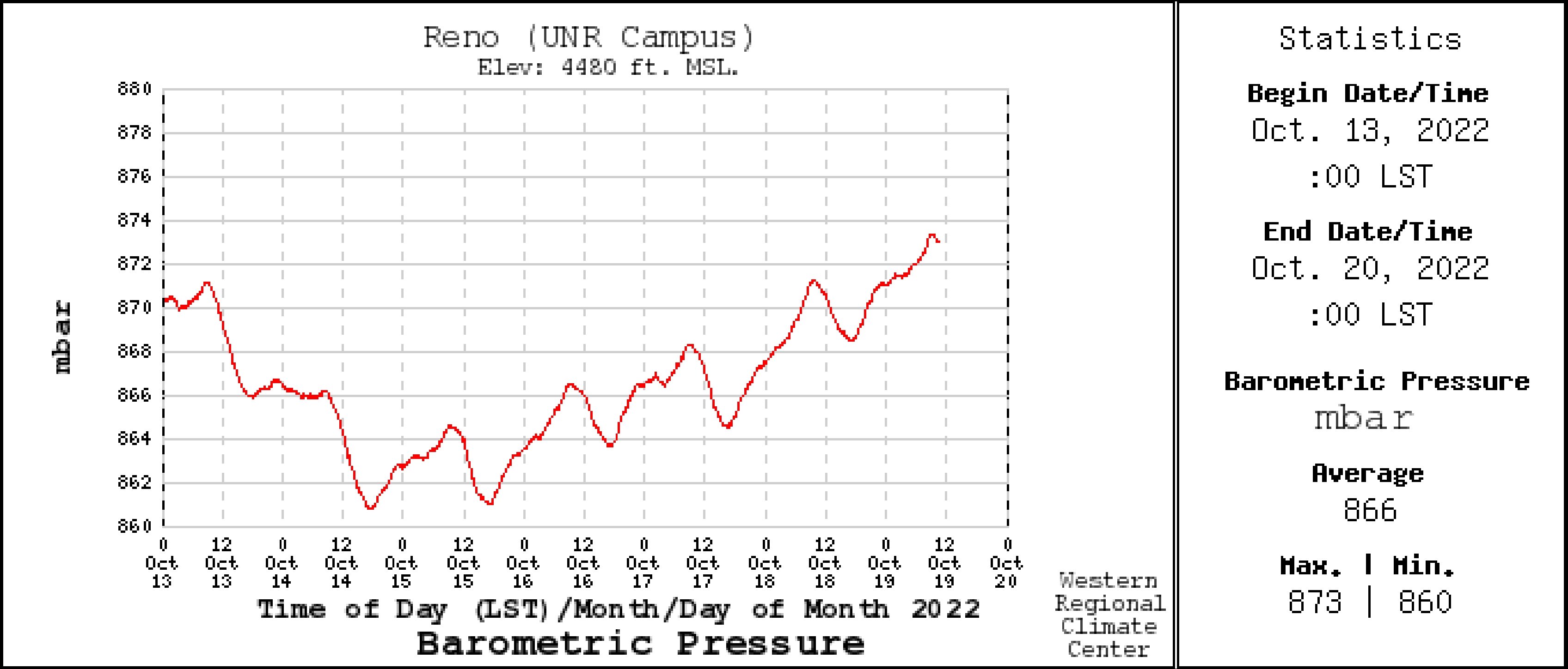
- Local Forecast Office
- More Local Wx
- 3 Day History
- Mobile Weather
- Hourly Weather Forecast

Extended Forecast for

Reno NV

Today	Tonight	Thursday	Thursday Night	Friday	Friday Night	Saturday
						50%
Sunny	Mostly Clear	Sunny	Mostly Clear	Sunny	Mostly Cloudy	Chance Rain and Breezy
High: 78 °F	Low: 45 °F	High: 78 °F	Low: 44 °F	High: 77 °F	Low: 46 °F	High: 59 °F

UNR Weather Station: Which pressure is this?



Statistics

Begin Date/Time
Oct. 13, 2022
:00 LST

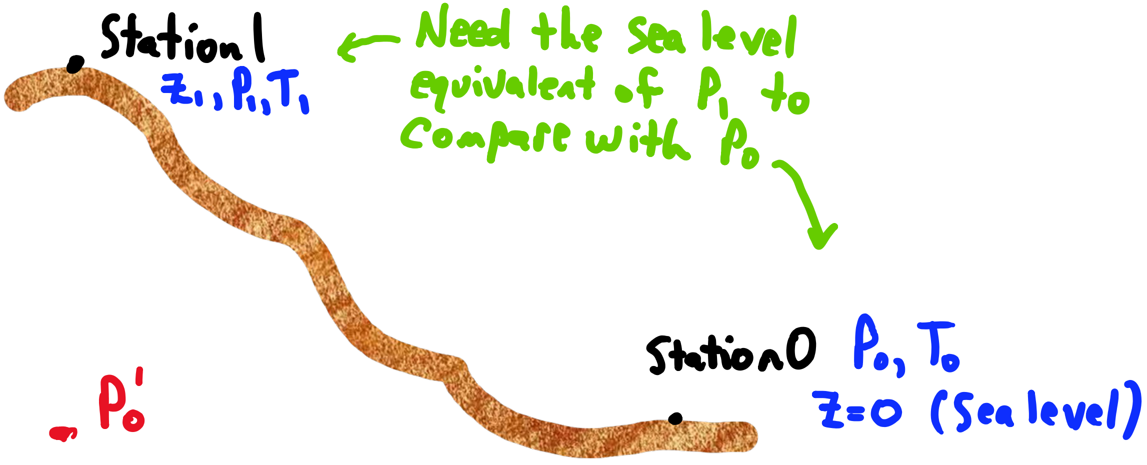
End Date/Time
Oct. 20, 2022
:00 LST

Barometric Pressure
mbar

Average
866

Max. | Min.
873 | 860

Reduction to Sea Level Pressure?



Need the sea level equivalent of P_1 to compare with P_0

One idea:

$$P_0' = P_1 e^{\frac{z_1 g_0}{R_0 \langle T_v \rangle}}$$

What to use for $\langle T_v \rangle$?

$\langle T_v \rangle \cong T_1$?

Rough guideline: Very near sea level, $\frac{dP}{dz} \simeq \frac{1\,\text{mb}}{8\,\text{meters}}$

Then $P_1(z) \approx P_0(z) - \frac{dP}{dz} z$

May also average T_1 from current value and value 12 hrs earlier

Pat Arnott

Altimeter: Measure P, estimate z

Choose $P_0 = 1013.25$ mb

$T_0 = 288$ K

Γ = lapse rate = 6.5°K/km

Assume $T_{\underline{v}}(z) = T_0 - \Gamma z$

Measure P_1 z=?

Then $$\int_{P_0}^{P_1} \frac{dP}{P} = -\int_0^z \frac{g_0\, dz'}{R_D\, T_{\underline{\underline{v}}}(z')}.$$

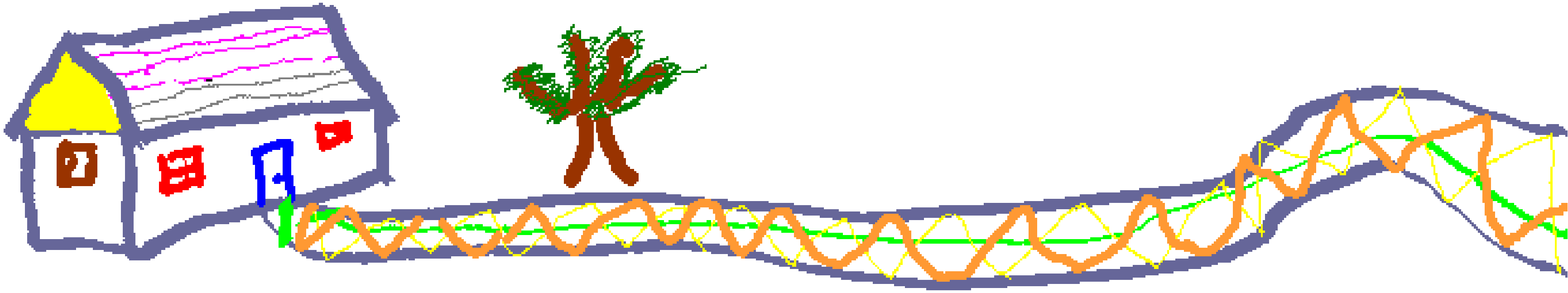
Solving,

$$\boxed{Z = \frac{T_0}{\Gamma}\left[1 - \left(\frac{P(z)}{P_0}\right)^{\frac{R_D \Gamma}{g_0}}\right]}$$

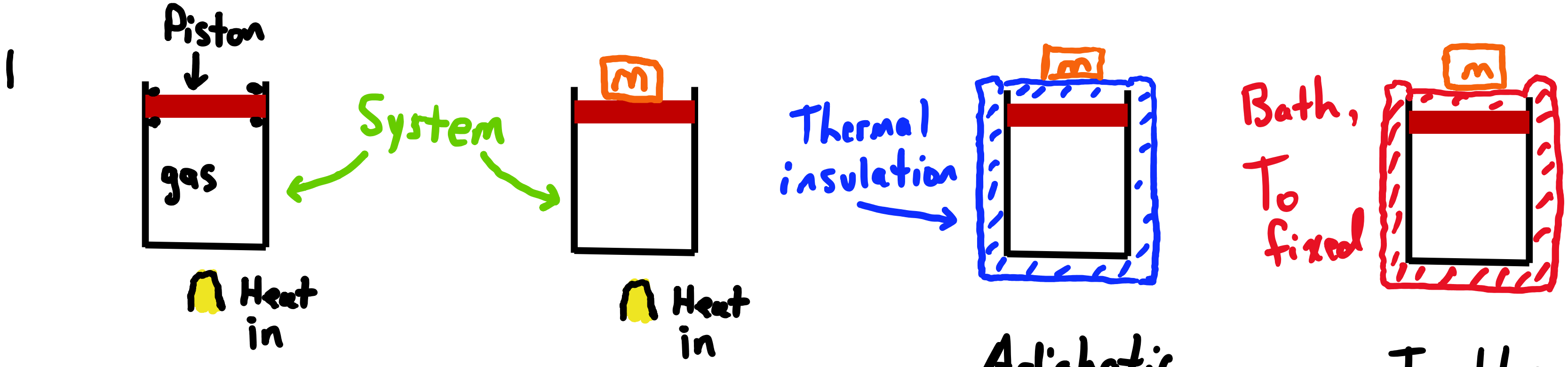
Set P_0 and T_0 to Starting Point values for a better estimate of z.

Pat Arnott

Types of thermodynamic Processes



Types of Thermodynamic Processes



Constant Volume (welded Piston) isochoric Process

Danger

Constant Pressure (isobaric Process)

Mass Provides fixed Pressure. Good for Many lab Conditions.

Adiabatic Process (No external heat in or out)

More or less the atmosphere when dry, nonradiative

Isothermal Process. Add mass to change pressure.

Walls at fixed temperature.

First law of thermodynamics : Energy is conserved

Basic law of Physics

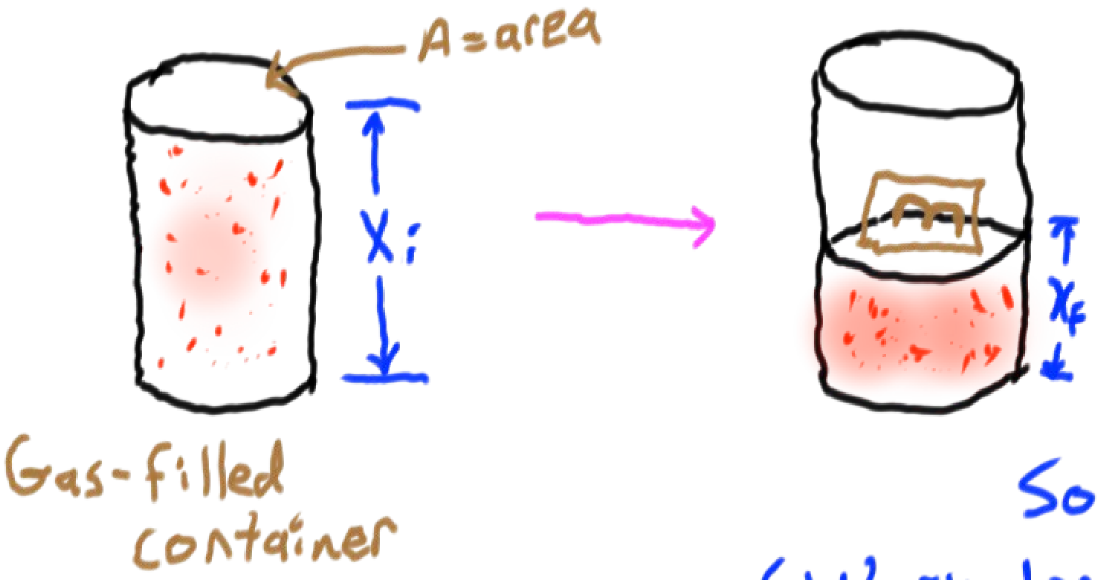
$du \equiv \frac{\text{Change in internal energy}}{\text{unit mass}}$ of a gas in some process

$$\boxed{du = dq - dw}$$

dq: $\frac{\text{heat}}{\text{mass}}$ into system

dw: $\frac{\text{Work}}{\text{mass}}$ done by the system in some process

Work Example:



$\text{Pressure} = \frac{mg}{A}$

$\text{Work by gas} = F(x_f - x_i)$

$dW = PA\,dx = PdV$

$\frac{\text{Work}}{\text{mass}} = Pd\alpha \qquad \alpha = V/m$

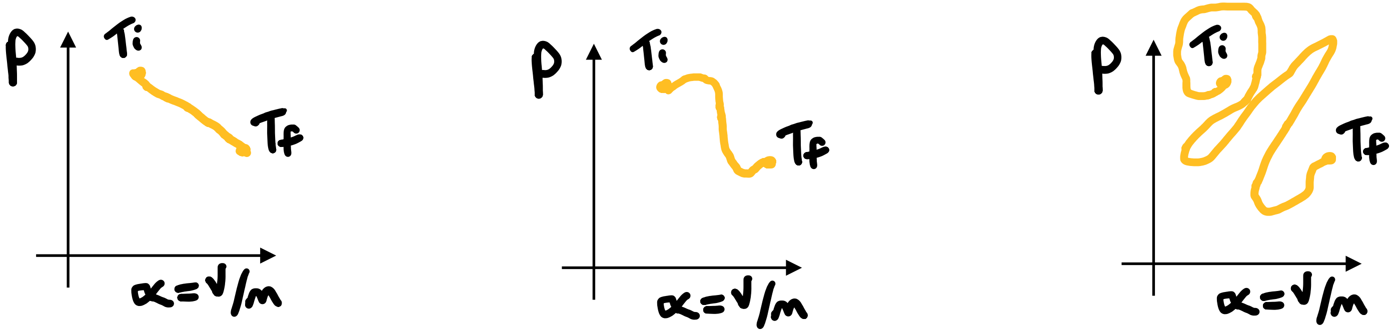
$dw < 0$ here,

So $du > 0$

(Work done by system < 0)

Pat Arnott

Ideal Gas: U depends only on T



For all paths, $\Delta U = C_v (T_f - T_i)$

$\frac{\text{Change of internal energy}}{\text{mass}}$

Heat Capacity at Constant Volume Per unit mass.

C_v is the heat needed to change the gas temperature for 1 Kg of gas by 1°K

For an isochoric Process, $d\alpha = 0$,

$$du = dq = c_v dT$$

Work done in moving the gas from T_i + T_f is the area under the curve on the P,α diagram

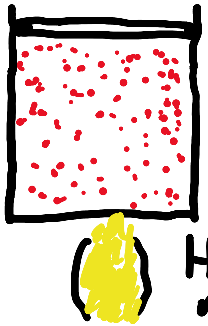
Specific Heat Capacity

$C_v \Rightarrow$ Units $\frac{\text{Joules}}{\text{Kg °K}}$

$$C_v = \frac{fR}{2} = \frac{f}{2}\,\frac{R^*}{M_w}$$

(Equi Partition theorem)

f = # degrees of freedom ~ Energy Pathways for the air molecules



Isochoric Process

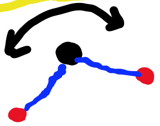
How much heat is needed to raise the temperature?

$$Q = M_{gas}\int_{T_i}^{T_f} C_v\,dT$$

$$Q \approx M_{gas}\,C_v(T_f - T_i)$$

Translation: 3 degrees of freedom

Molecule with K.E. (Kinetic Energy)



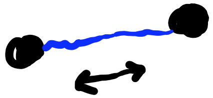
Rotation: 3 degrees of freedom for H_2O, O_3, N_2O, etc

triatomic molecules, non linear

CO_2 (linear)

N_2, O_2, CO

2 rotational degrees of freedom



Vibration

"frozen out" at atmospheric temperature

$$P(E_{vib}) \propto \exp(-h\nu_{vib}/kT)$$

<<1

Probability thermal energy kT excite vibrational state $h\nu_{vib}$

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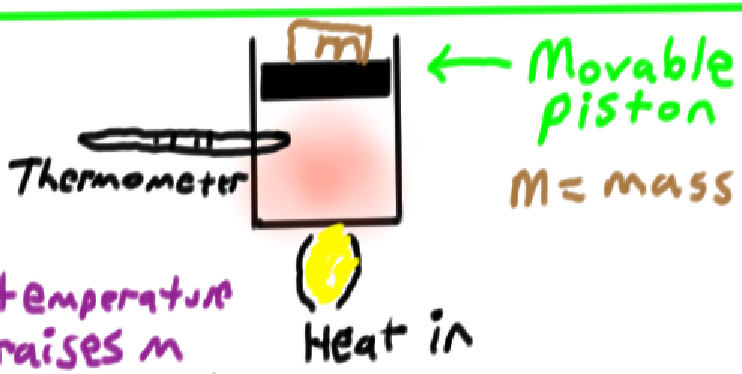
C_v from the equipartition theorem

$C_v = \frac{f}{2} R$ $du = C_v \, dT$

Example	molecule	translation	vibration	rotation	f	Picture
H, Ar, He	Monatomic	3	0	0	3	
O_2, N_2, CO	diatomic	3	2 frozen out	2	5	
H_2O, O_3	triatomic Nonlinear	3	6 frozen out	3	6	

Isobaric Process ⇒ Constant Pressure

$dP = 0$



$$\boxed{\left.\frac{dq}{dT}\right)_P \equiv C_p = C_v + R}$$

for ideal gas.

Heat in changes temperature and does work, raises m

Proof: Use the I.G.L. and first law of thermodynamics.

$dq = du + P \, d\alpha$

$dq = C_v dT + R dT - \cancel{\alpha dP}$ isobaric

$dq \sim KE +$ work done to expand

$P\alpha = RT$ $P d\alpha + \alpha dP = R dT$ so

$P d\alpha = R dT - \alpha dP$

$$\boxed{\left.\frac{dq}{dT}\right)_P = C_p = C_v + R}$$

For Air

$C_v = \frac{5}{2} R$ $C_p = \frac{7}{2} R$ $\frac{C_p}{C_v} = \frac{7}{5}$

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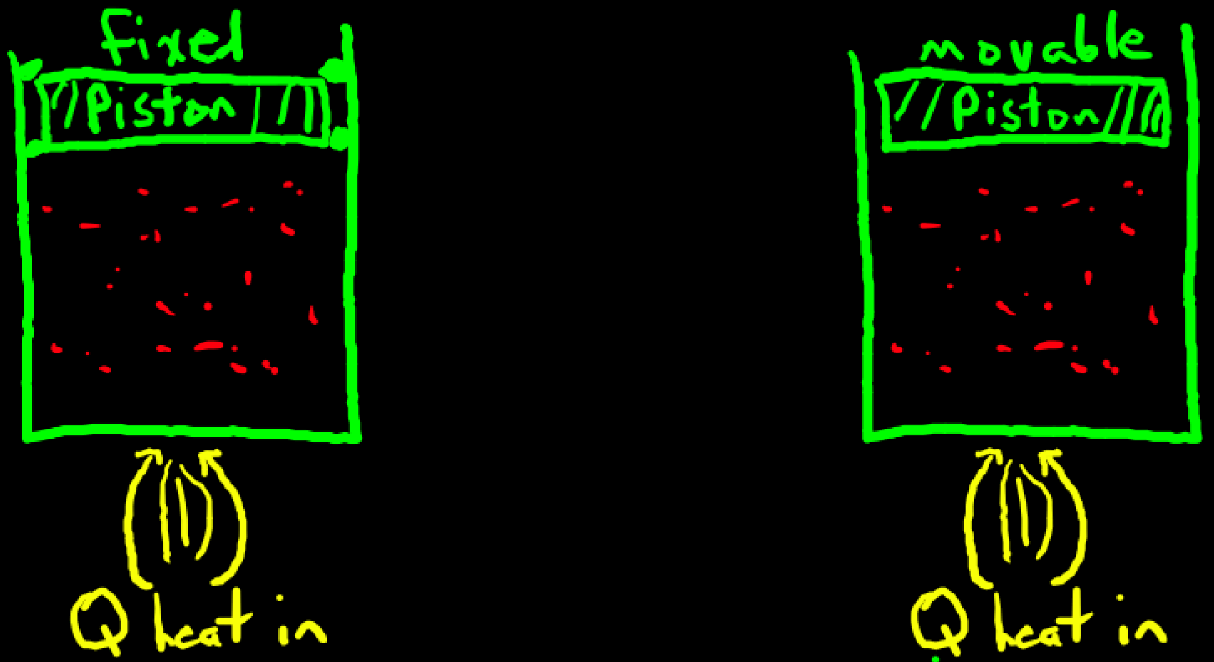
Quiz:

Which system comes to a higher temperature?



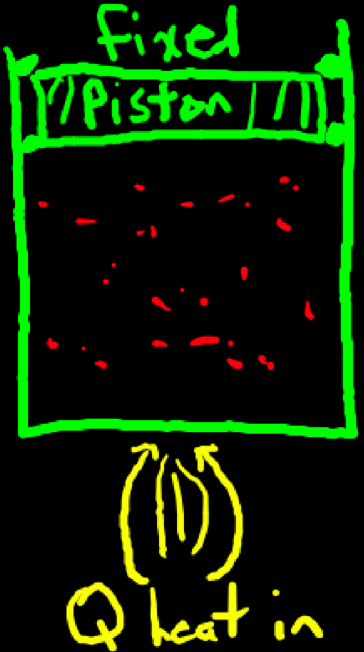
isochoric - Constant Volume

isobaric Constant Pressure

Which one is more like an air parcel?

Quiz:

Which system comes to a higher temperature?

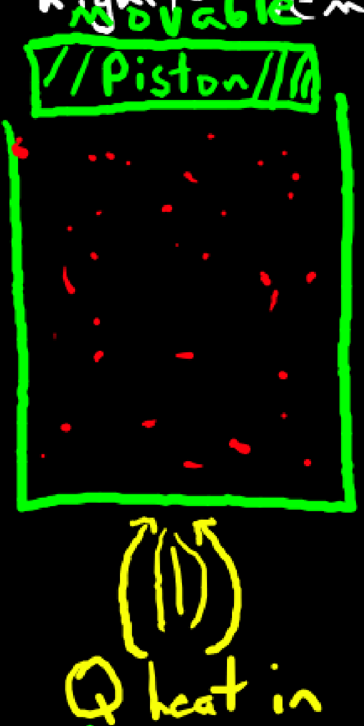


$du = Q = C_v dT$

$dT = \frac{Q}{C_v}$

isochoric - Constant Volume

$dT > dT'$ since $dV > 0$.



$du = Q - PdV$

$du = C_v dT'$

$Q = C_p dT'$

$\therefore$

$$\boxed{C_p - C_v = P \left.\frac{\partial V}{\partial T}\right|_p}$$

$C_p - C_v = R$ (I.G.L.)

isobaric Constant Pressure

$$dT' = \frac{Q - PdV}{C_v}$$

Which one is more like an air parcel?

Adiabatic ($dq=0$) Lapse Rate Γ_0

Adiabatic Atmosphere:

$$T(z) = T_0 - \Gamma_0 z$$

$$dq = \underline{du} + \underline{\underline{P d\alpha}}$$

1) Use $\underline{du} = \underline{C_v dT}$

2) Use I.G.L. $P\alpha = RT$

$$P d\alpha + \alpha dP = R dT \quad \text{so} \quad \underline{\underline{P d\alpha}} = \underline{\underline{R dT - \alpha dP}}$$

3) Use hydrostatic equation: $dP = -\rho g dz = -g/\alpha \, dz$ $\alpha = 1/\rho$

Assemble the Parts:

$$dq = du + P d\alpha = C_v dT + R dT + g dz = 0$$

$$\underbrace{(C_v + R)}_{C_p} dT = -g dz$$

$$\boxed{C_p = \frac{7}{2} R \text{ for air}} \quad \boxed{C_v = \frac{5}{2} R \text{ for air}} \quad \boxed{\frac{dT}{dz} = -\frac{g}{C_p} = -\Gamma_0}$$

Numerically:

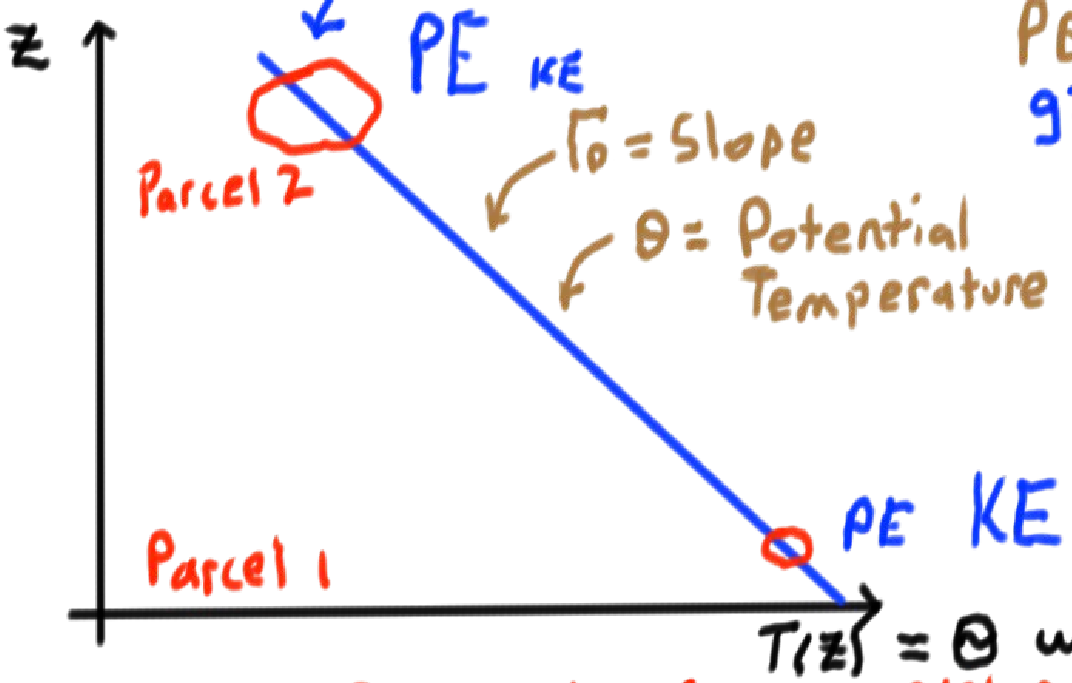
$$\boxed{\Gamma_0 = \frac{g}{C_p} = \frac{9.8 \, m/s^2 * 29 \, Kg/Kmole}{7/2 * 8314.3 \, J/Kmole \, {}^\circ K} = \frac{9.8 \, {}^\circ K}{km}}$$

Pat Arnott

Dry Static Energy $gz+c_pT$

Adiabatic Lapse Rate and Potential Temperature

must do work on environment to expand



= θ when P(z) = 1000 mb

PE + KE = constant

$gz + c_pT$ = constant

Dry Static Energy

$$\boxed{\Gamma_D = \text{Slope} = -\frac{dT}{dz} = \frac{g}{c_p}}$$

Dry adiabatic lapse rate.

Parcel 1 and 2 can be in equilibrium.

θ = Potential Temperature = constant on adiabat.

$$\theta = T\left(\frac{1000\,\text{mb}}{P}\right)^{2/7}$$

Θ Potential Temperature Derivation Θ

1st Law of Thermodynamics

$$dq = du + P\,\underline{\underline{d\alpha}}$$

Ideal gas law

$P\alpha = RT$ so

$$P\,d\alpha + \alpha\,dP = R\,dT$$

$$P\,\underline{\underline{d\alpha}} = R\,dT - \alpha\,dP$$

1

Combining

$$dq = du + R\,dT - \alpha\,dP$$

Aside: Let $dh = du + R\,dT$

$\underline{\underline{\text{Enthalpy}}} \longrightarrow dh = C_v\,dT + R\,dT$

$$dh = C_p\,dT$$

$$dq = dh - \alpha\,dP$$

$$dq = C_p\,dT - \alpha\,dP$$

2

Adiabatic Process $dq = 0$

$$dq = 0 = C_p\,dT - \alpha\,dP$$

Ideal gas law

$$\alpha = RT/P$$

So $C_p\,dT - RT\,dP/P = 0$

$$dT/T = R/C_p\,dP/P$$

3

Integrating

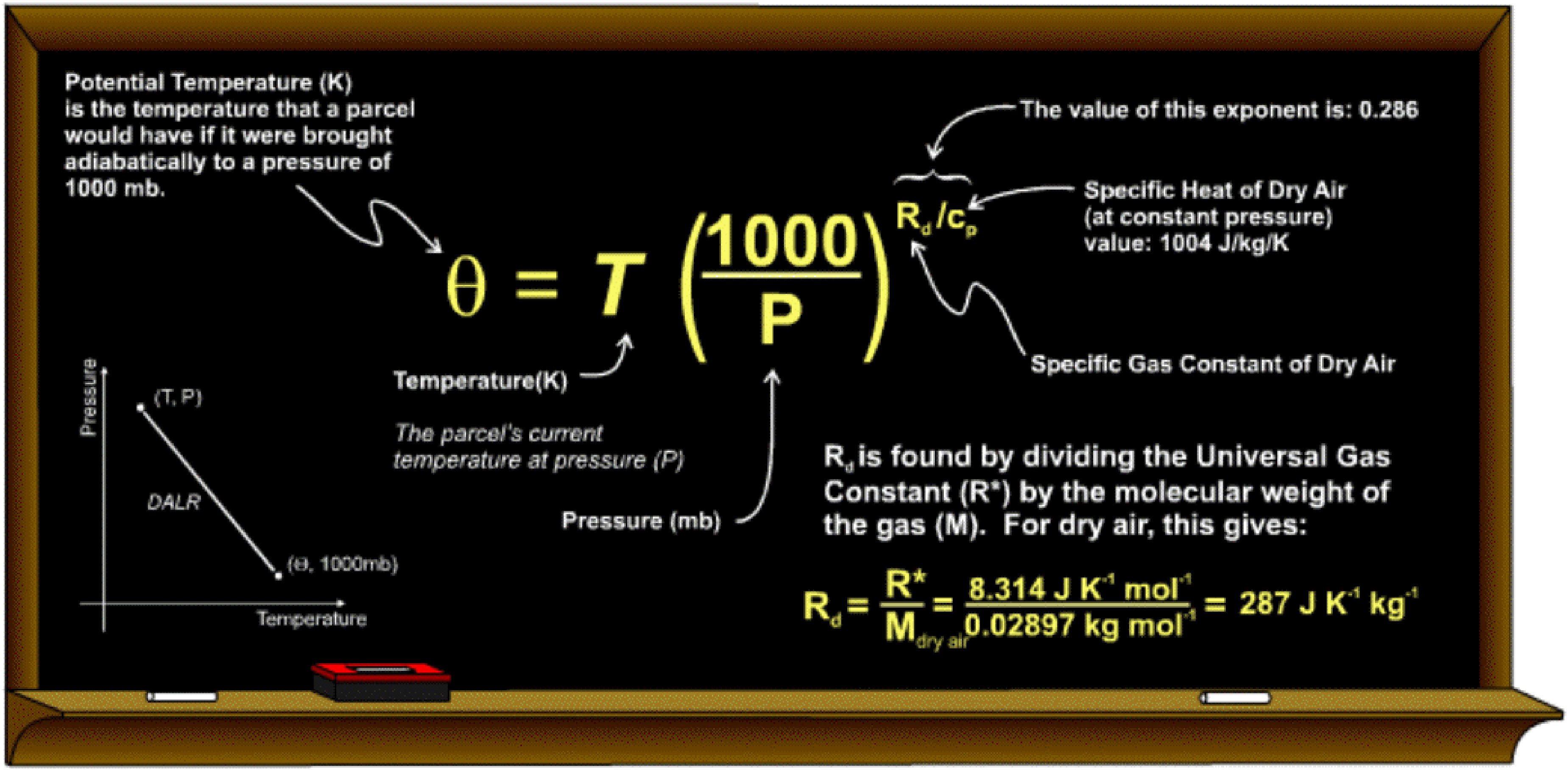
$$\int_{T(z)}^{\Theta} \frac{dT}{T} = \int_{P(z)}^{1000\,mb} \frac{R}{C_p}\frac{dP}{P} \qquad R/C_p = \frac{2}{7}$$

$$\underline{\underline{\Theta}} = T\left(\frac{1000\,mb}{P}\right)^{2/7}$$

4

Pat Ar

Potential Temperature



Potential Temperature (K) is the temperature that a parcel would have if it were brought adiabatically to a pressure of 1000 mb.

$$\theta = T\left(\frac{1000}{P}\right)^{\overbrace{R_d/c_p}}$$

The value of this exponent is: 0.286

Specific Heat of Dry Air (at constant pressure) value: 1004 J/kg/K

Specific Gas Constant of Dry Air

Temperature(K)

The parcel's current temperature at pressure (P)

Pressure (mb)

R_d is found by dividing the Universal Gas Constant (R^*) by the molecular weight of the gas (M). For dry air, this gives:

$$R_d = \frac{R^*}{M_{\text{dry air}}} = \frac{8.314\ \text{J K}^{-1}\ \text{mol}^{-1}}{0.02897\ \text{kg mol}^{-1}} = 287\ \text{J K}^{-1}\ \text{kg}^{-1}$$

Adiabatic compressional warming (and cooling by expansion) lends itself to predicting large-scale weather patterns, because air motions in large weather systems are, for all practical purposes, generally adiabatic in nature.

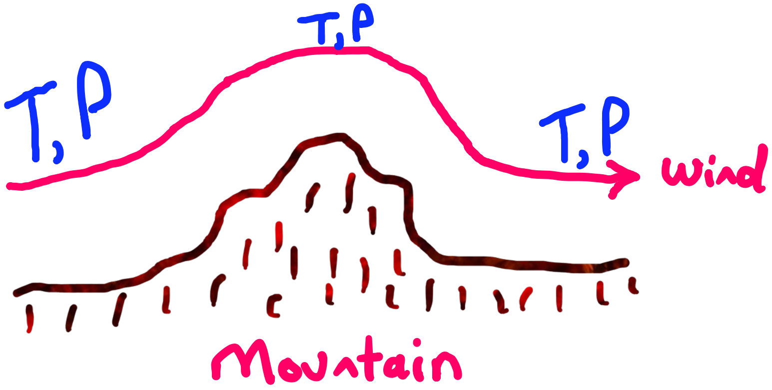
Significance of Potential Temperature

$$\Theta = T(z)\left(\frac{P_0}{P(z)}\right)^{R/c_p}$$

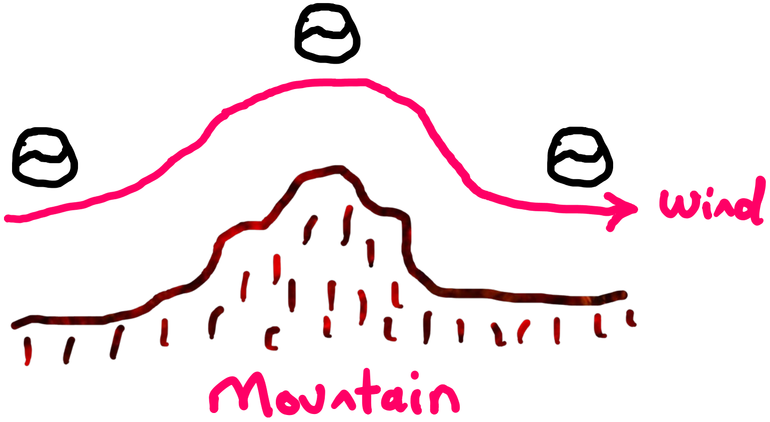
$R/c_p = 2/7$ for air

$P_0 = 1000$ mb

(Known also as Poisson's Eq.)



Looks like warm air under Cool air? Is convection going to happen?

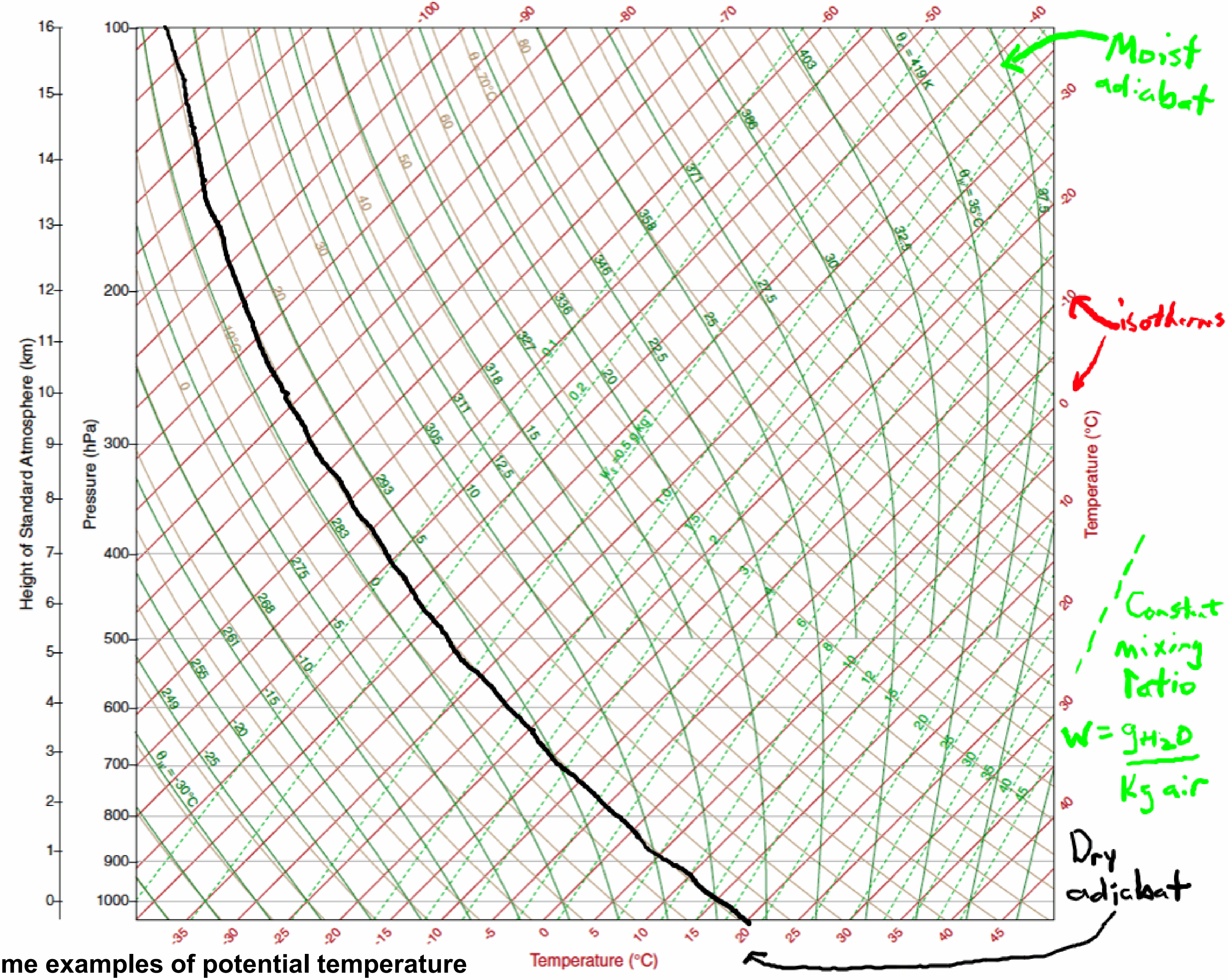


No, Θ=constant, air is just being adiabatically lifted over the mountain.

Nonadiabatic Processes:

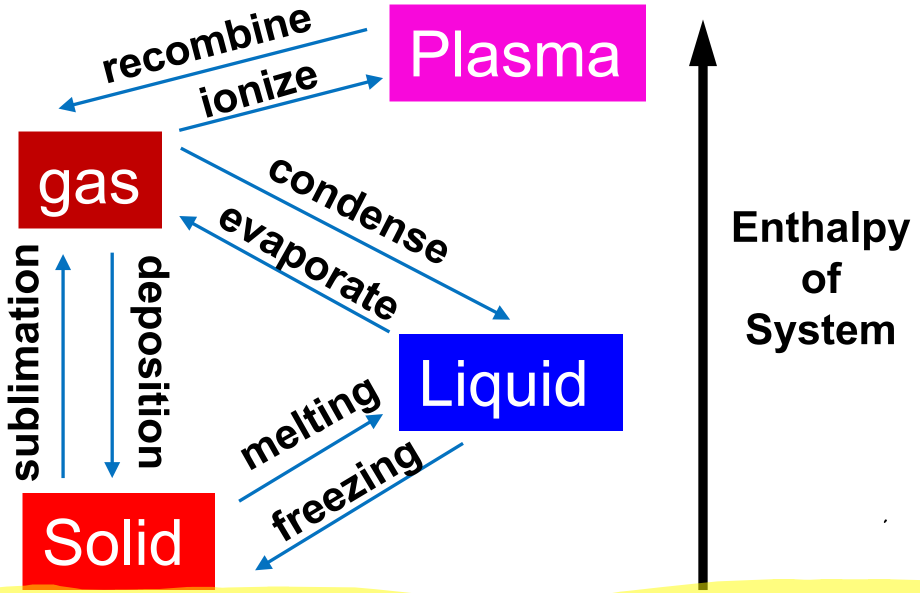
1) Radiative heating or cooling.
2) Water condensation or evaporation.

Θ contours are great for mapping mountain induced gravity waves!

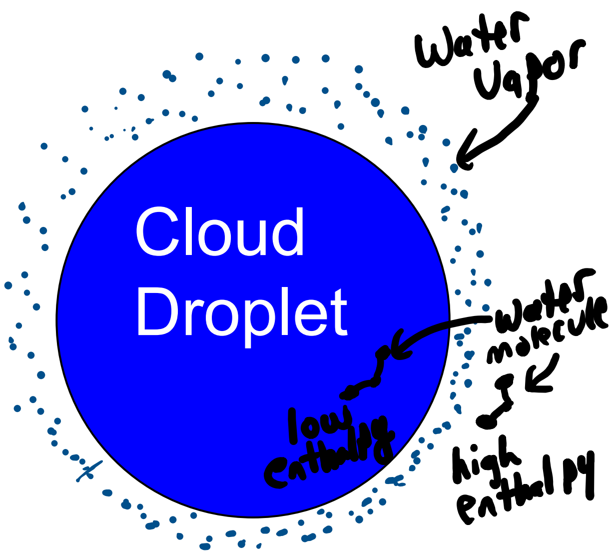


Do some examples of potential temperature

Latent (Hidden) Heat And Phase Transformation



Latent Heat!



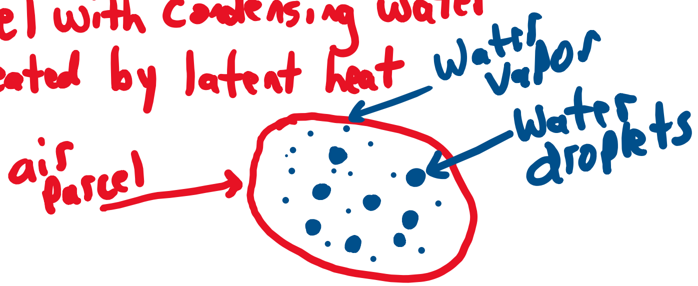
Latent Heat

Heat needed or released for a Phase Change.

Solid→Liquid
Liquid→Vapor

Vapor→liquid
liquid→Solid

An air Parcel with condensing water in it is heated by latent heat release



Exothermic Process

Enthalpy difference between the vapor and liquid phase must be released from the droplet to surroundings for the water molecule to stay in the liquid Phase - Condensation.

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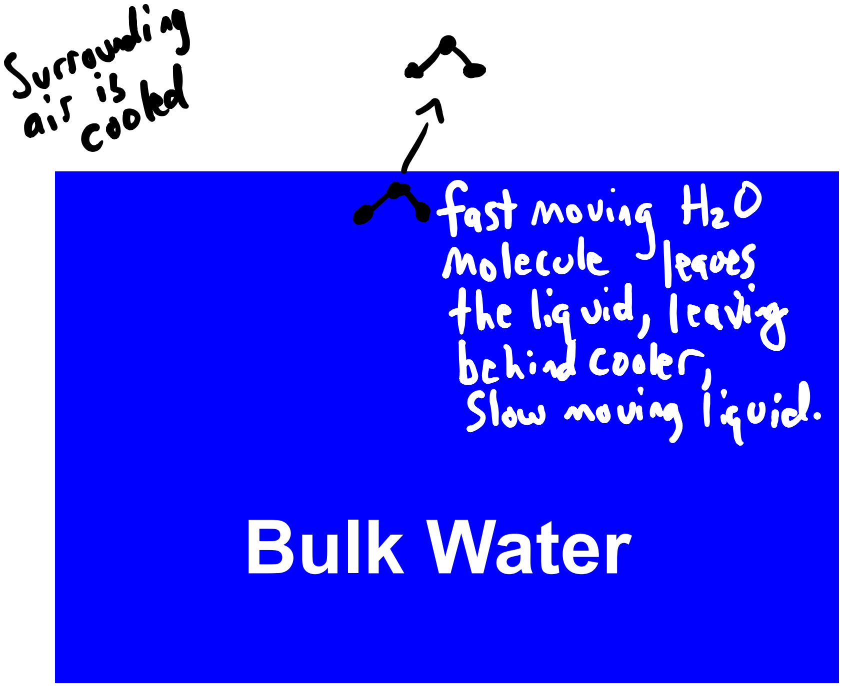
Evaporation – Evaporative Cooling – Endothermic Process



Surrounding air is cooled

fast moving H_2O molecule leaves the liquid, leaving behind cooler, slow moving liquid.

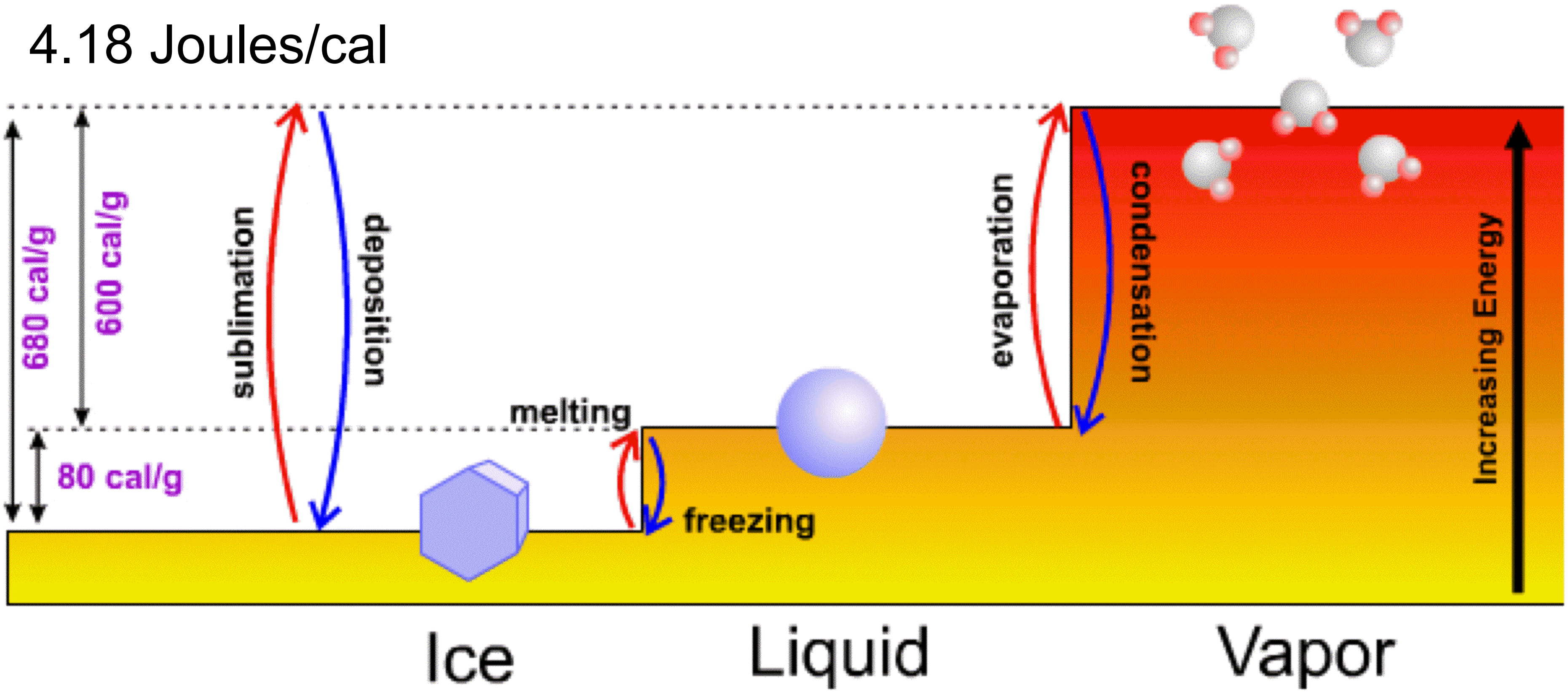
Bulk Water

Phase Change	Latent Heat
Solid-liquid	3.34×10^5 J/kg
Liquid-vapor	2.5×10^6 J/Kg

Well-stirred, Stays near 0°C Until all ice is melted

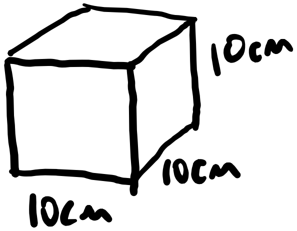
Energy Change On Phase Transformatoin

4.18 Joules/cal



Example of Latent Heat Release

Determine the temperature increase of 1 m^3 of air at sea level when the latent heat of formation of 1 kg water is added to it.



1 Kg of water fits in this volume

$L = 2.6\times10^{6}\,\mathrm{J/kg}$ Latent heat of condensation

$m C_v \Delta T$ ~ Temperature Change $= L * 1\,\mathrm{kg}$

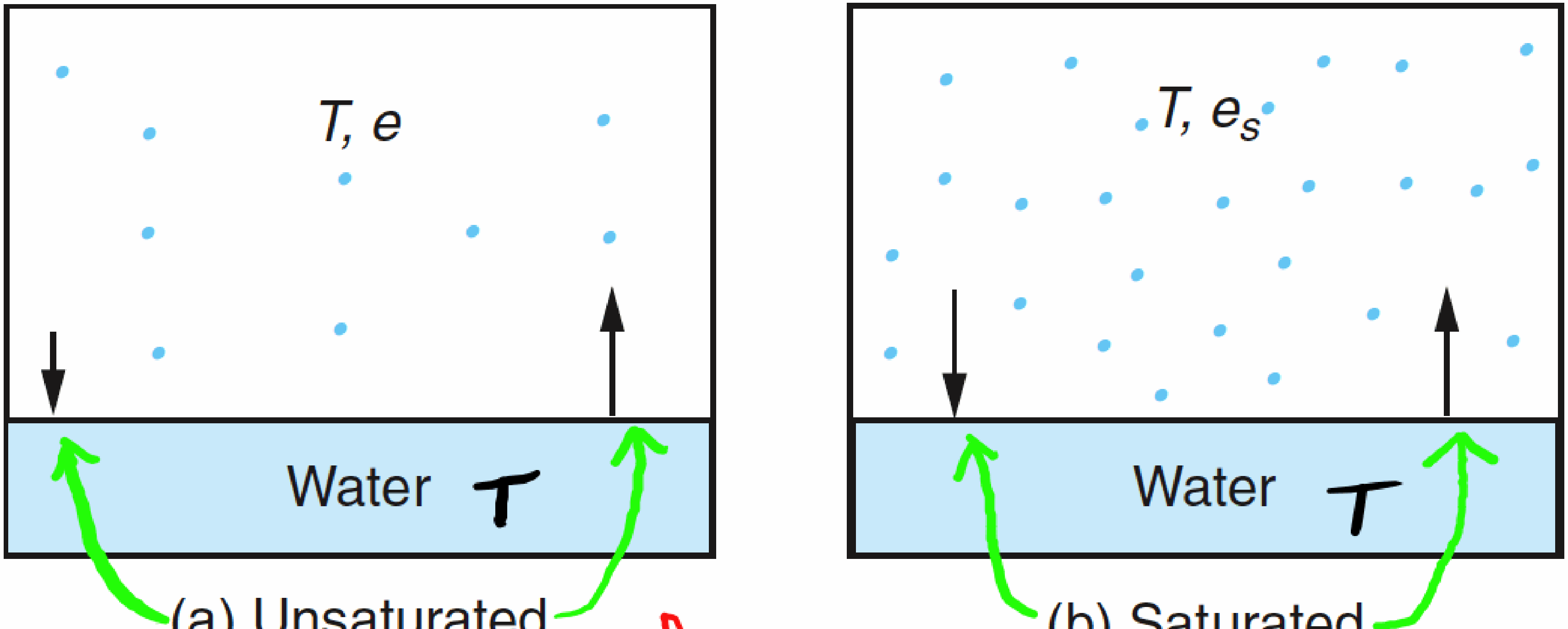
(Specific heat at constant volume $= \frac{5}{2} R = 717\,\mathrm{J/kg\,K}$

mass of air in $1\,\mathrm{m^3} = 1.25\,\frac{\mathrm{kg}}{\mathrm{m^3}} * 1\,\mathrm{m^3} = 1.25\,\mathrm{kg}$

$$\Delta T = \frac{L}{m C_v} = 2.6\times10^{6}\,\frac{\mathrm{J}}{\mathrm{kg}} \cdot 1\,\mathrm{kg}\ \frac{\mathrm{kg\,K}}{1.25\,\mathrm{kg}\ 717\,\mathrm{J}} = 2901\,\mathrm{K}\,!$$

That's a lot of condensed H_2O, but an impressive ΔT!

Saturation Vapor Pressure, $e_s(T)$



(a) Unsaturated

(b) Saturated

Review: $e = \rho_v R_v T$ = Partial Pressure of water vapor.

New idea: $e_s \approx 6.11\,\mathrm{mb}\,\exp\left[\frac{L}{R_v}\left(\frac{1}{273}-\frac{1}{T}\right)\right]$

Note: $e_s = e_s(T)$!!!

Saturation vapor pressure is only a function of T.

$RH \equiv e/e_s$

L = latent heat of ...

liquid to vapor,

$L_{ev} = 2.501 \times 10^{6}$ J/kg

Solid ice to vapor

$L_{sv} = 2.834 \times 10^{6}$ J/kg

$R_v = R/mw_v = \frac{8314.3\ \mathrm{J/K \cdot kmole}}{18\ \mathrm{kg/kmole}}$

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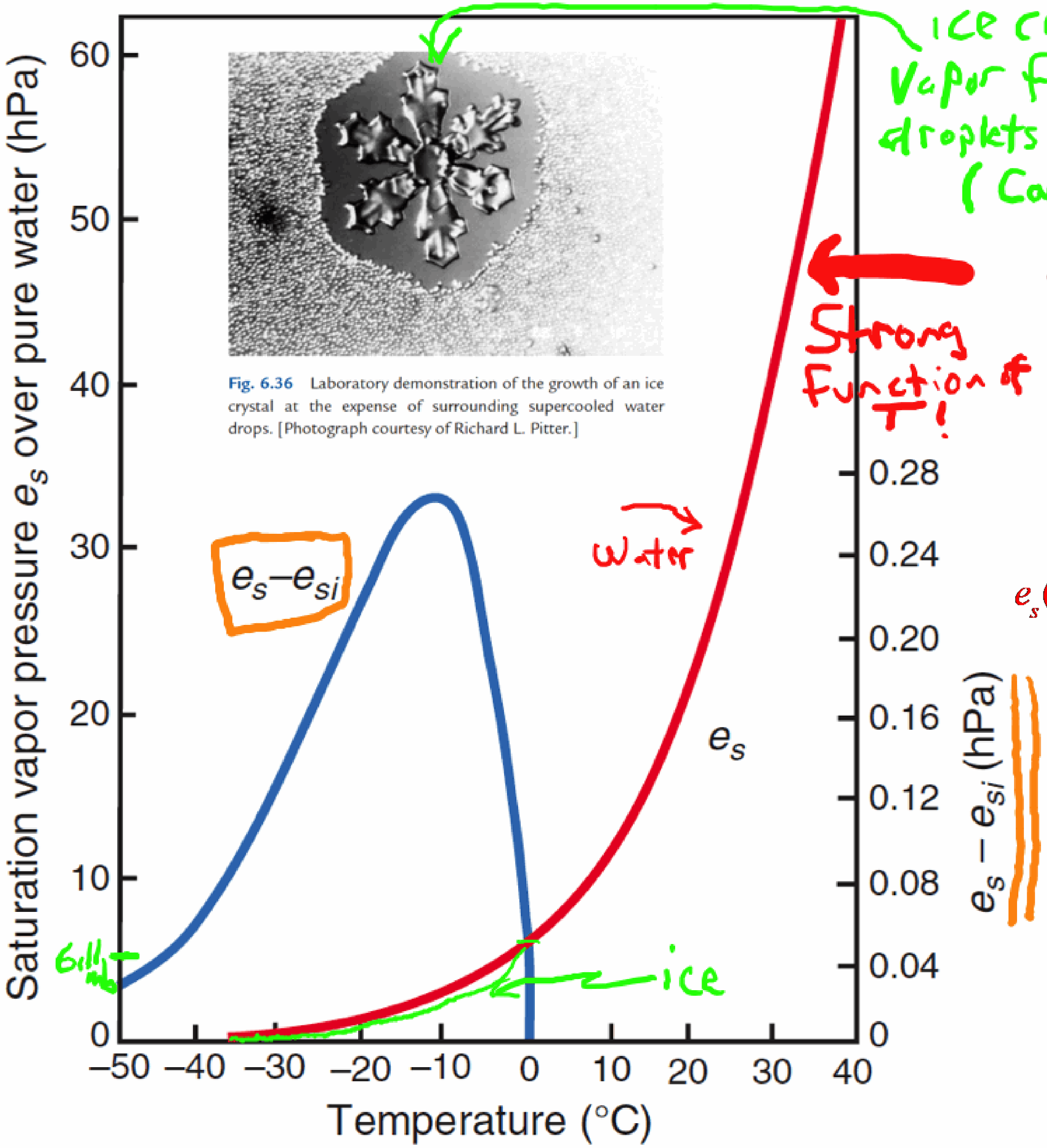


Fig. 6.36 Laboratory demonstration of the growth of an ice crystal at the expense of surrounding supercooled water drops. [Photograph courtesy of Richard L. Pitter.]

ice crystal 'eats' water vapor from supercooled water droplets because $e_s^{water} > e_s^{ice}$ (Can easily do this)

Strong Function of T!

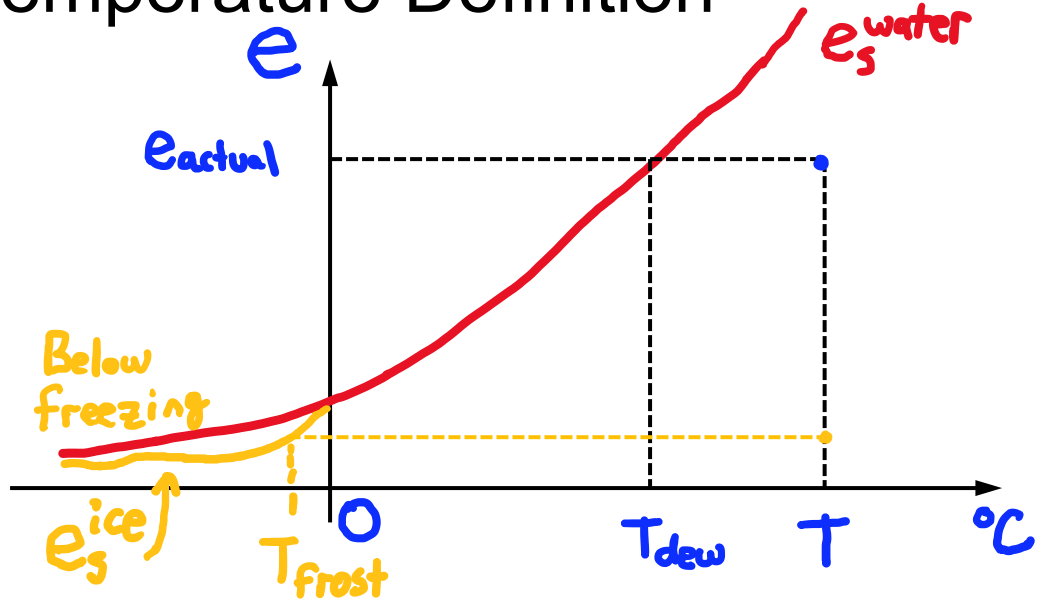
$e_s(T) \simeq 6.11 mb \times \exp[\frac{L}{R_v}(\frac{1}{273}-\frac{1}{T})]$

$L_{LV} = 2.501 \times 10^6$ J/kg

$L_{SV} = 2.834 \times 10^6$ J/kg

$$e_s(T) = 6.11\,mb\,\exp\left[\frac{L}{R_v}\left(\frac{1}{273}-\frac{1}{T}\right)\right]$$

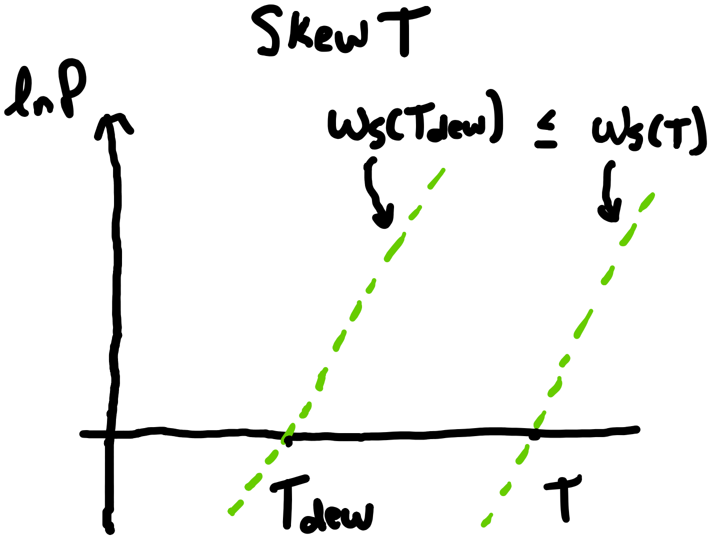
Dew Point Temperature Definition



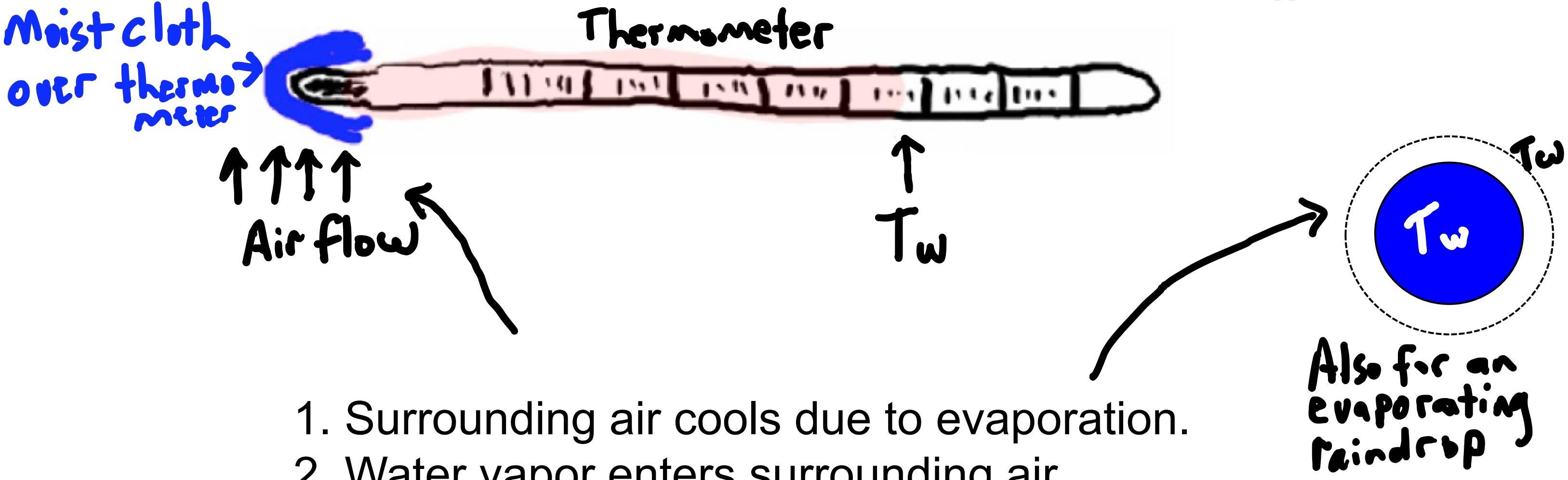
$$W = \frac{\epsilon \overset{\text{Actual value}}{e}}{P} = \frac{\epsilon \, \overset{\text{Defines } T_{dew}}{e_s(T_{dew})}}{P} = W_s(T_{dew})$$

$$RH = \frac{W}{W_s(T)} = \frac{W_s(T_{dew})}{W_s(T)}$$

Note: $T_{dew} \le T$



Wet Bulb Temperature, T_w



1. Surrounding air cools due to evaporation.
2. Water vapor enters surrounding air.
3. T_w is reached when the surrounding air becomes saturated.

Comparison: T_{dew} and T_w

T_{dew}

w=mixing ratio

Cool at constant pressure until saturation at the dew point temperature

T_w

w'=mixing ratio.

w'=w + evaporated water vapor.

Now cool at constant pressure.

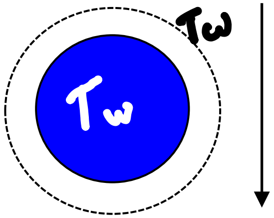
until saturation occurs at Tw.

Notes: $w' > w$ so $T_{dew} \leq T_w \leq T$, equal at the LCL.

Calculation of T_w, the Wet Bulb Temperature

T_o = ambient temperatual

W_o = ambient mixing ratio



falling raindrop

Also for an evaporating raindrop

Enthalpy

→ 0 Constant Pressure Process

$$dq = C_p dT - \alpha \cancel{dP} = -L_{ev}\, dw$$

Evaporated water vapor mixing ratio.

Latent heat

dq=heat required to overcome the H_20 liquid state potential energy of attraction. dq comes from the surrounding air.

Integrating, $C_p(T_w - T_o) = -L_{ev}\left(W_s(T_w) - \underline{\underline{W_o}}\right)$

Solve for T_w.

Recall: $w_s(T) = \frac{\epsilon e_s(T)}{p}$, $e_s(T_c) = 6.112\,\text{mb}\,\exp\left(\frac{17.67T}{T+243.5}\right)$

Compare:

T_{dew}

$e_s(T_{dew}) = e_{observed}(T_o)$

Solve for T_{dew}.

Note:

$\underline{\underline{W_o}} = \frac{\epsilon e_{sat}(T_{dew})}{p}$

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Analysis of T_{dew} from T and T_w

Analysis For T_{dew} :

From the first law of thermodynamics:

$$w_0(T_{dew},P_0)=w_s(T_w,P_0)-\frac{c_p}{L_{lv}}(T_0-T_w)$$

General Definition:

$$w_0(T_{dew},P_0)=\frac{\varepsilon e_{sat}(T_{dew})}{P_0}$$

From Bolton,

$$e_{sat}(T_c)=e_0\exp\left(\frac{17.67\,T_c}{243.5+T_c}\right),\ e_0=6.112\ mb$$

So

$$\frac{e_{sat}(T_{dew})}{e_0}=\frac{e_s(T_w)}{e_0}-\frac{P_0\,c_p}{\varepsilon\,e_0\,L_{lv}}(T_0-T_w)\equiv R=\exp\left(\frac{17.67\,T_{dew}}{243.5+T_{dew}}\right)$$

Solving,

$$T_{dew}=\frac{243.5\ln(R)}{17.67-\ln(R)}\qquad \textit{Celsius units,}$$

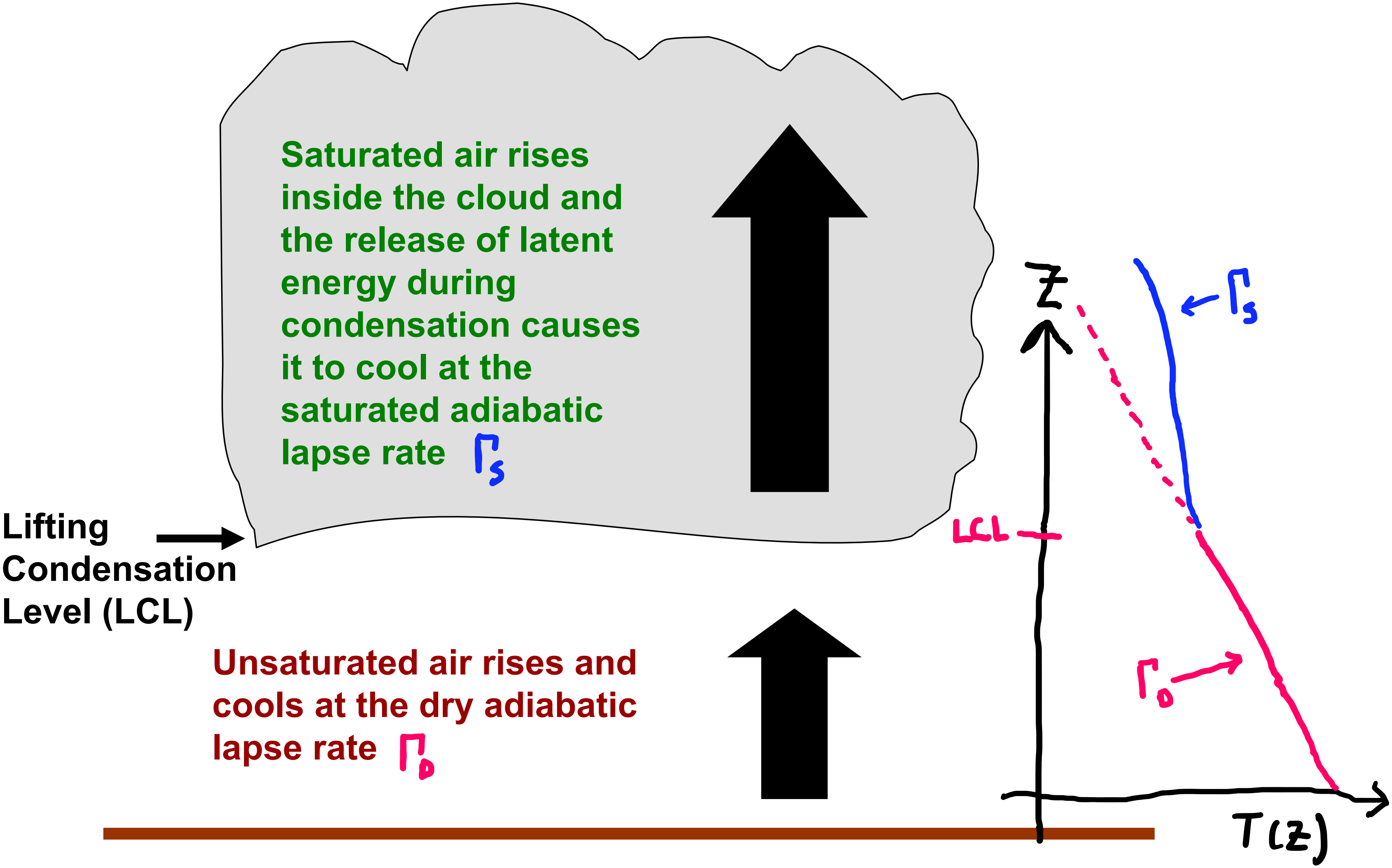
where

$$R=\frac{e_s(T_w)}{e_0}-\frac{P_0\,c_p}{\varepsilon\,e_0\,L_{lv}}(T_0-T_w).$$

$$\varepsilon=0.622,\ c_p=1004\frac{J}{kg\,K},$$

$$L_{lv}(T)=1000\left(2500.8-2.36T+0.0016T^2-0.00006T^3\right)\frac{J}{kg}\qquad T \text{ in Celsius.}$$

Saturated Adiabatic Lapse Rate



Saturated air rises inside the cloud and the release of latent energy during condensation causes it to cool at the saturated adiabatic lapse rate Γ_s

Lifting Condensation Level (LCL)

Unsaturated air rises and cools at the dry adiabatic lapse rate Γ_D

Pat Arnott

The Saturated Adiabatic Lapse Rate

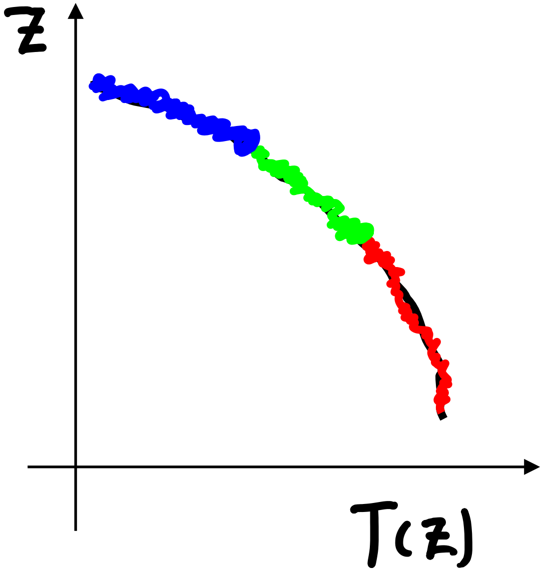
The saturated adiabatic lapse rate is always *less* than the dry adiabatic lapse rate because the cooling caused by adiabatic expansion *is partially offset by* *the release of latent energy during condensation*.

Tropopause, $\Gamma_s \approx \Gamma_{DRY} = 9.8$ K/km

Mid troposphere, $\Gamma_s \approx 6$ K/km

For humid air near ground level,

$\Gamma_s \approx 4$ K/km



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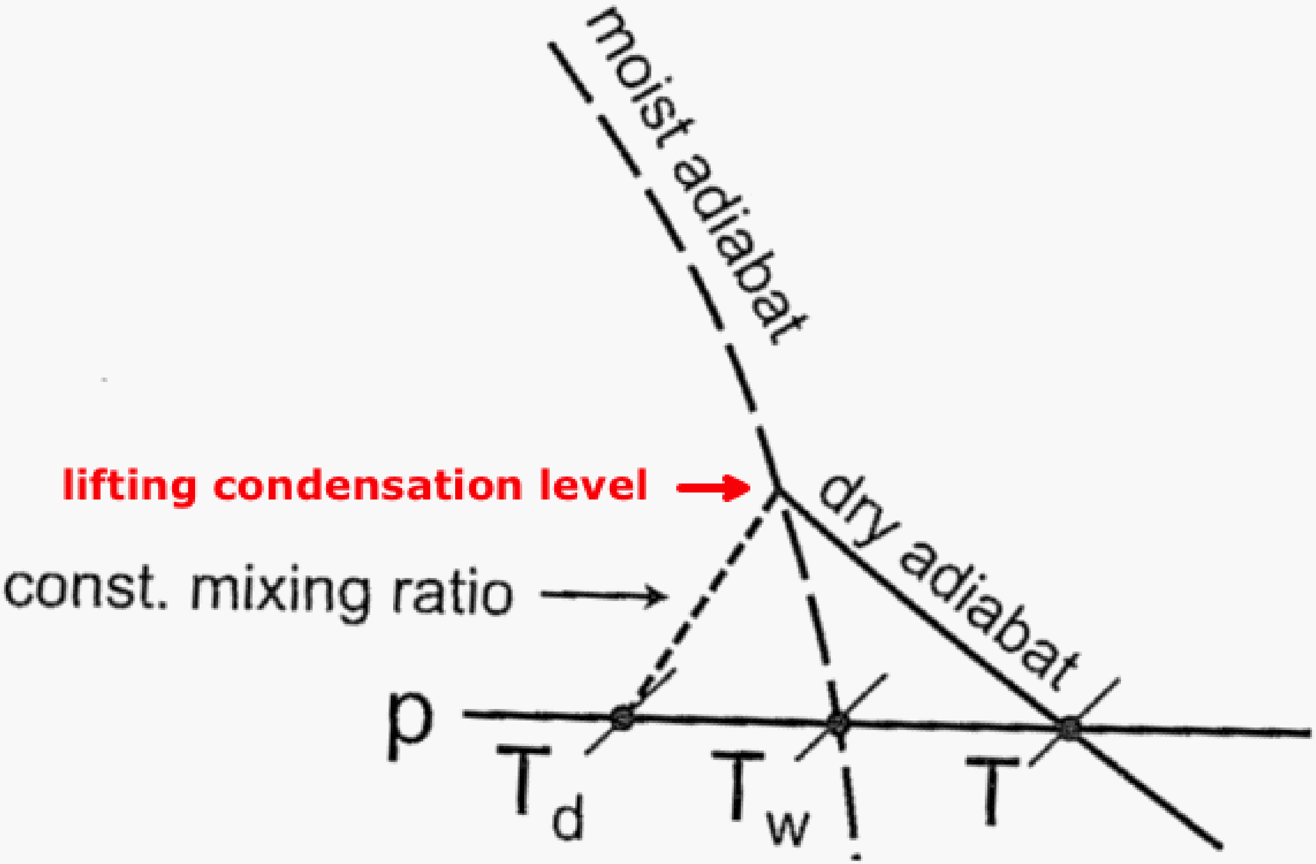
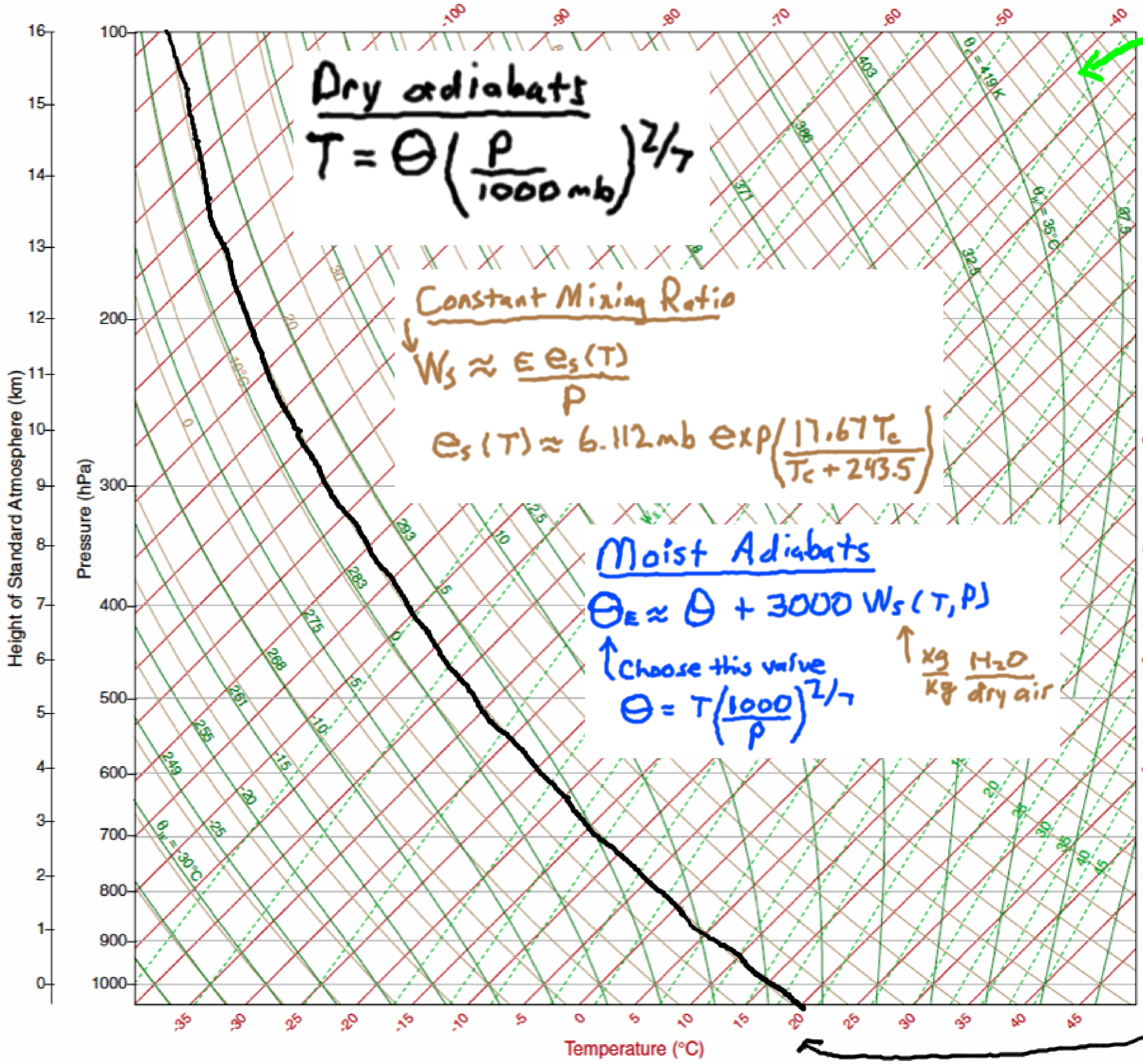
Normand’s Rule

Relationship between dew point, wet-bulb and dry bulb temperatures

NORMAND's RULE



Do an example from room conditions and T, T_w measurements



Dry adiabats

$$T = \theta\left(\frac{p}{1000\,mb}\right)^{2/7}$$

Constant Mixing Ratio

$$W_s \approx \frac{\epsilon\, e_s(T)}{p}$$

$$e_s(T) \approx 6.112\,mb\,\exp\left(\frac{17.67\,T_c}{T_c + 243.5}\right)$$

Moist Adiabats

$$\theta_E \approx \theta + 3000\,W_s(T,p)$$

Choose this value

$$\theta = T\left(\frac{1000}{p}\right)^{2/7}$$

$\frac{kg\ H_2O}{kg\ dry\ air}$

Moist adiabat

isotherms

Constant mixing Ratio

$$W = \frac{g_{H_2O}}{kg\ air}$$

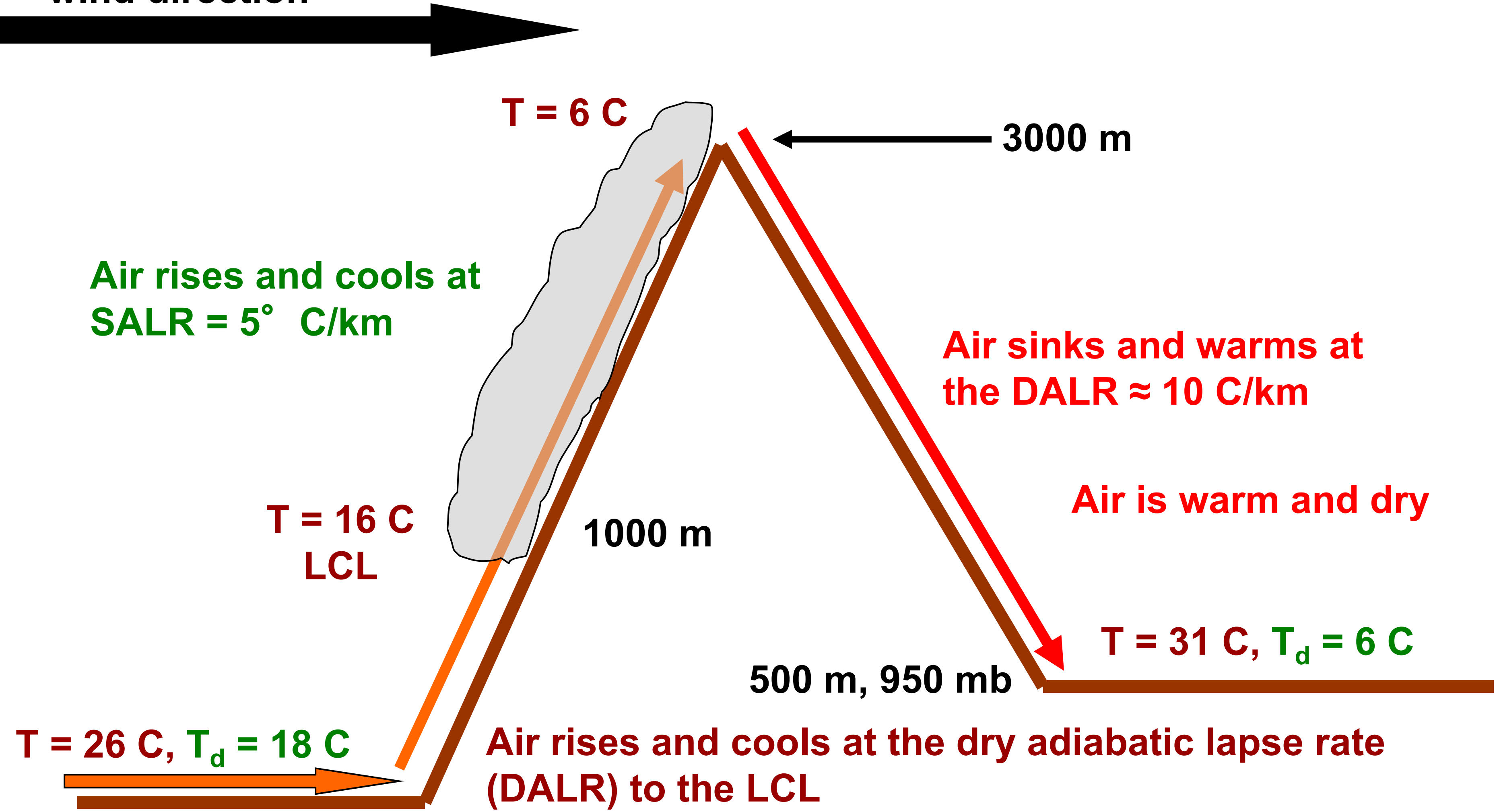
Dry adiabat

Flow Over a Mountain: 100% Precipitation Efficiency

Windward side

wind direction

Leeward side



T = 6 C

3000 m

Air rises and cools at SALR = 5° C/km

Air sinks and warms at the DALR ≈ 10 C/km

Air is warm and dry

T = 16 C
LCL

1000 m

$T = 31\ C, T_d = 6\ C$

500 m, 950 mb

$T = 26\ C, T_d = 18\ C$

Air rises and cools at the dry adiabatic lapse rate (DALR) to the LCL

0 meters, 1010 mb

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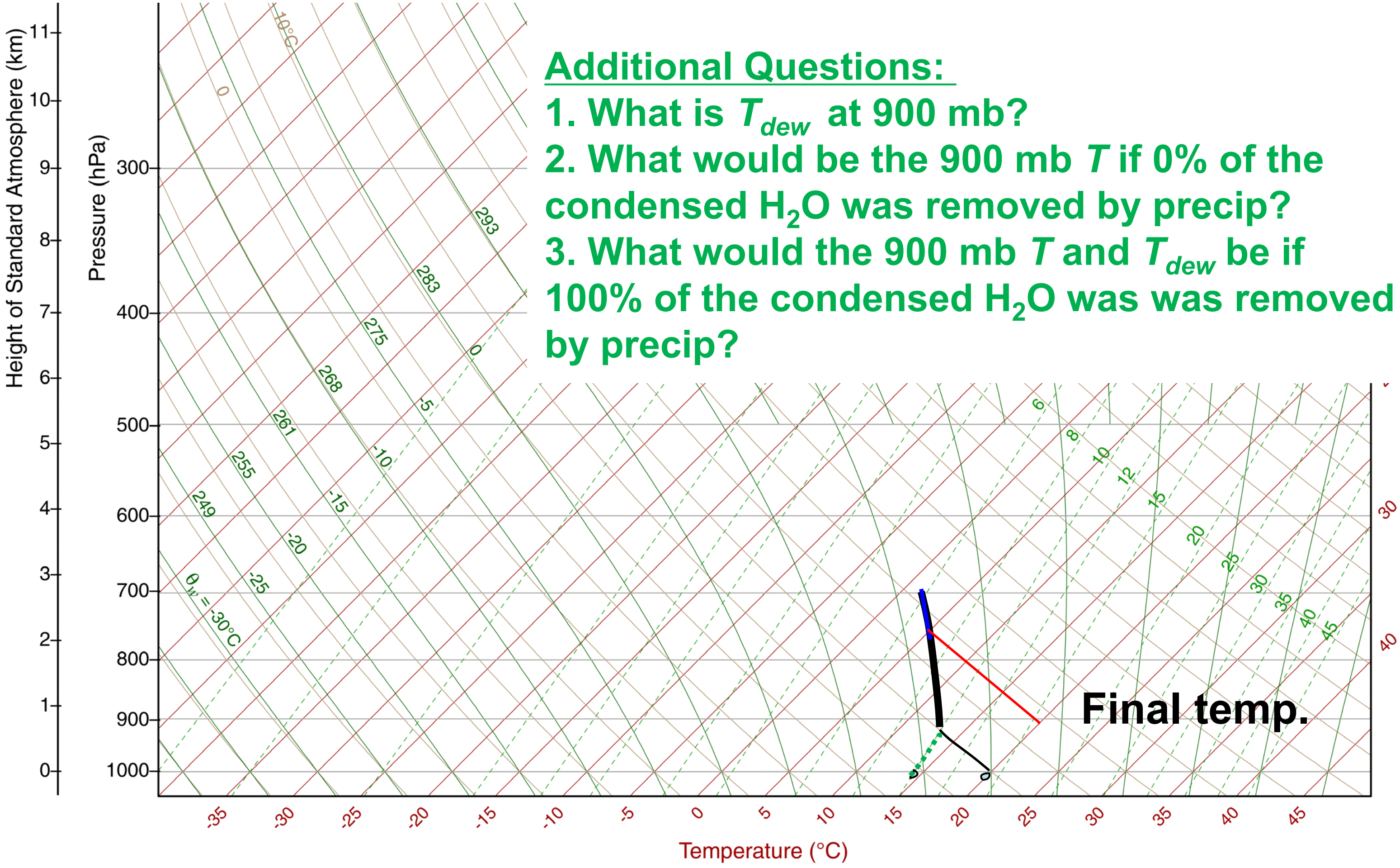
3.48 Air at a temperature of 20 °C and a mixing ratio of 10 g kg^{-1} is lifted from 1000 to 700 hPa by moving over a mountain. What is the initial dew point of the air? Determine the temperature of the air after it has descended to 900 hPa on the other side of the mountain if 80% of the condensed water vapor is removed by precipitation during the ascent. (**Hint**: Use the skew $T - \ln p$ chart.)

T_{dew}=14.5 C.
W_s=6g/kg is water vapor @ 700 mb.
0.2*(10g/kg-6g/kg)=0.8 g/kg is cloud.
Follow moist adiabat down from 700 mb to evaporate cloud to W_s=6.8g/kg.
Follow the dry adiabat from there to 900 mb.
The final T is about 20 C.
Ascending patch in black.
Descending path in blue and red.

Additional Questions:

1. What is T_{dew} at 900 mb?
2. What would be the 900 mb T if 0% of the condensed H_2O was removed by precip?
3. What would the 900 mb T and T_{dew} be if 100% of the condensed H_2O was was removed by precip?

Chinook Winds Example (also known as Föhn)



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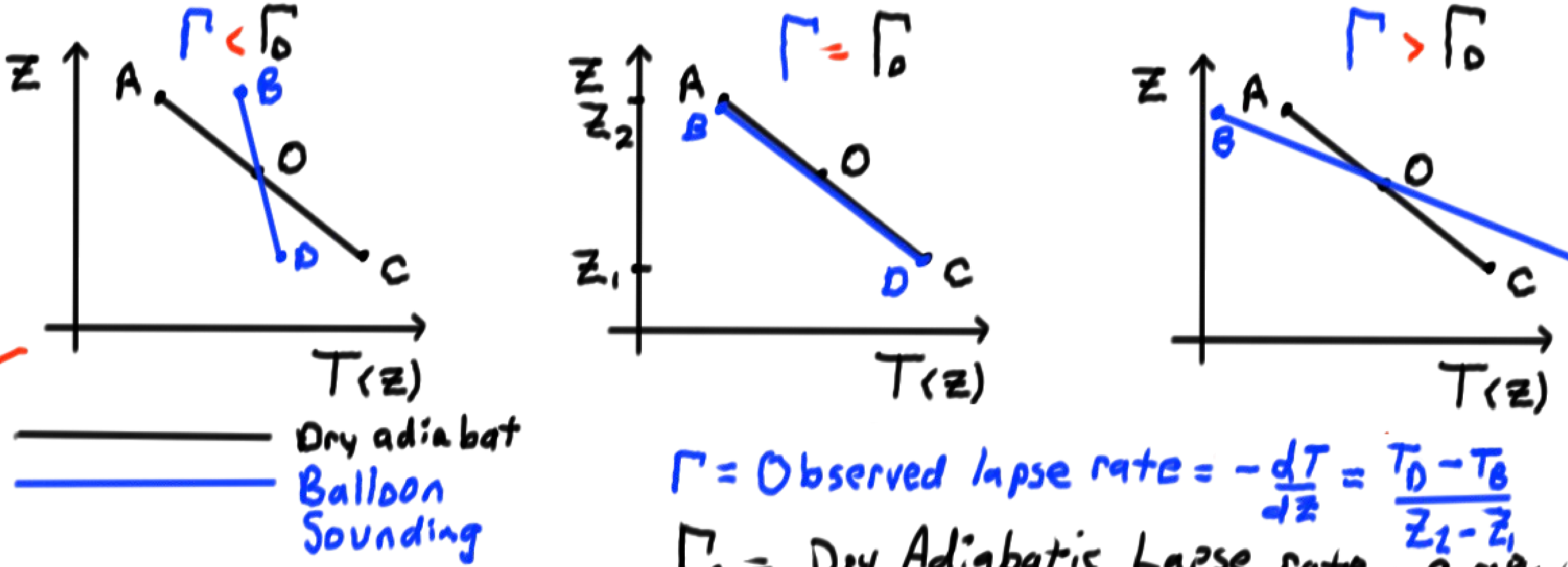
Stability of An Unsaturated Air Parcel



Γ = Observed lapse rate = $-\frac{dT}{dz} = \frac{T_D - T_B}{Z_2 - Z_1}$

Γ_0 = Dry Adiabatic Lapse rate = 9.8°K/km

$\Gamma < \Gamma_0$

Raise parcel from O to A. $T_A < T_B$, Parcel Sinks since $\rho_A > \rho_B$

Stable $\Gamma < \Gamma_0$

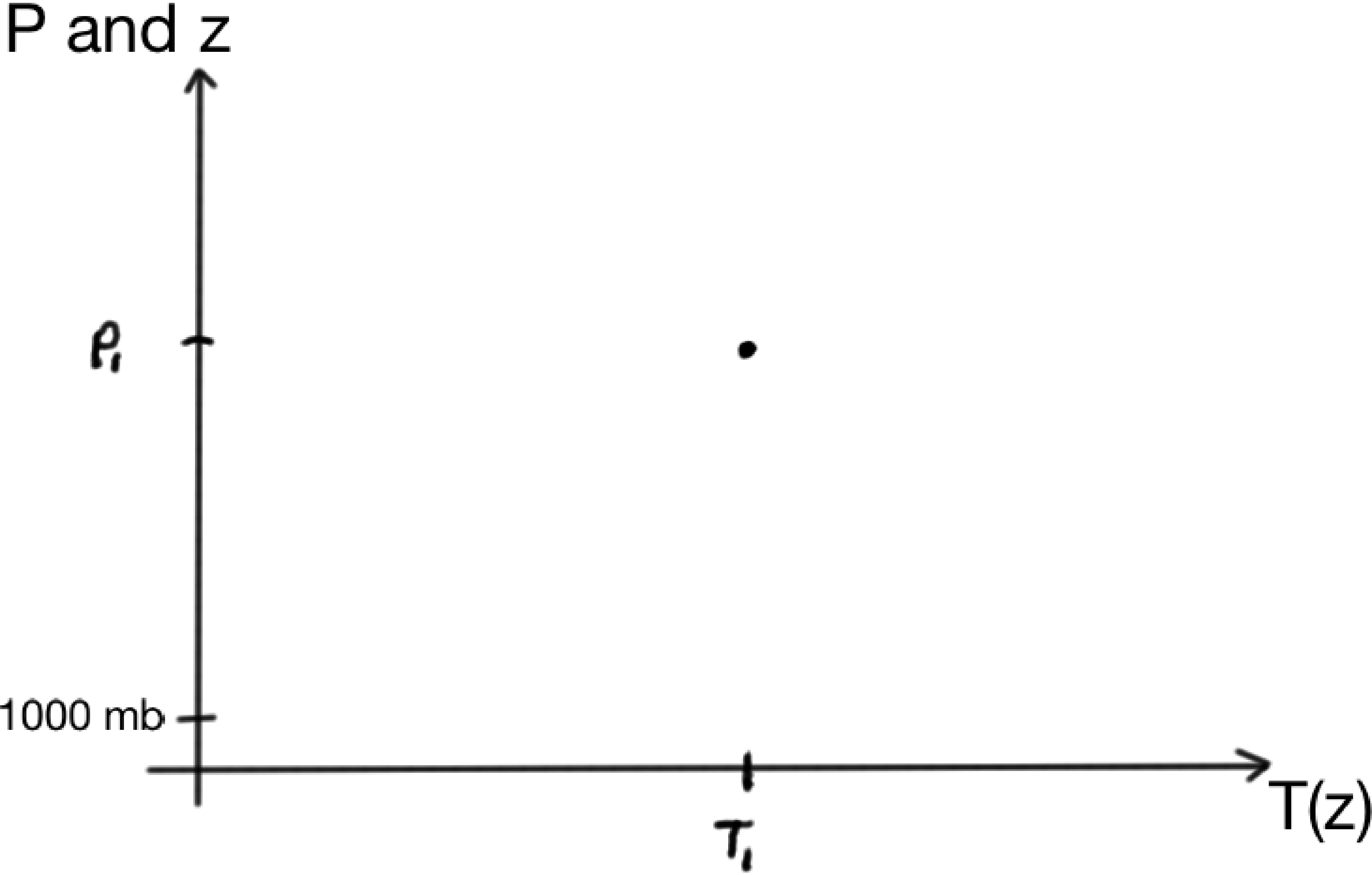
$\Gamma < \Gamma_0$ Stable
$\Gamma > \Gamma_0$ unstable
$\Gamma = \Gamma_0$ neutrally Stable

Raise Parcel from O to A. $T_A > T_B$. $\rho_A < \rho_B$. Parcel Continues to rise!

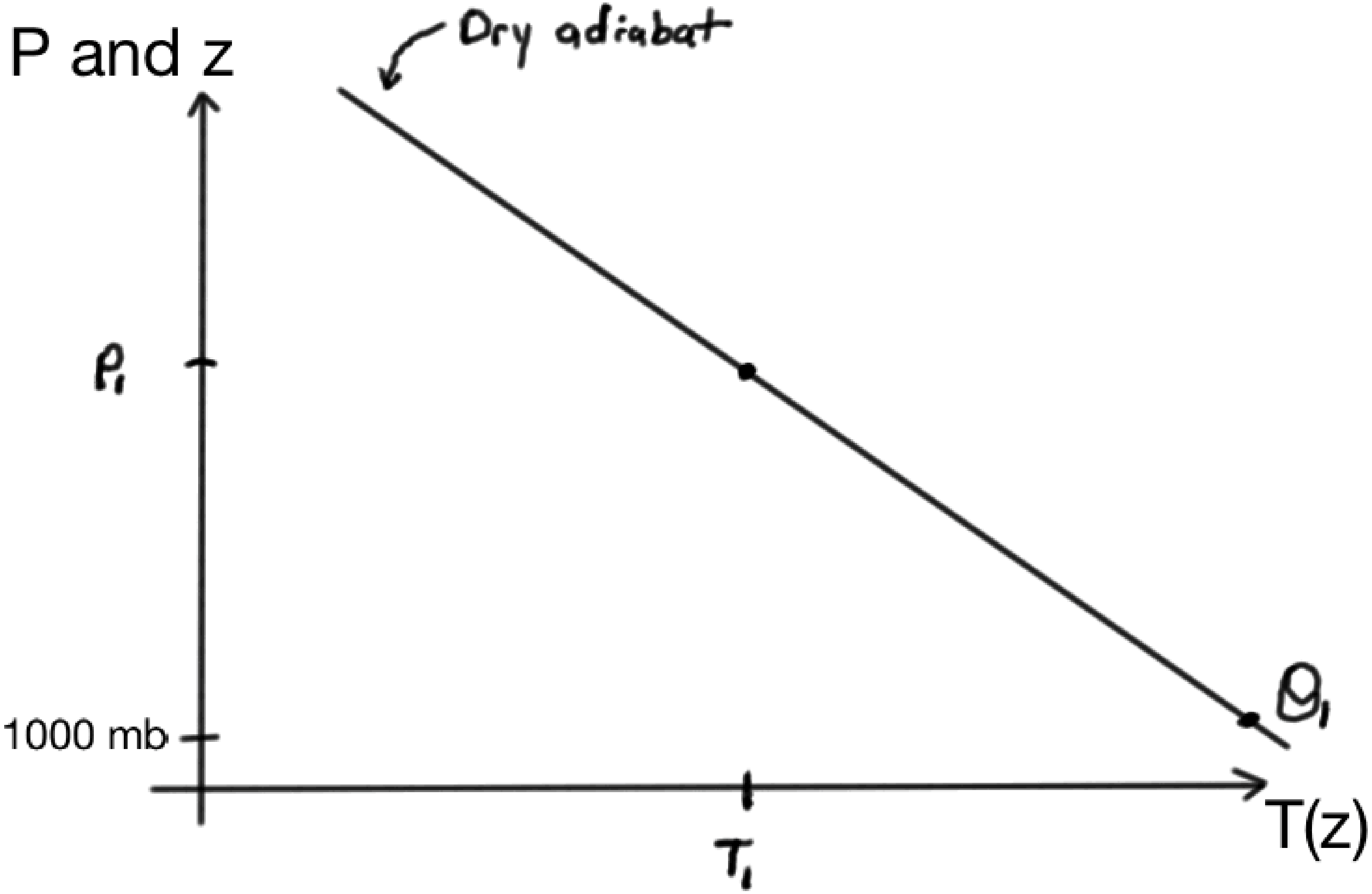
$\Gamma > \Gamma_0$ Unstable

Pat Arnott

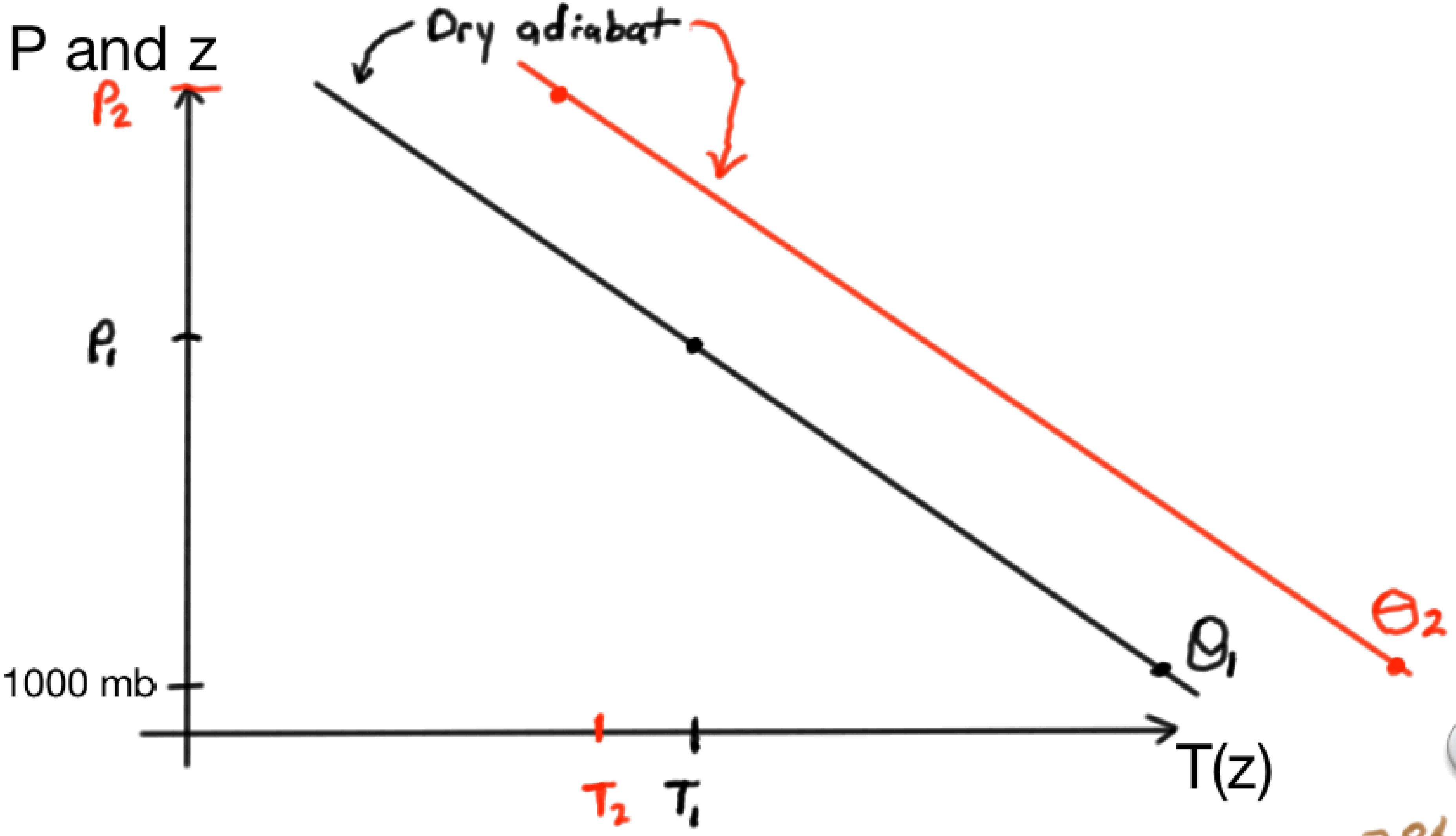
Step by Step Discussion of Stability



Step by Step Discussion of Stability

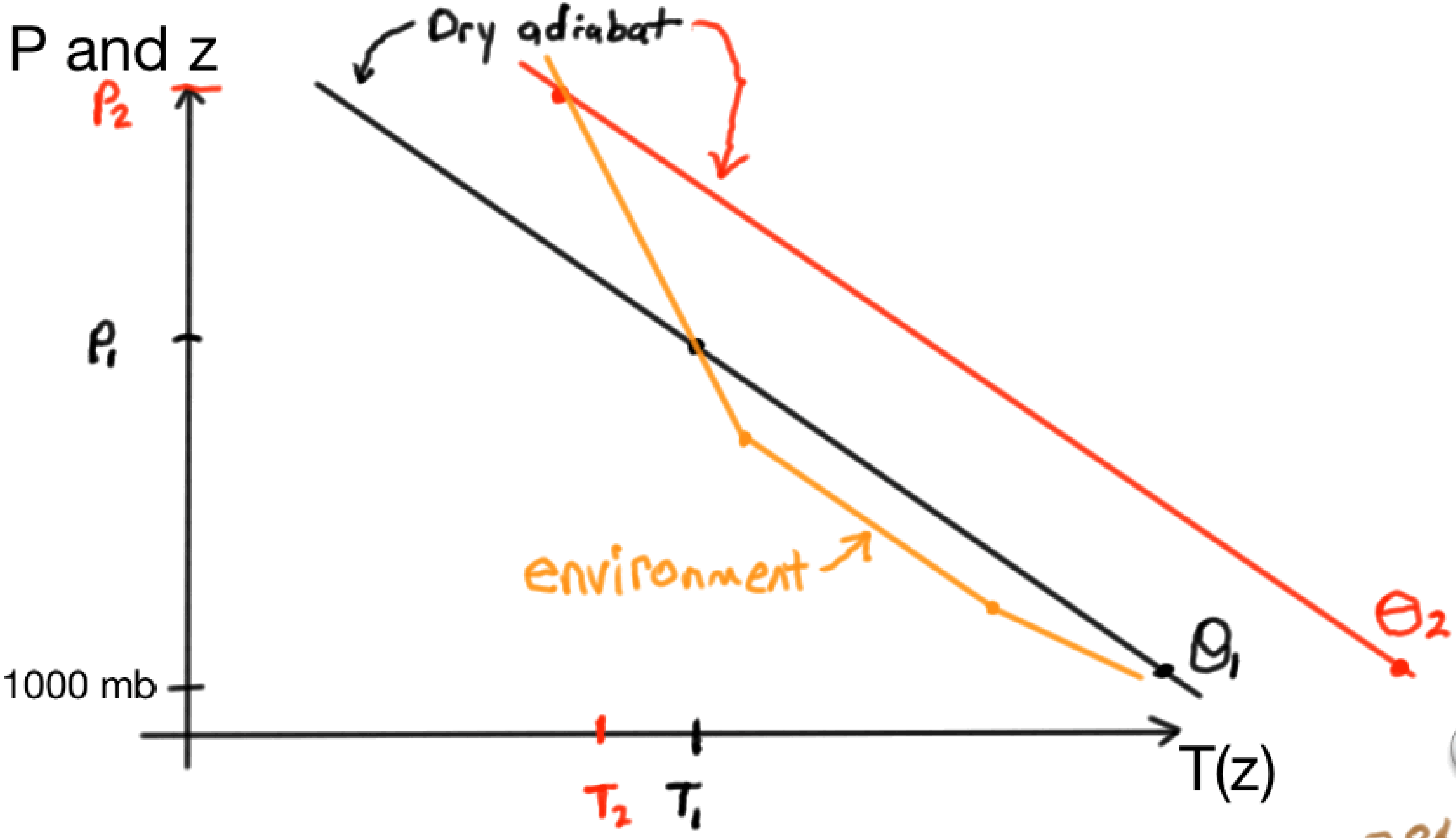


Step by Step Discussion of Stability



$$\theta_1 = T_1\left(\frac{1000}{P_1}\right)^{0.286}$$

Step by Step Discussion of Stability



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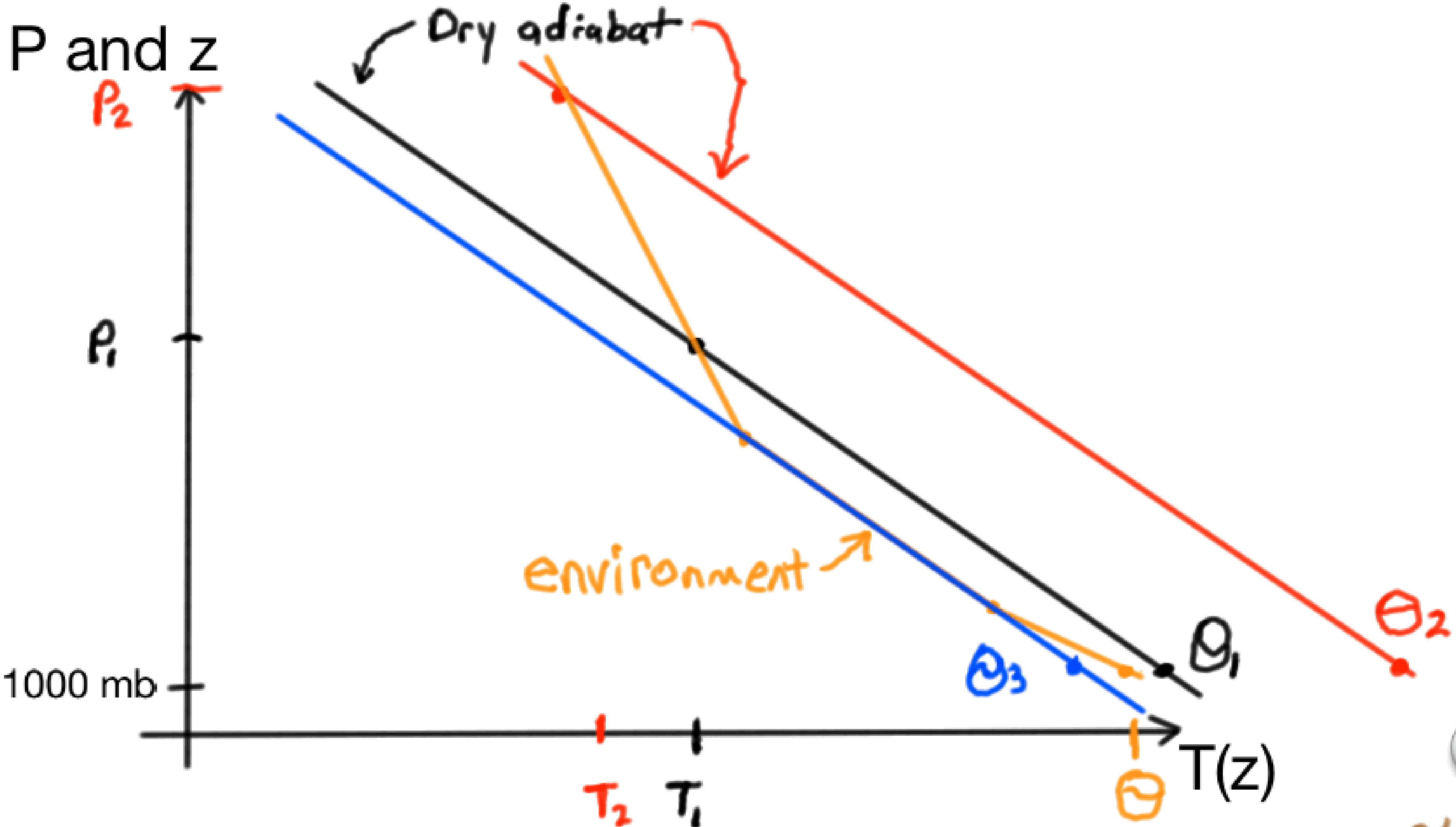
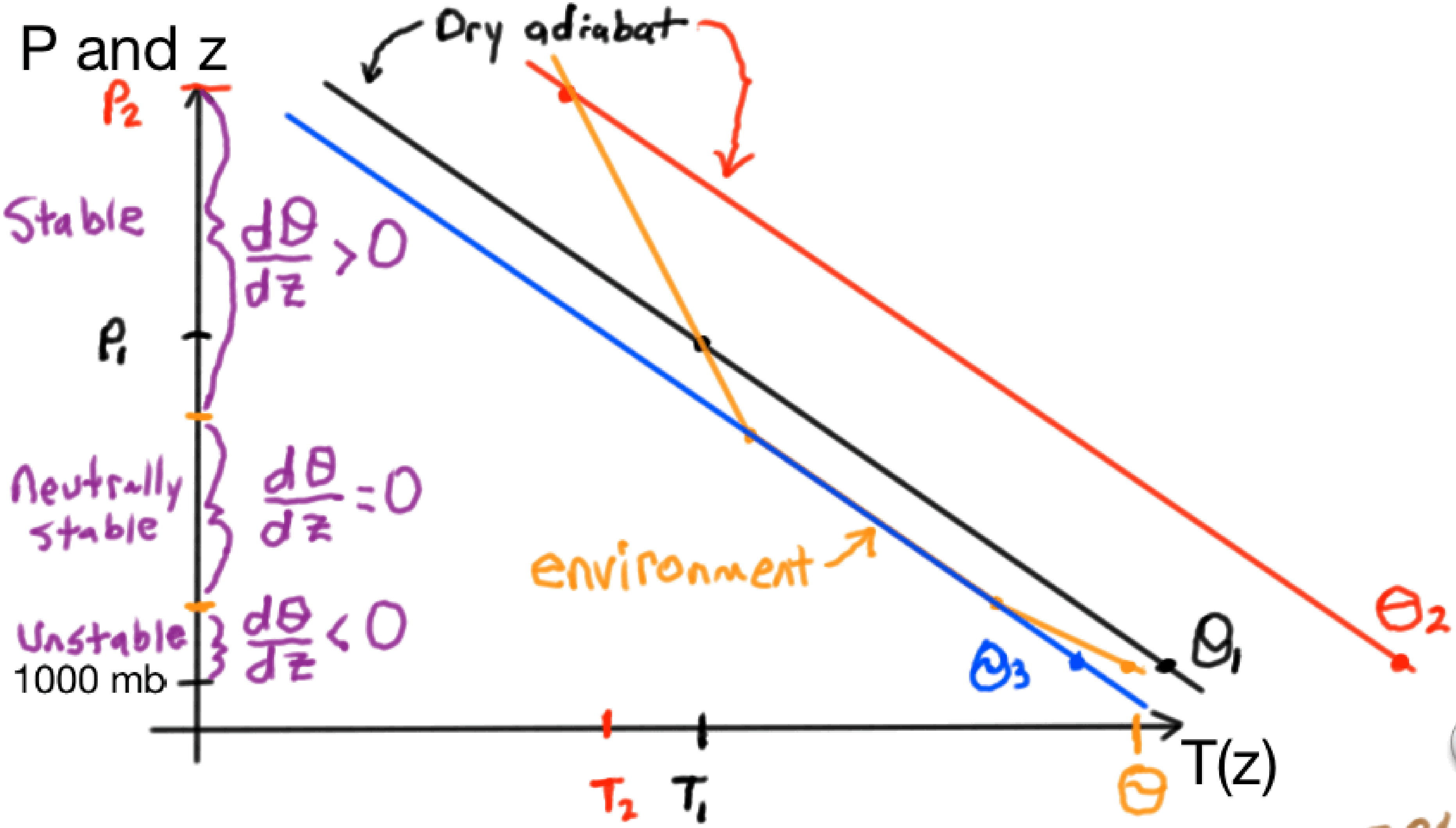
Step by Step Discussion of Stability

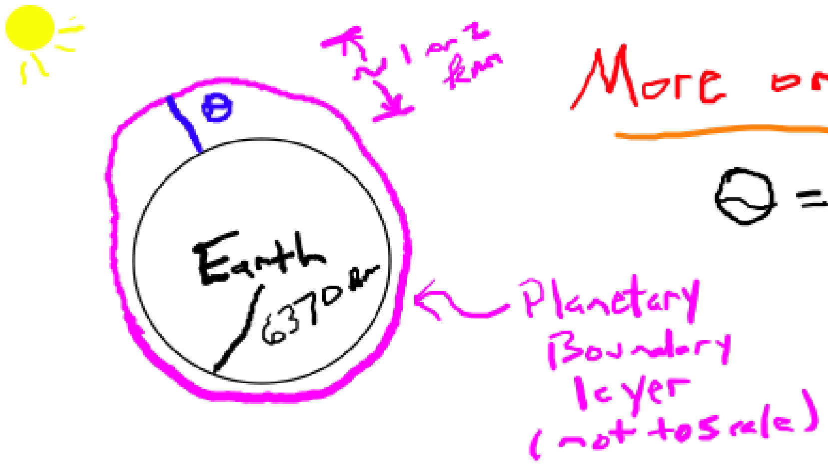


$$\Theta_1 = T_1 \left(\frac{1000}{P_1}\right)^{0.286}$$

Step by Step Discussion of Stability



$$\theta_1 = T_1 \left(\frac{1000}{P_1}\right)^{0.286}$$

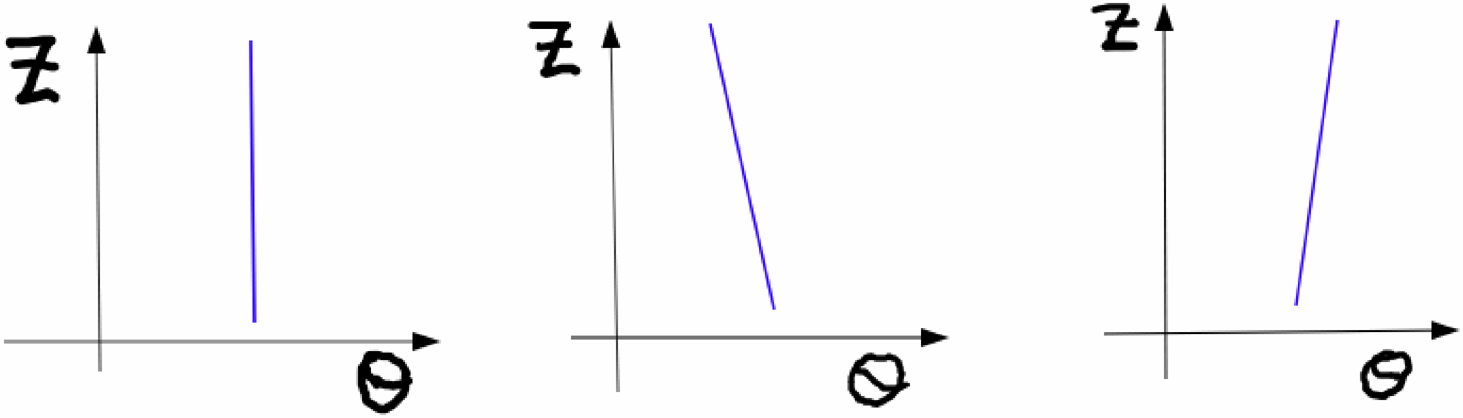


More on the Potential Temperature

$$\Theta = T(z)\left(\frac{P_o}{P(z)}\right)^{R/C_p}$$

$P_o = 1000$ mb

$R/C_p = 2/7$

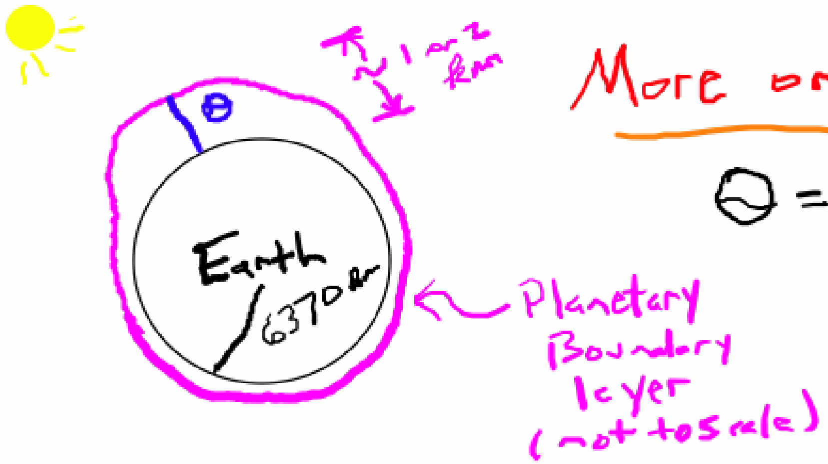


Match: A) Neutrally stable.

B) Stable.

C) Unstable.

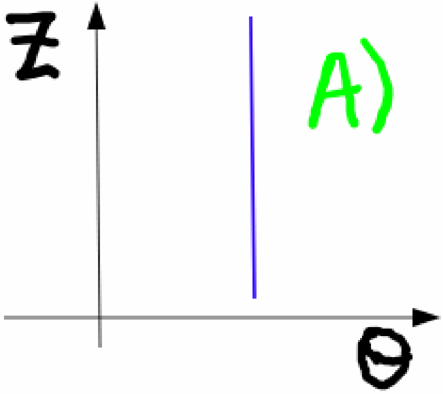
More on the Potential Temperature

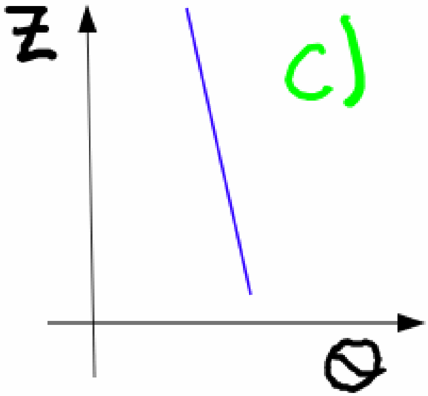


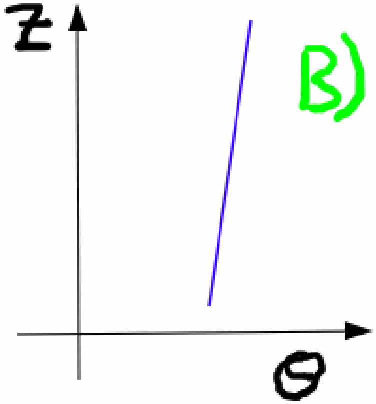
$$\Theta = T(z)\left(\frac{P_o}{P(z)}\right)^{R/c_p}$$

$P_o = 1000\,\text{mb}$

$R/c_p = 2/7$







Match: A) Neutrally stable.

B) Stable.

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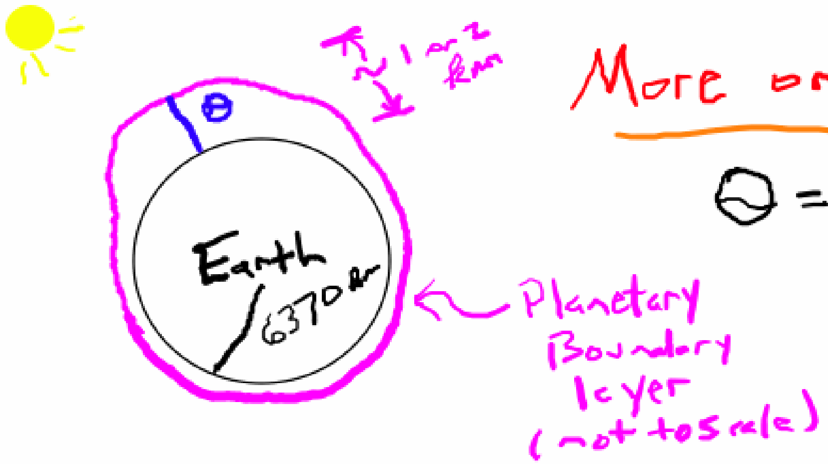
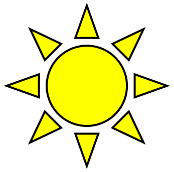
More on the Potential Temperature

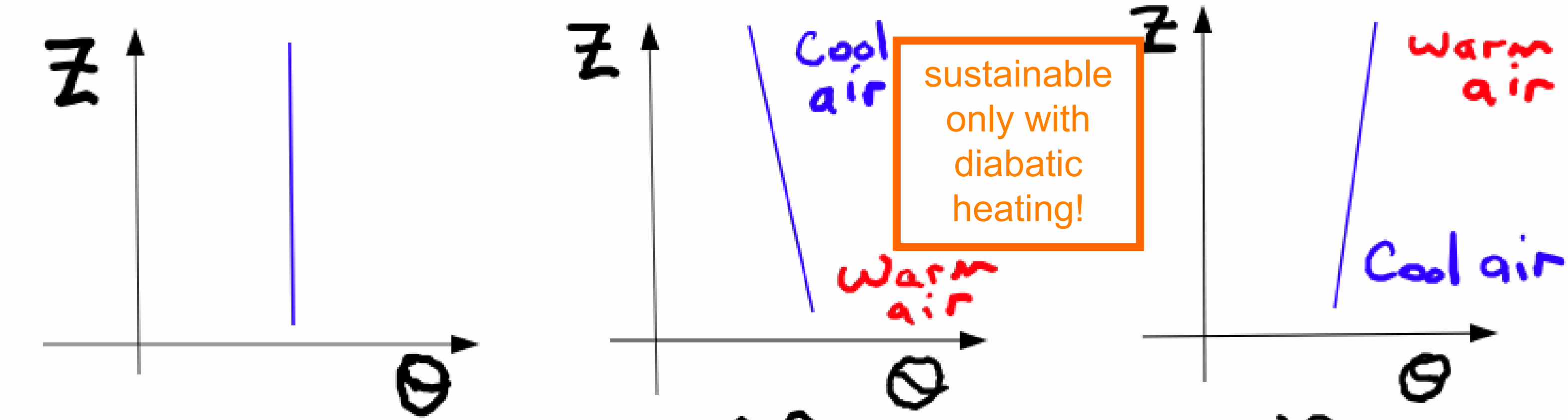
$$\Theta = T(z)\left(\frac{P_0}{P(z)}\right)^{R/c_p}$$

$P_0 = 1000\,\text{mb}$

$R/c_p = 2/7$







sustainable only with diabatic heating!

$\frac{\partial\theta}{\partial z} = 0$ Neutrally Stable

$\frac{\partial\theta}{\partial z} < 0$ Unstable

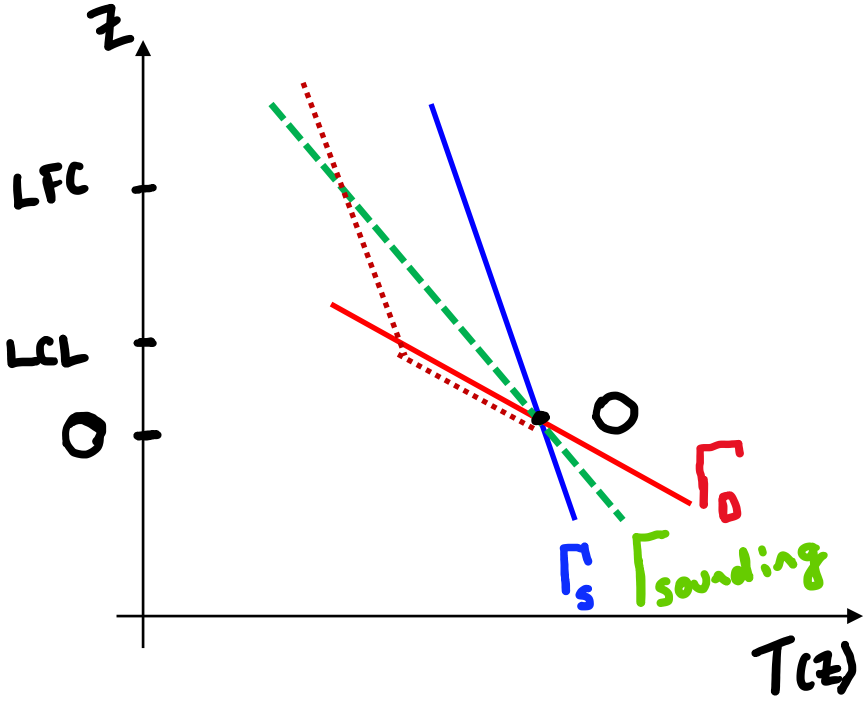
$\frac{\partial\theta}{\partial z} > 0$ Stable

Warm air is less dense than cool air.

Archimedes says the warm air wants to rise!

Pat Arnott

Conditionally Unstable Atmosphere



O - Starting Point for lifting air Parcel

LCL - lifting condensation level of lifted parcel

LFC - level of free Convection of lifted air Parcel

Γ_D = dry adiabatic lapse rate

Γ_s = Saturated lapse rate

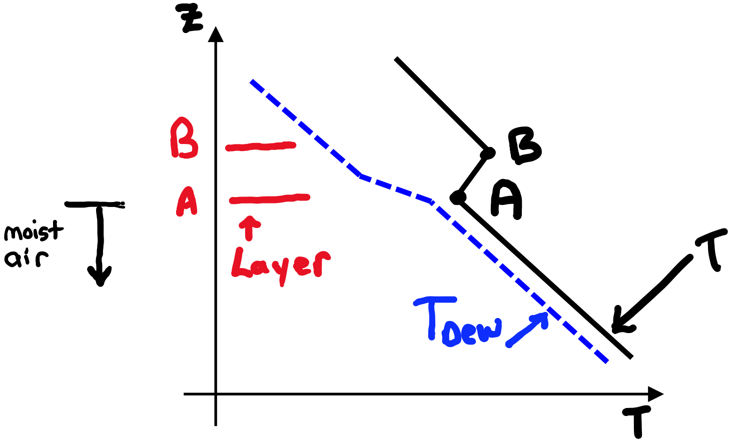
$\Gamma_{sounding}$ = Observed values

...... Path of parcel lifted From O

With enough lifting from O, a parcel can reach its LFC and take off.

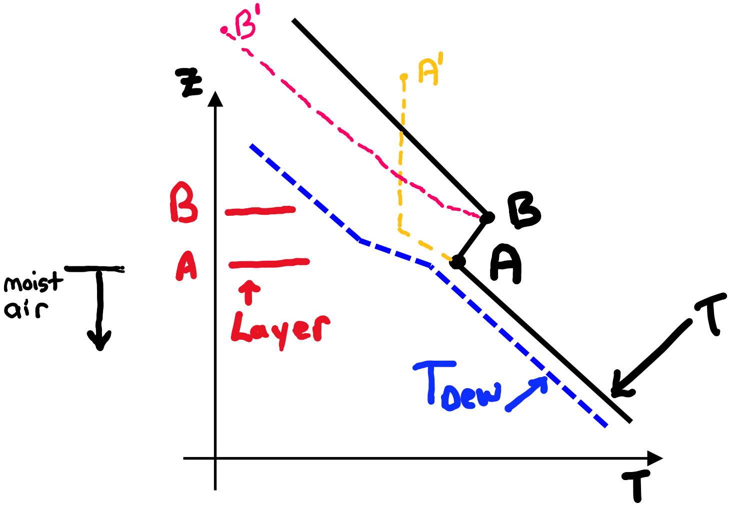
$$\Gamma_s < \Gamma_{sounding} < \Gamma_D$$

Convective Instability



Inverted green onion

← Sounding typical of the tropics



$T_{B'} < T_{sounding}$ at the same level, sinks

$T_{A'} > T_{sounding}$ at the same level

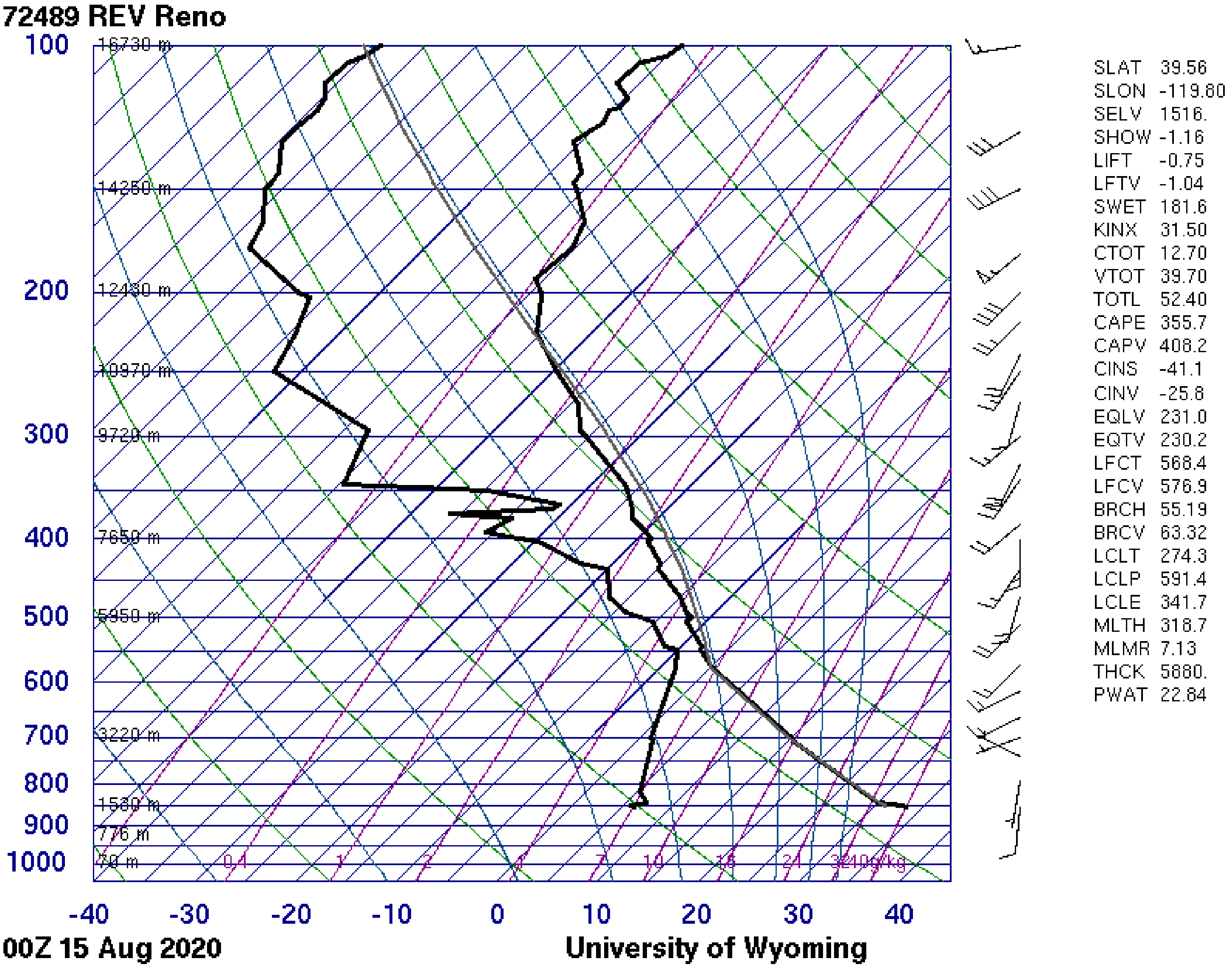
Unstable

Convection happens, parcels overturn.

Note: $\Theta_{EB} < \Theta_{EA}$ So $\frac{d\Theta_E}{dz} < 0$ for Convective instability

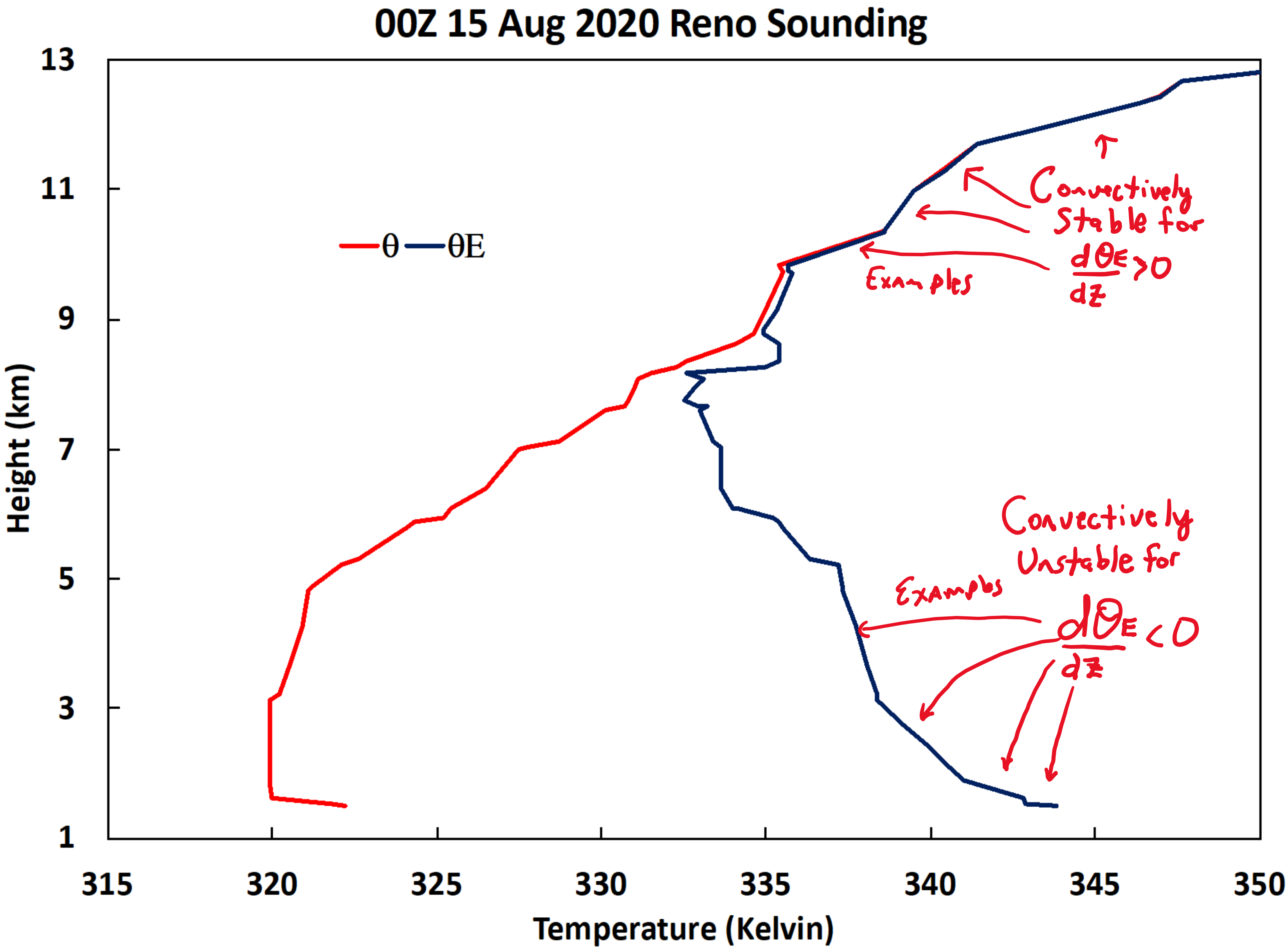
Θ_E = equivalent Potential temperature $\approx \Theta + 3000 W$

Stability Analysis: Plot θ and θ_E as a function of z and interpret in terms of stability for this sounding



SLAT	39.56
SLON	-119.80
SELV	1516.
SHOW	-1.16
LIFT	-0.75
LFTV	-1.04
SWET	181.6
KINX	31.50
CTOT	12.70
VTOT	39.70
TOTL	52.40
CAPE	355.7
CAPV	408.2
CINS	-41.1
CINV	-25.8
EQLV	231.0
EQTV	230.2
LFCT	568.4
LFCV	576.9
BRCH	55.19
BRCV	63.32
LCLT	274.3
LCLP	591.4
LCLE	341.7
MLTH	318.7
MLMR	7.13
THCK	5880.
PWAT	22.84

Stability Analysis: Plot θ and θ_E as a function of z and interpret in terms of stability for this sounding



Convectively Stable for $\frac{d\theta_E}{dz} > 0$

Examples

Convectively Unstable for $\frac{d\theta_E}{dz} < 0$

Examples

Pat Arnot

Pseudoadiabatic Chart

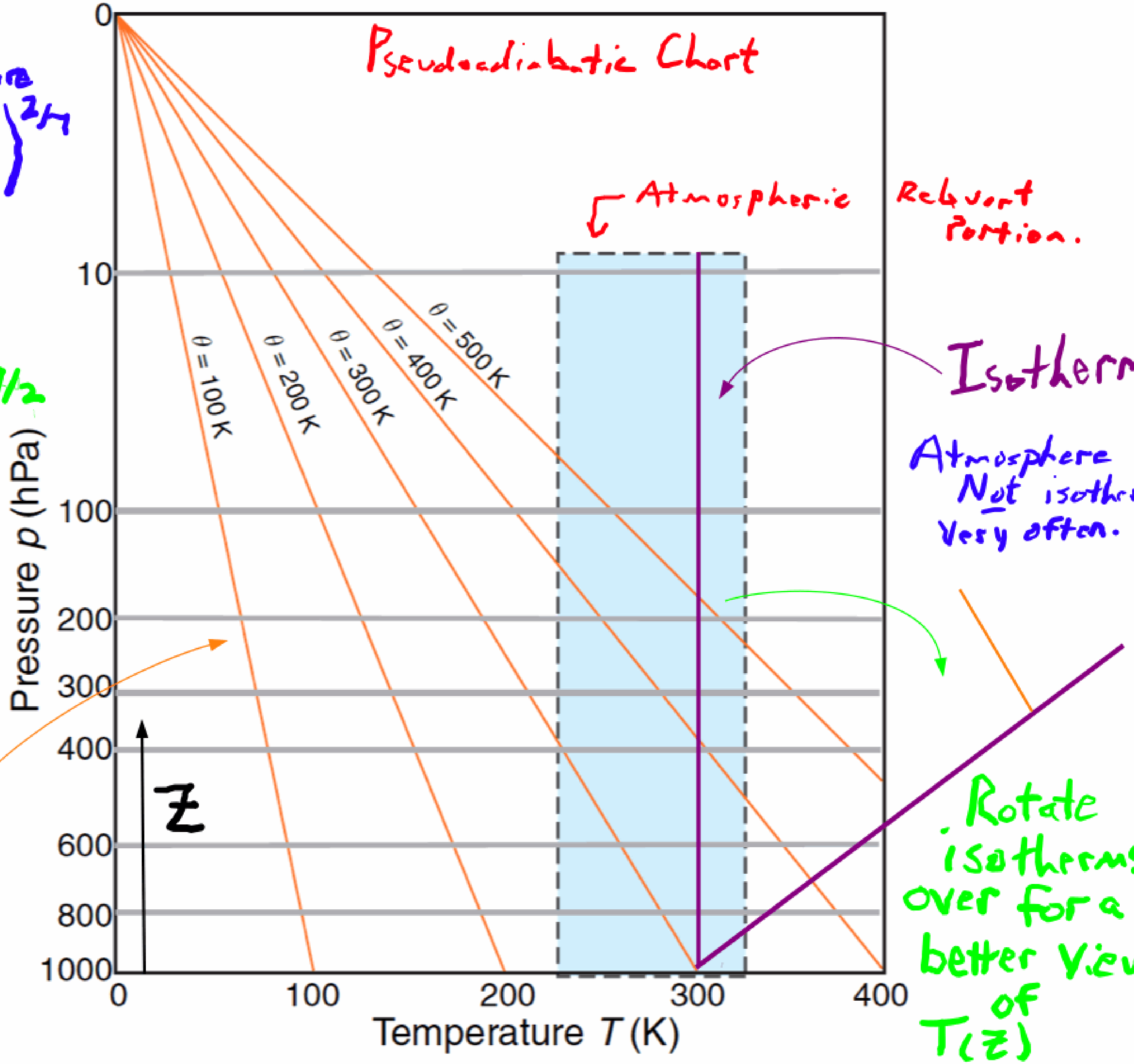
Θ = potential temperature

$$\theta = T(z)\left(\frac{P_0}{P(z)}\right)^{2/7}$$

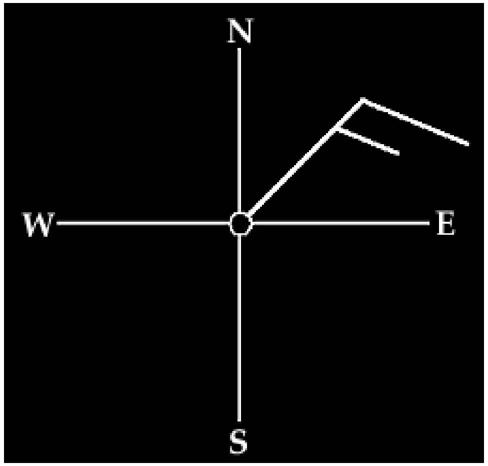
$P_0 = 1000$ hPA

Solve for P(z).

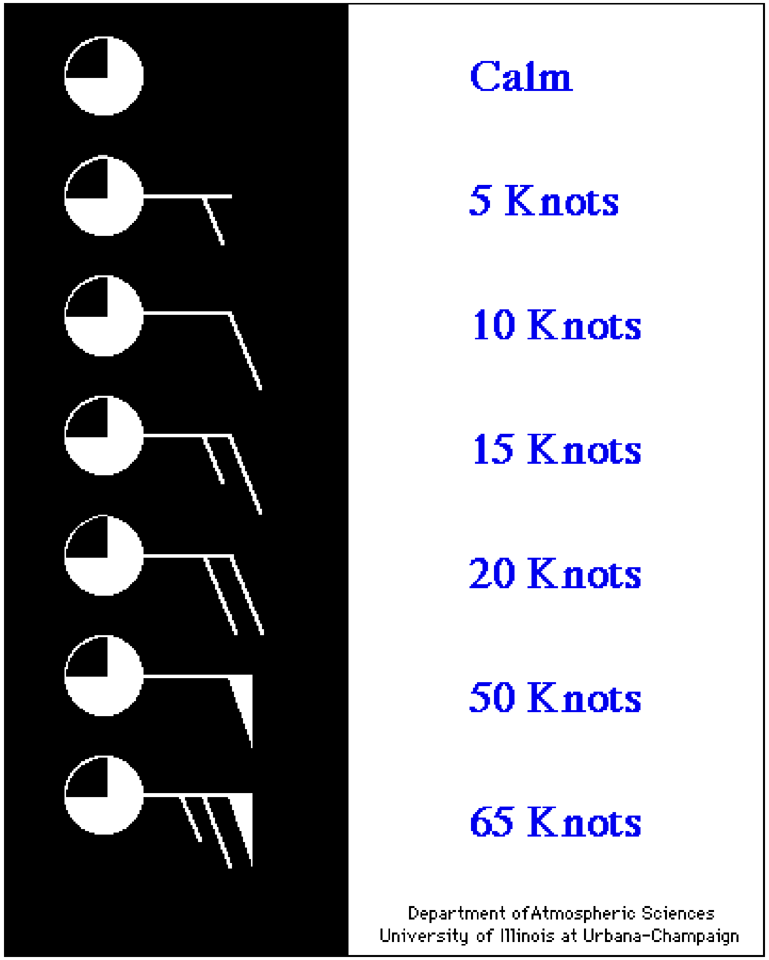
$$P(z) = P\left(\frac{T(z)}{\theta}\right)^{7/2}$$



Winds on Skew T Log P Charts



wind is 15 knots
coming ***<u>from</u>*** the northeast.

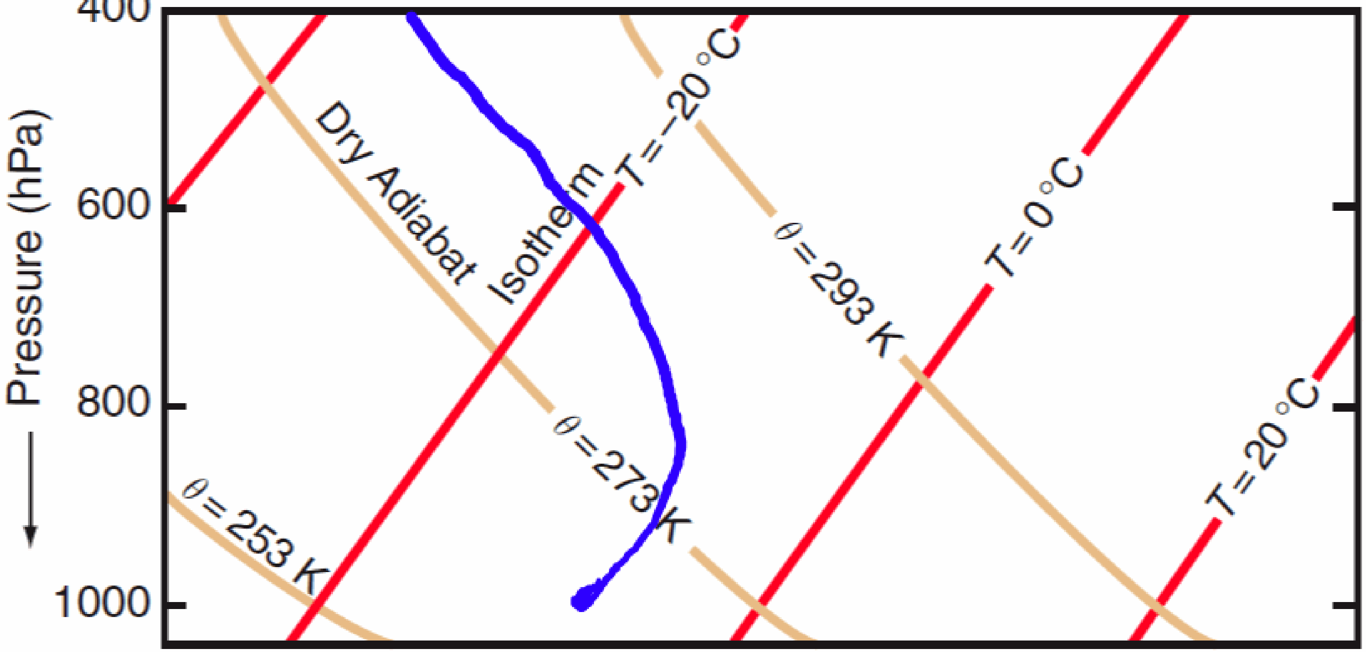


1 Knot = 1.15 Miles Per Hour (MPH)

1 Knot = 1.9 Kilometers Per Hour (KM/HR)

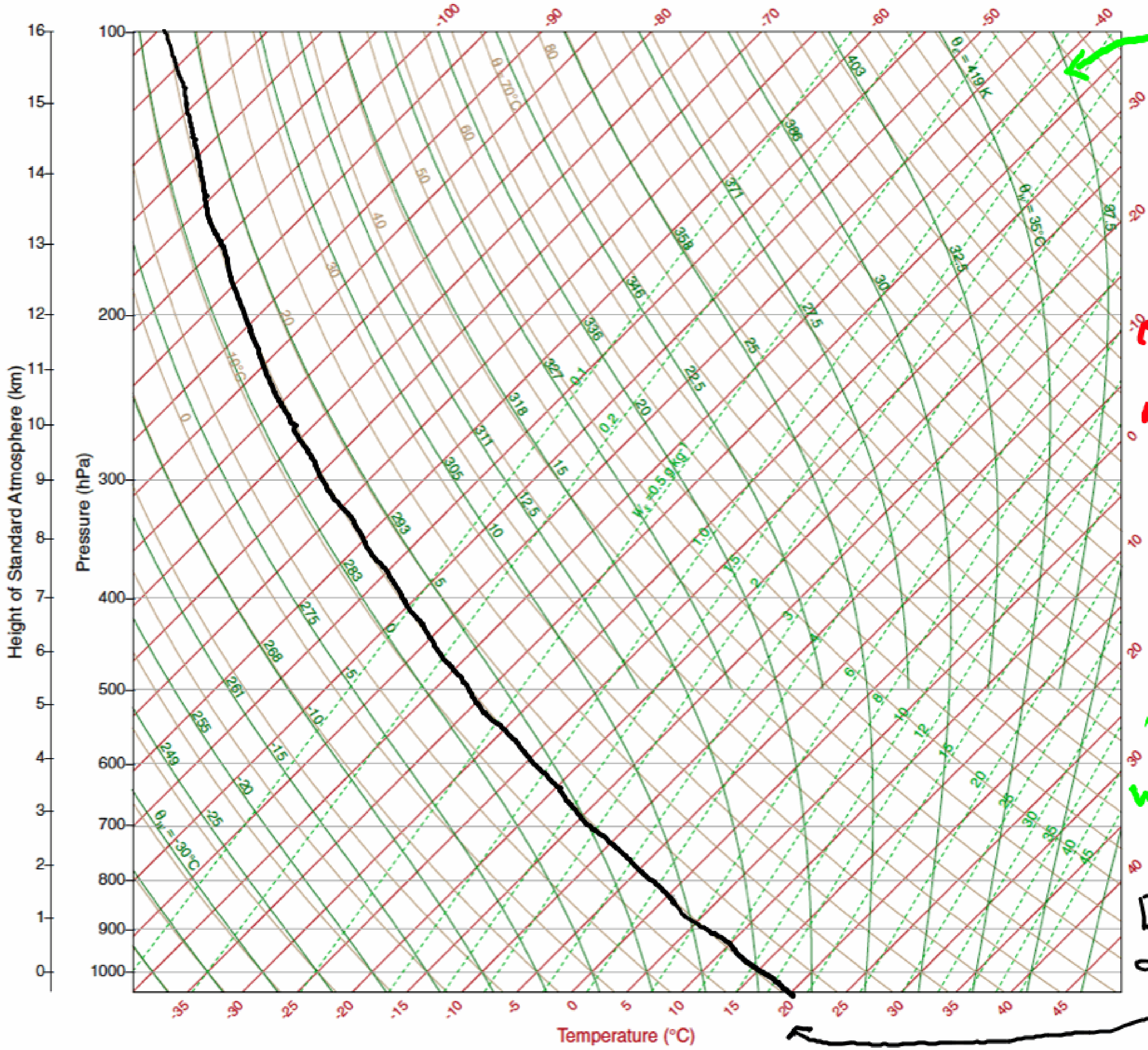
Skew T log P

How to read: compare sounding with nearby isotherms.



What is the 800 mb temperature for this sounding?

↑ Example sounding with a temperature inversion down low, and adiabatic conditions aloft.



Moist adiabat

isotherms

Constant mixing ratio

$w = \frac{g_{H_2O}}{kg\ air}$

Dry adiabat

Example: Use the SkewT log P Chart

1) Start in Reno, 860 mb, T = 15 C.

Follow the dry adiabat through this point. What are the temperatures at these pressures along the adiabat?

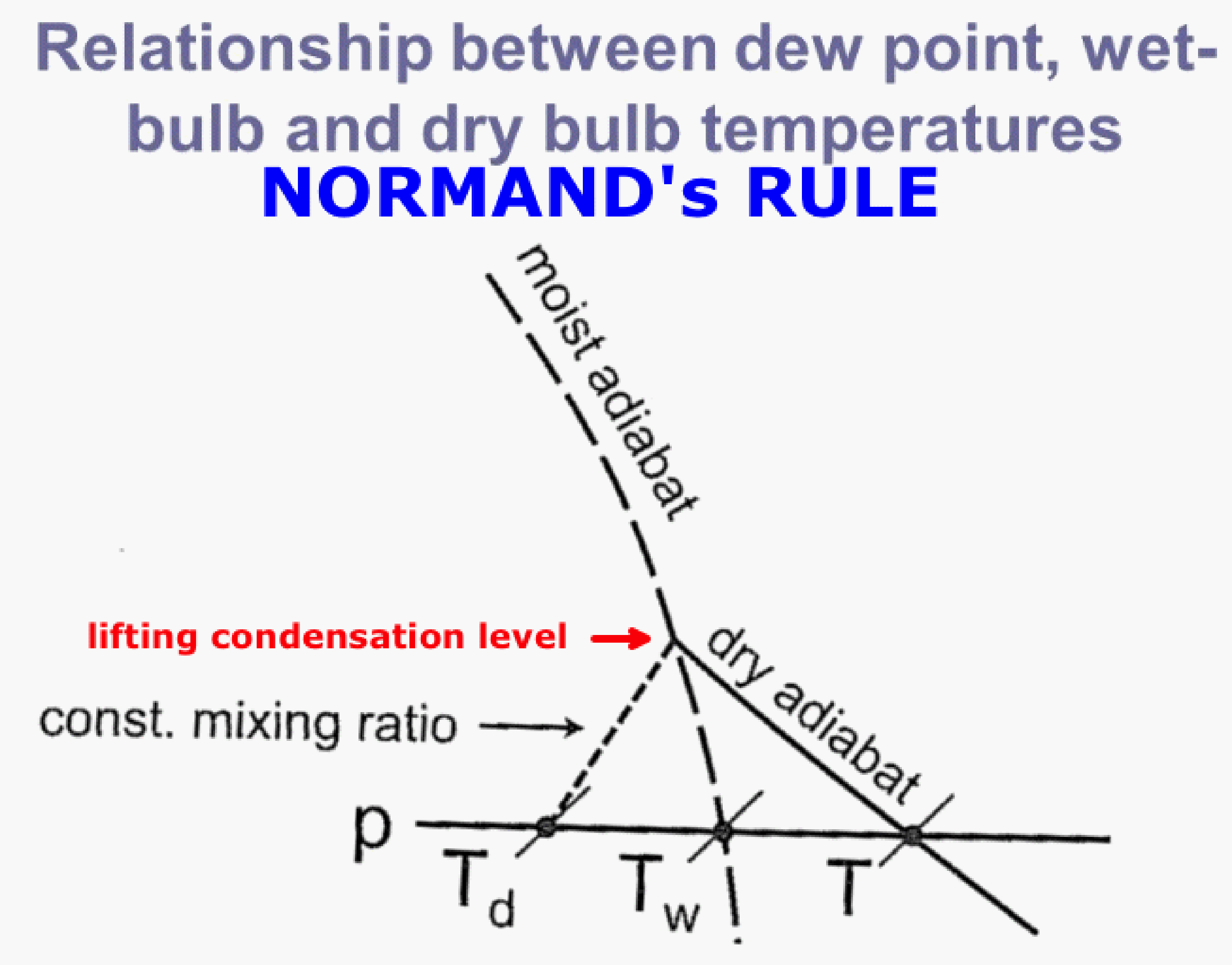
	P(mb)	T
P_o	1000	Θ =
	860	15C
	700	
	500	
	200	

Check using

$$T(z) = \left(\frac{P(z)}{P_o}\right)^{2/7} \Theta$$

↑ Equation of the dry adiabat

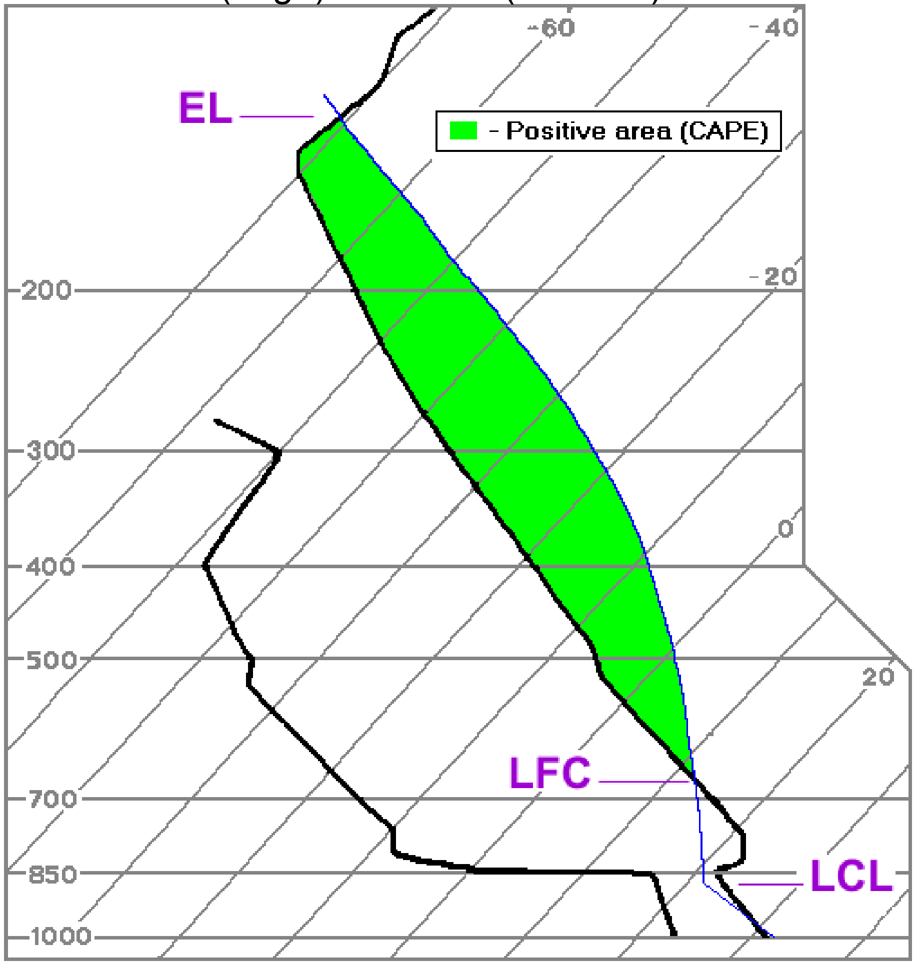
Normand’s Rule



Relationship between dew point, wet-bulb and dry bulb temperatures

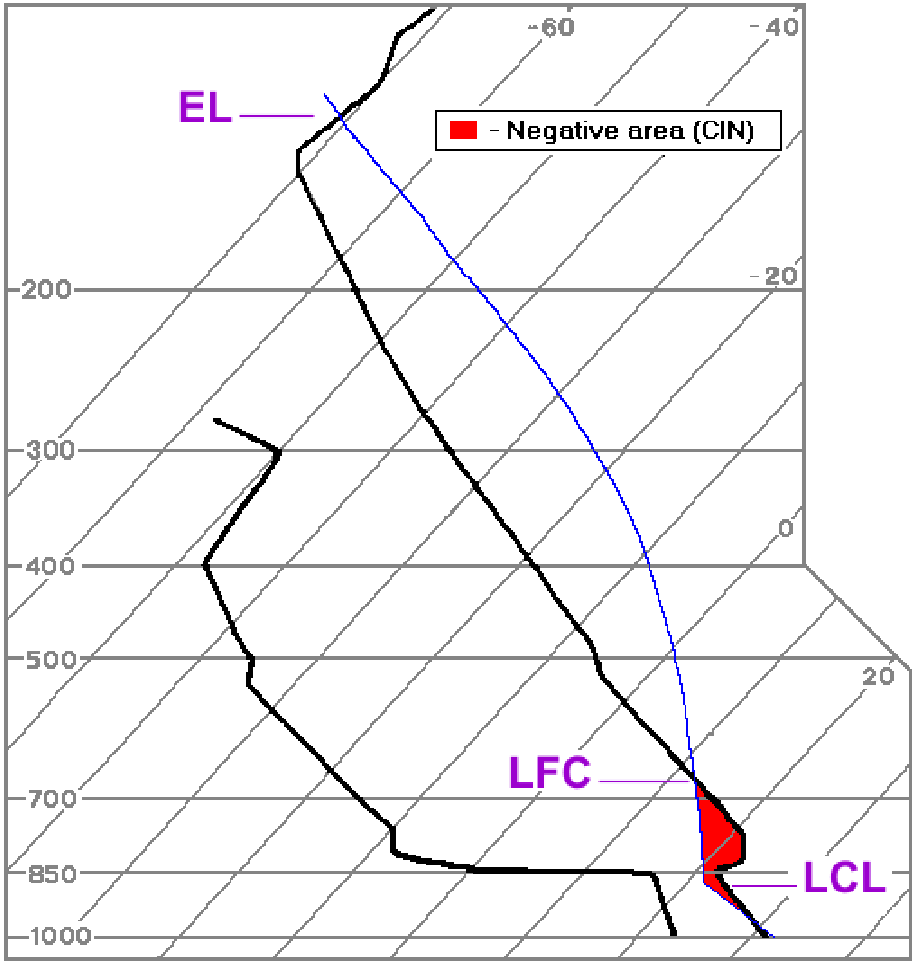
NORMAND's RULE

CAPE (J/kg): 0-1000 (small) 1000-2500 (moderate)
2500-4000 (large) > 4000 (extreme).



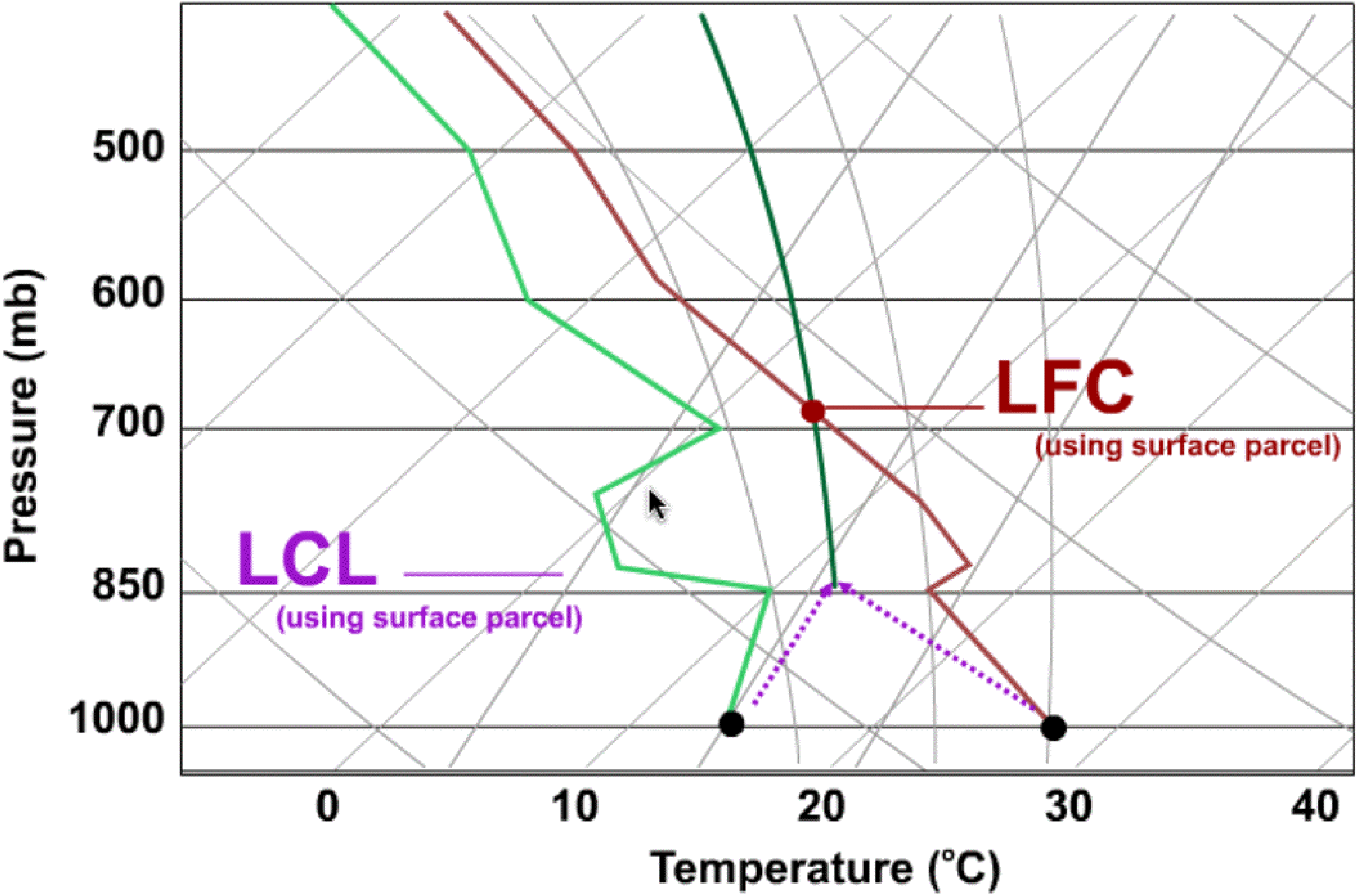
LCL: Lifting condensation level.
LFC: Level of free convection.
EL: Equilibrium level.
CAPE: Convective available potential energy.
CIN: Convective inhibition.

CIN (J/kg): 0 to -25 (weak) -25 to -50 (moderate)
- 50 to -100 (strong convective inhibition)



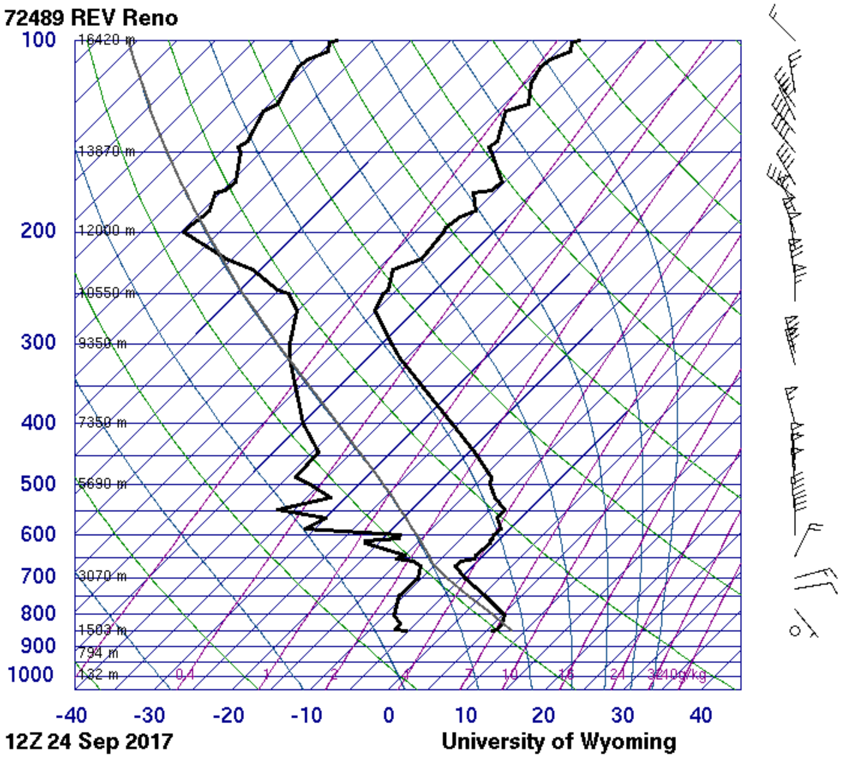
Solar heating, surface convergence promote parcels to the LFC: Must pass above the inversion in the CIN area.

Pat Arnott

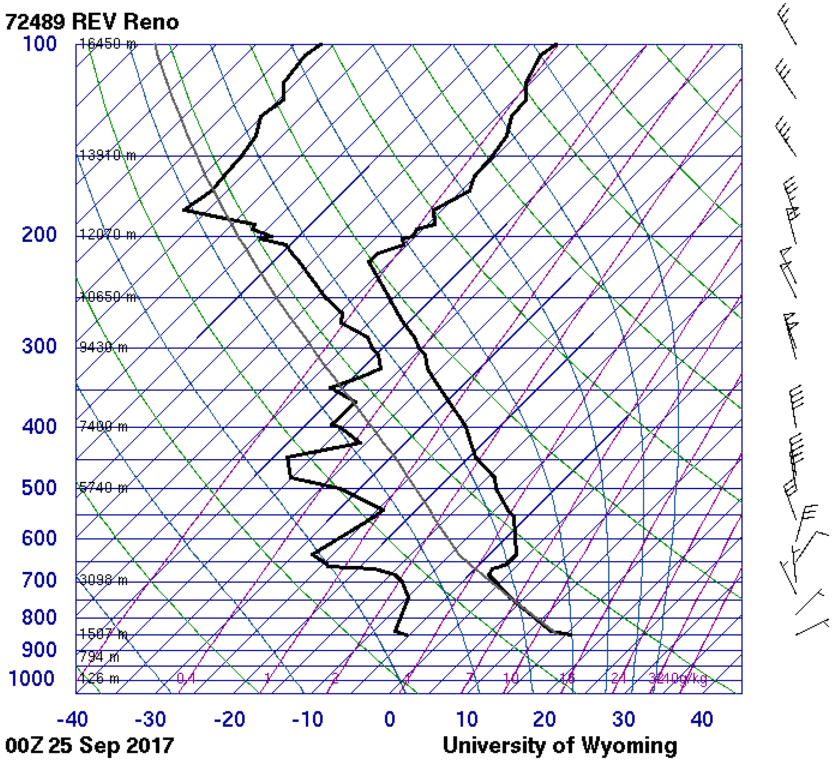

Pressure (mb)
500
600
700
850
1000
LFC
(using surface parcel)
LCL
(using surface parcel)
0
10
20
30
40
Temperature (°C)

Pat Arnott

Reno Examples



```
SLAT 39.56
SLON -119.80
SELV 1516.
SHOW -9999
LIFT 13.67
LFTV 13.59
SWET -9999
KINX -9999
CTOT -9999
VTOT -9999
TOTL -9999
CAPE 0.00
CAPV 0.00
CINS 0.00
CINV 0.00
EQLV -9999
EQTV -9999
LFCT -9999
LFCV -9999
BRCH 0.00
BRCV 0.00
LCLT 263.1
LCLP 670.1
MLTH 295.0
MLMR 2.68
THCK 5558.
PWAT 6.60
```



```
SLAT 39.56
SLON -119.80
SELV 1516.
SHOW -9999
LIFT 10.22
LFTV 10.11
SWET -9999
KINX -9999
CTOT -9999
VTOT -9999
TOTL -9999
CAPE 0.00
CAPV 0.00
CINS 0.00
CINV 0.00
EQLV -9999
EQTV -9999
LFCT -9999
LFCV -9999
BRCH 0.00
BRCV 0.00
LCLT 262.0
LCLP 615.5
MLTH 300.9
MLMR 2.68
THCK 5614.
PWAT 6.38
```

MORNING
5 a.m. Local Daylight Time
24 Sept 2017

AFTERNOON
5 p.m. Local Daylight Time
24 Sept 2017

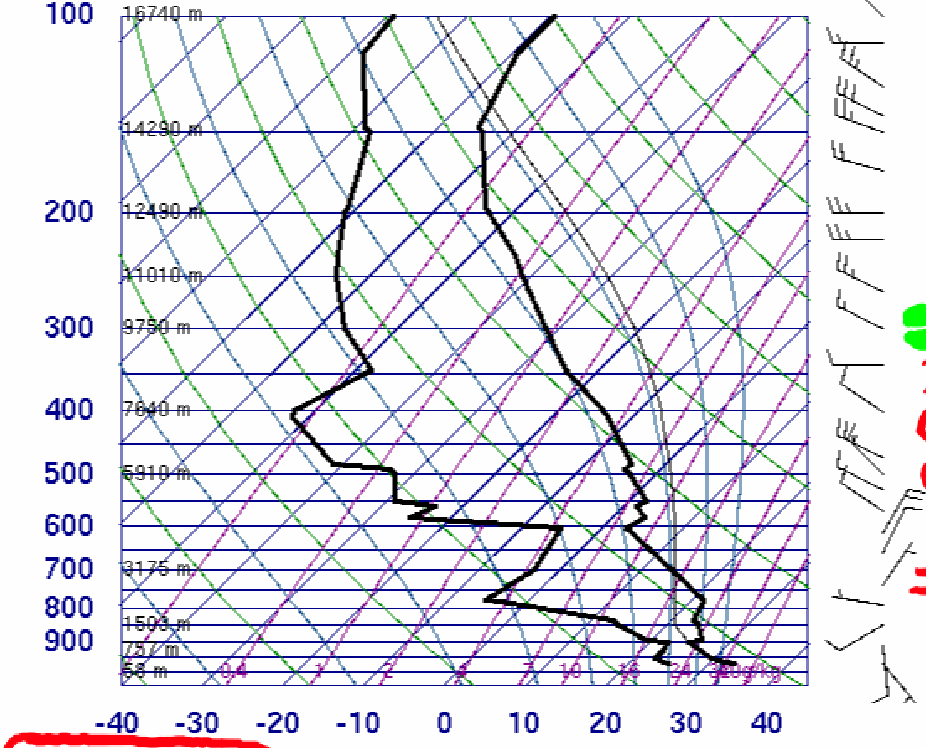
Goto
LMN
Lamont
Oklahoma.

DoE operates a
climate facility there.

LMN
Soundings
Much
more
frequent.

Summer 2010 Example

74646 LMN Lamont Oklahoma

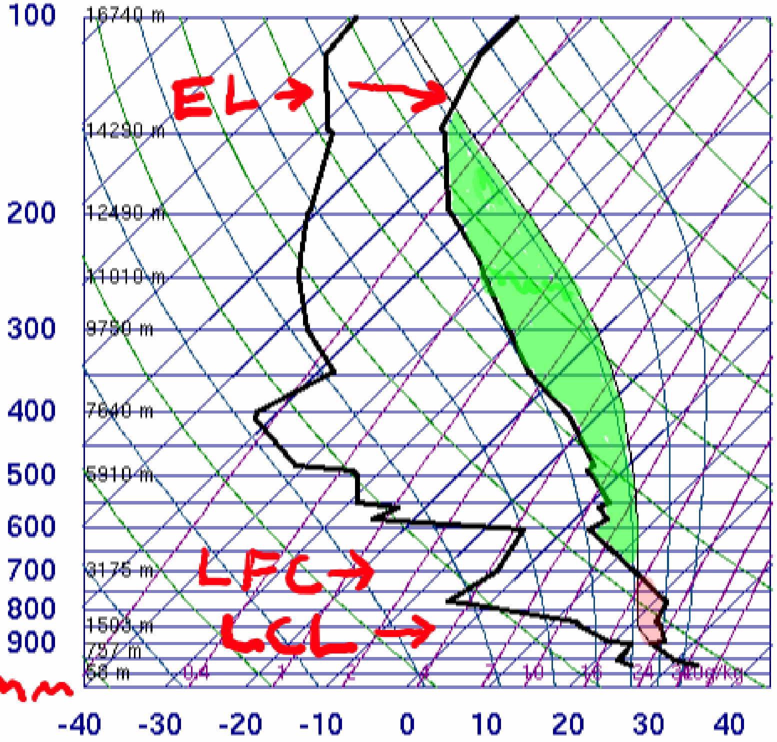


18Z 13 Jul 2010

University of Wyoming

SLAT	36.62
SLON	-97.48
SELV	317.0
SHOW	0.39
LIFT	-5.26
LFTV	-6.65
SWET	201.9
KINX	24.30
CTOT	17.30
VTOT	27.30
TOTL	44.60
CAPE	3340.
CAPV	3661.
CINS	-158.
CINV	-59.3
EQLV	136.4
EQTV	136.4
LFCT	710.1
LFCV	752.1
BRCH	192.7
BRCV	211.2
LCLT	294.7
LCLP	864.7
MLTH	307.2
MLMR	19.27
THCK	5852.
PWAT	36.07 mm

Same Sounding: Color CAPE + CIN



CAPE = Convective available Potential Energy

CIN = Convective inhibition.

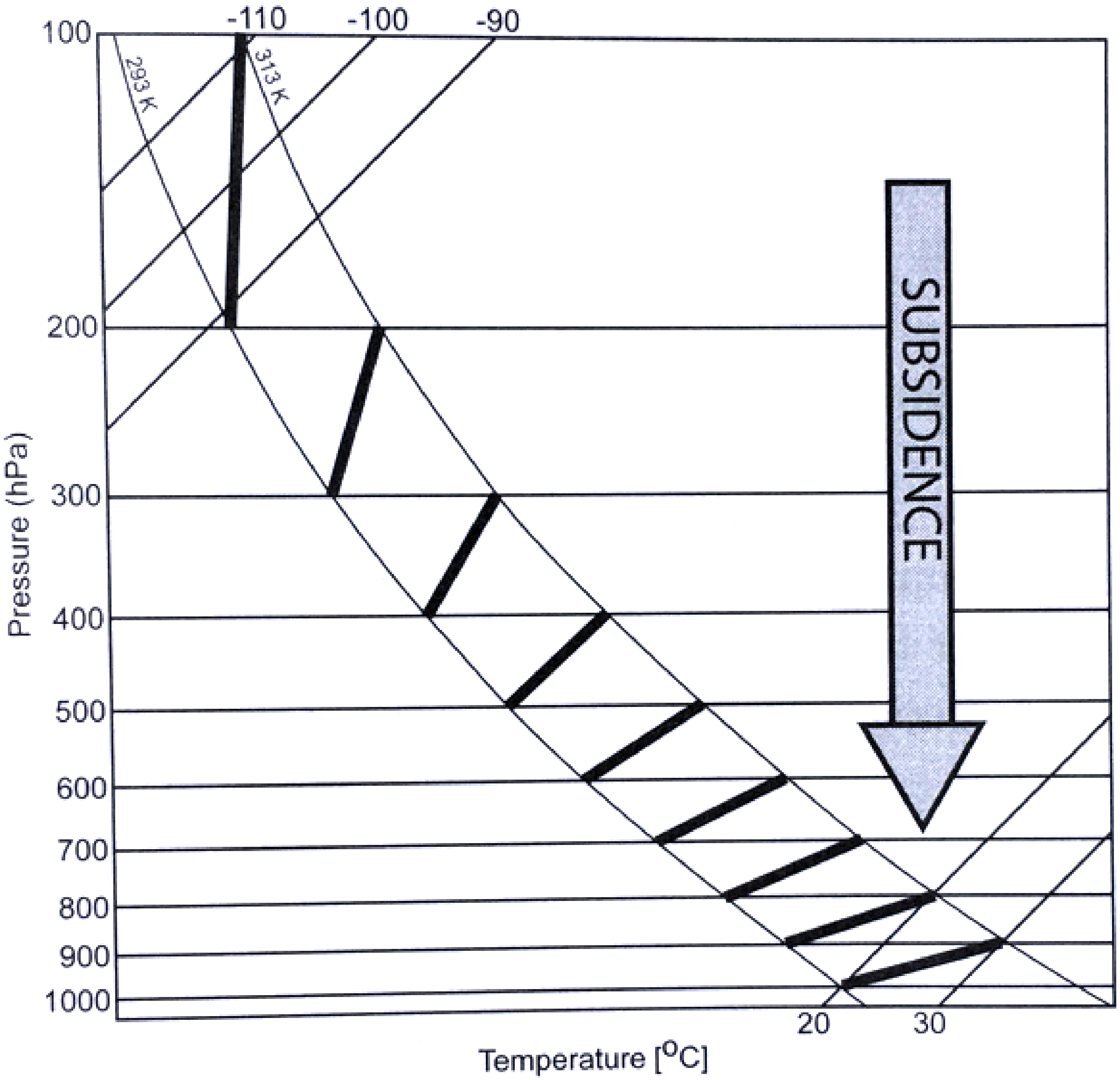
LCL = lifting condensation level

LFC = Level of free convection

EL = Equilibrium Level

Sounding indices given at
http://weather.uwyo.edu/upperair/indices.html

Subsidence Inversion: High pressure and a 100 mb thick adiabatically descending layer

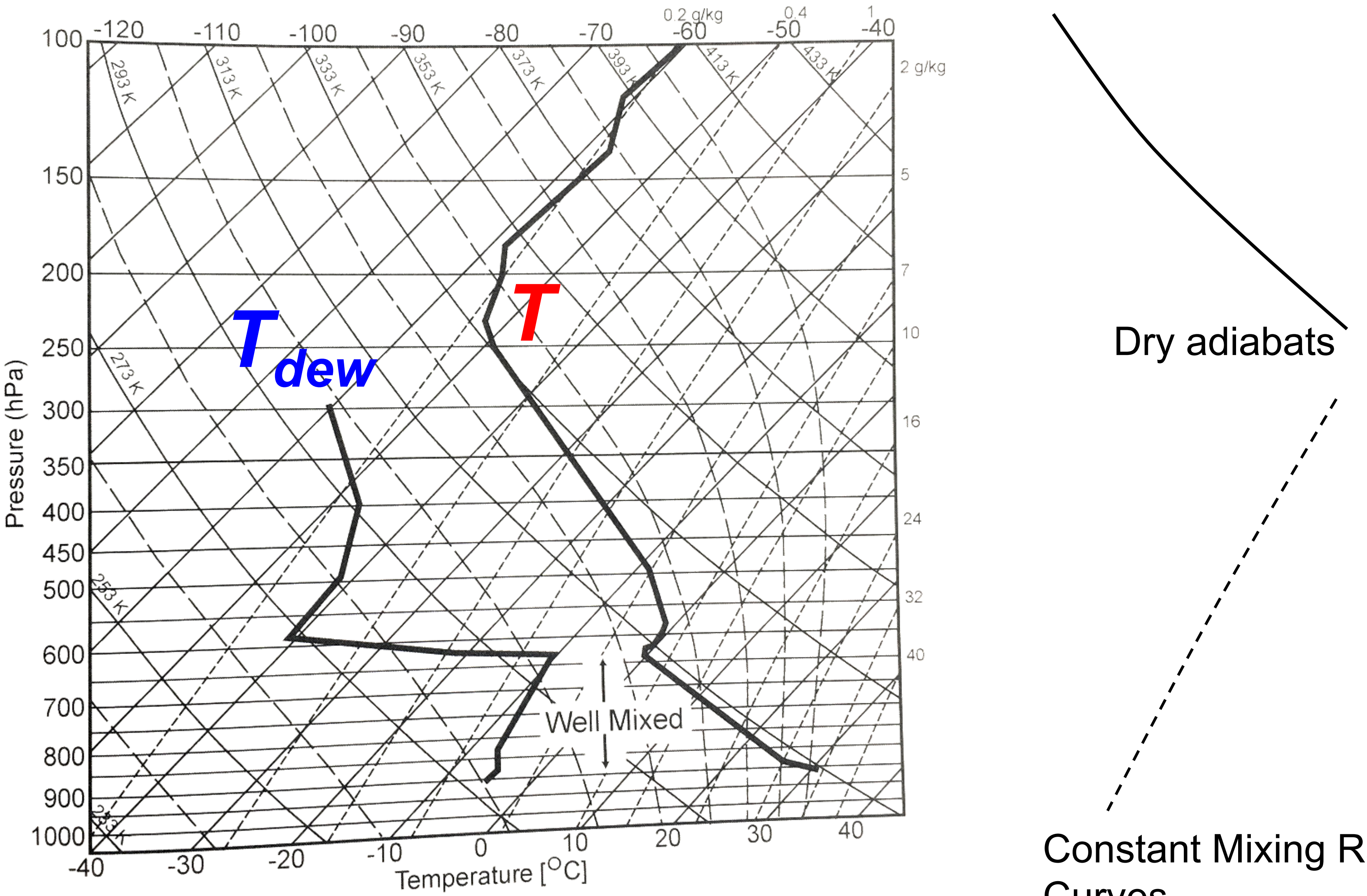


$$\Gamma = -\frac{dT}{dz}$$

$$\Gamma < 0$$

From Petty

What a well-mixed boundary layer looks like



Dry adiabats

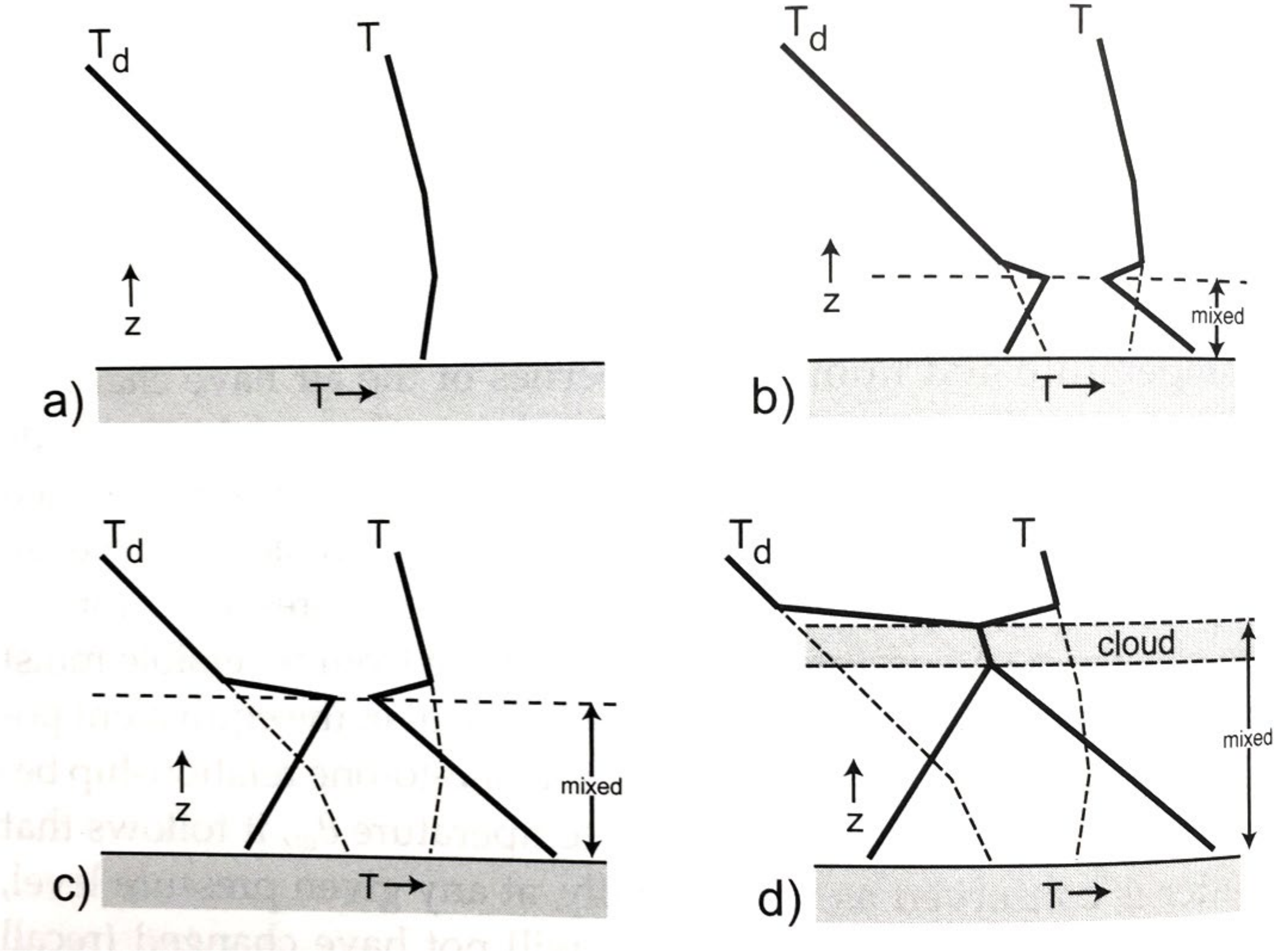
Constant Mixing Ratio Curves

Air temperature follows the dry adiabat.

Dewpoint temperature follows the constant mixing ratio curve.

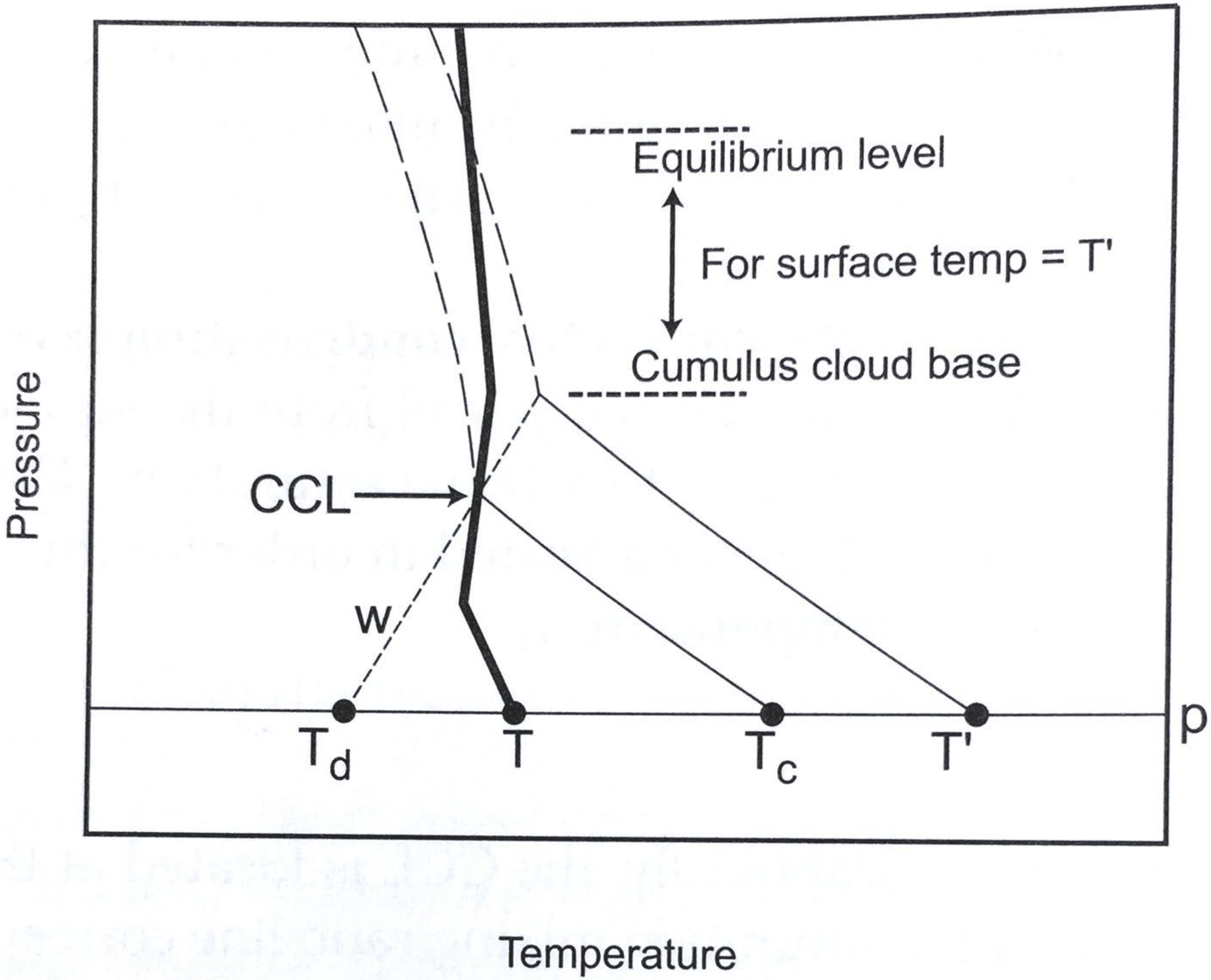
From Petty

Cloud formation as the mixed level increases



From Petty

Convective Condensation Level T_c



Heat the surface air from T to T' during the day and get a cumulus cloud. From Petty.

Hot Air Balloons – Just like air parcel analysis

ρ, T

$p = \rho R T = \rho_b R T_b$ Pressure same inside balloon and outside

So $\boxed{\rho_b = \frac{T}{T_b}\rho}$

Nonadiabatic process

Downward Force Boyant force Upward

$$\underbrace{(m_p + m_b)}_{\text{Payload and balloonshell}} g = (\rho - \underbrace{\rho_b}_{\text{air inside balloon}}) V_b g = \rho\left(\frac{T_b - T}{T_b}\right) V_b g$$

Solving,

$$\boxed{\underbrace{\frac{p}{RT}\left(\frac{T_b - T}{T_b}\right) V_b}_{\text{Lift}} = \underbrace{m_p + m_b}_{\text{Load}}}$$

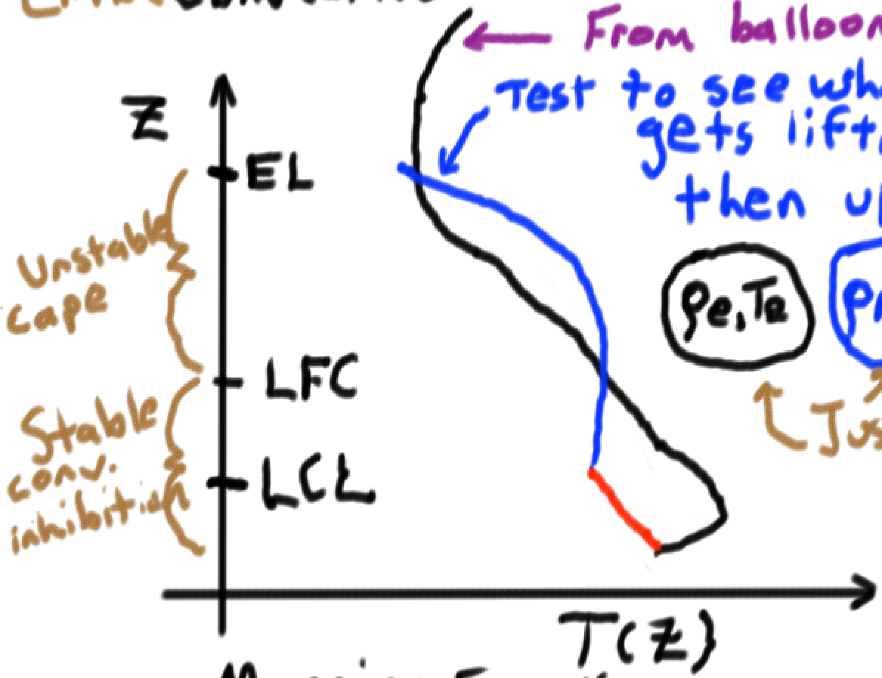
Limits: $T_b \to \infty$, $\rho_b \to 0$ $\frac{p}{RT} V_b = m_p + m_b$ is the max payload.

Generally as p goes down, T_b has to go up.

Temperature inversion effect?

Pat Arnott

CAPE = Convective Available Potential Energy, and peak Kinetic Energy



← From balloon sounding measurements.

Test to see what happens if surface air parcel gets lifted, first up dry adiabat, and then up moist adiabat.

$\boxed{\rho_e, T_e}$ $\boxed{\rho_p, T_p}$ Subscript p = parcel
Subscript e = environment

↑ Just like the hot air balloon problem

Morning Sounding
LCL = lifting condensation level
LFC = level of free convection
EL = equilibrium level
Remember $\rho = P/RT$

Math $W = \frac{dz}{dt}$ = Vertical velocity.

$W(z,t)$ at some z.

$\frac{DW}{Dt} = \frac{\partial w}{\partial t} + w\frac{dw}{dz}$ = Vertical acceleration

Steady state, $\frac{\partial w}{\partial t} = 0$

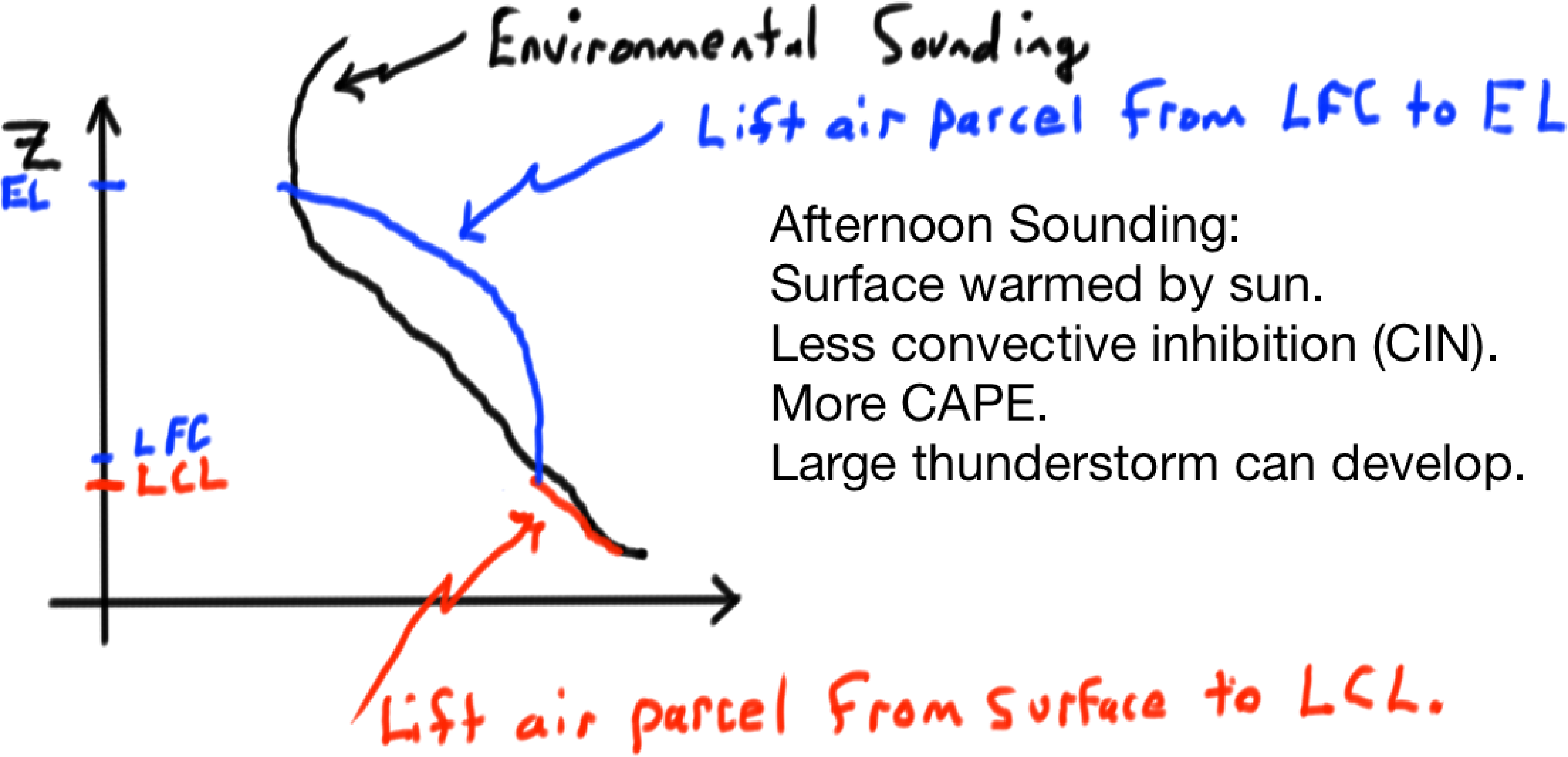
Newton's 2nd Law. (neglect "hydrostatic" part)

Bouyant Force → $\rho_p \frac{DW}{Dt} = (\rho_e - \rho_p) g$

$$\int_{LFC}^{EL} w\,dw = \boxed{\int_{LFC}^{EL} \left(\frac{T_p - T_e}{T_e}\right) g\,dz \equiv CAPE}$$

So $\boxed{W_{EL} = \sqrt{2CAPE}}$

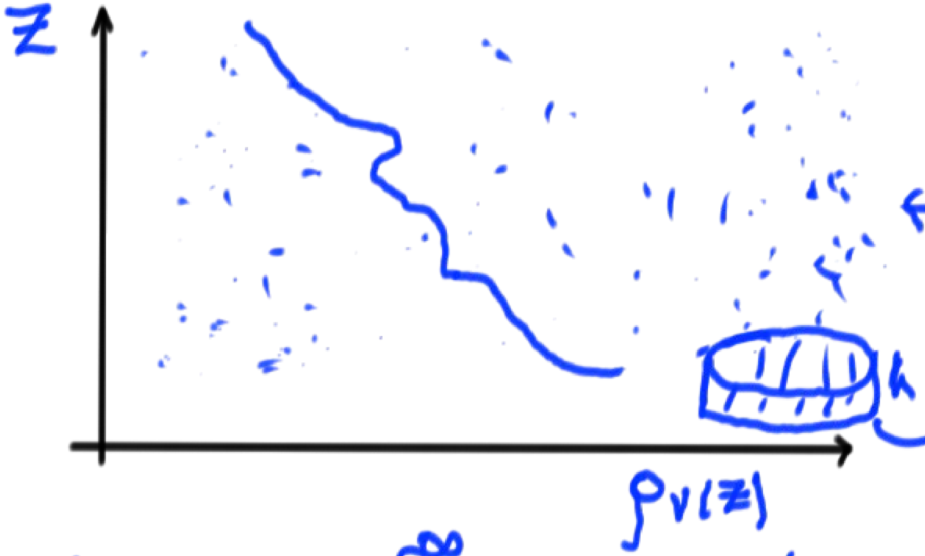
Pat Arnott



Afternoon Sounding:
Surface warmed by sun.
Less convective inhibition (CIN).
More CAPE.
Large thunderstorm can develop.

LCL=Lifting Condensation Level.
LFC=Level of Free Convection.
EL=Equilibrium Level.

Precipitable Water Vapor



$\rho_v(z) = \frac{e}{R_v T}$ = water vapor density

← Condense water vapor for the whole column into liquid water of height h.

Liquid water of density ρ_L

$\rho_L = 1000\ \mathrm{kg/m^3} = 1\ \mathrm{g/cc}$

As an integral, $\int_0^\infty \rho_v(z)\,dz = \rho_L h$

h = Precipitable water vapor amount if it were condensed into liquid water.

As a sum, $\boxed{h \approx \sum_{i=1}^{N} \frac{\rho_v^i \Delta z_i}{\rho_L}}$.

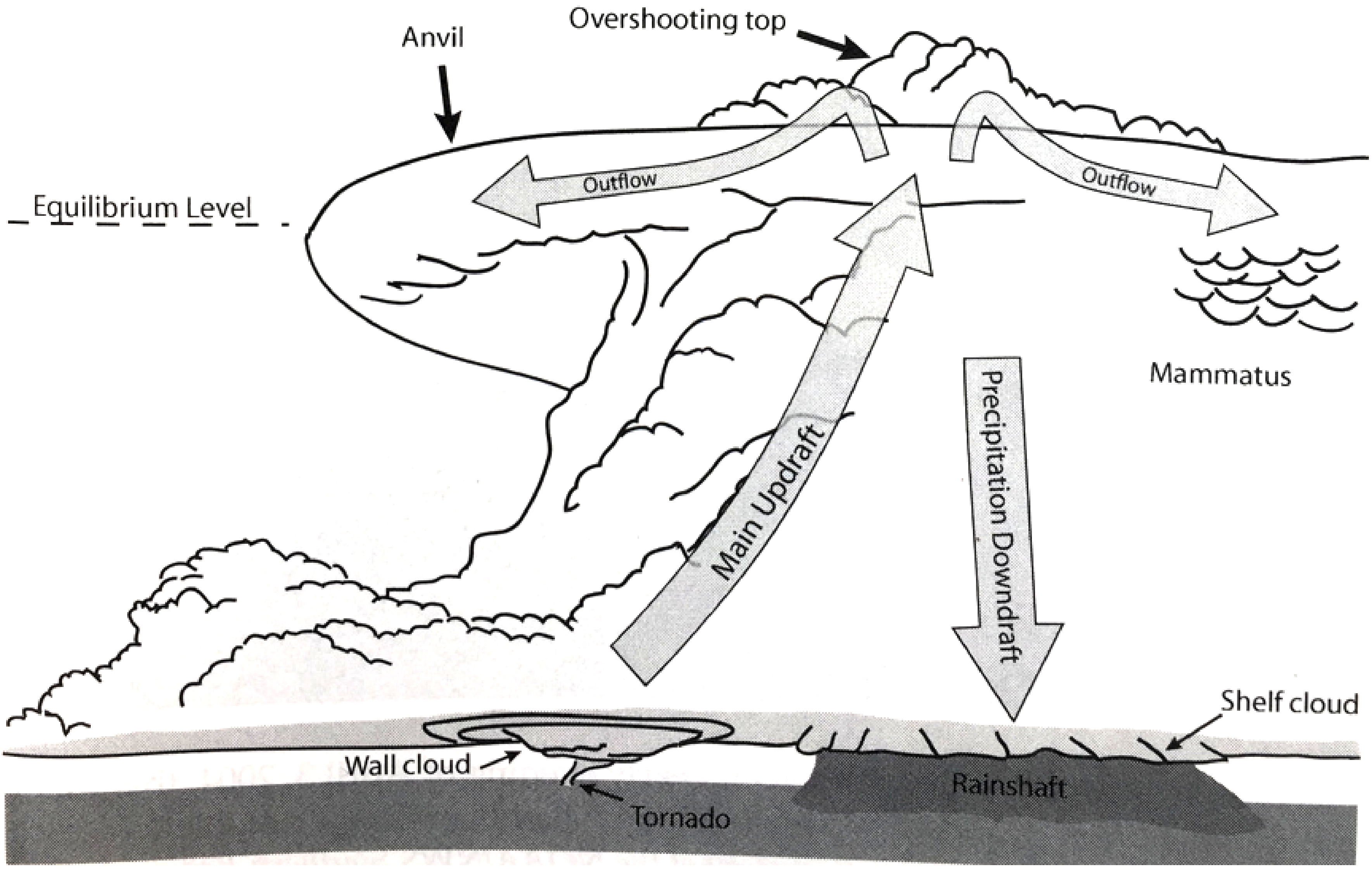
$e_s(T) \approx 611\ \mathrm{Pa}\ \exp\left[\frac{L_v}{R_v}\left(\frac{1}{273} - \frac{1}{T}\right)\right]$

$\rho_v = \frac{e}{R_v T}$. $e = \frac{RH}{100} e_s(T)$. RH = % Relative humidity

$R_v = 461.9 \frac{J}{kg°K}$ = Water vapor gas constant.

L_v = latent heat ≈ $2.5\times10^6 \frac{J}{kg}$

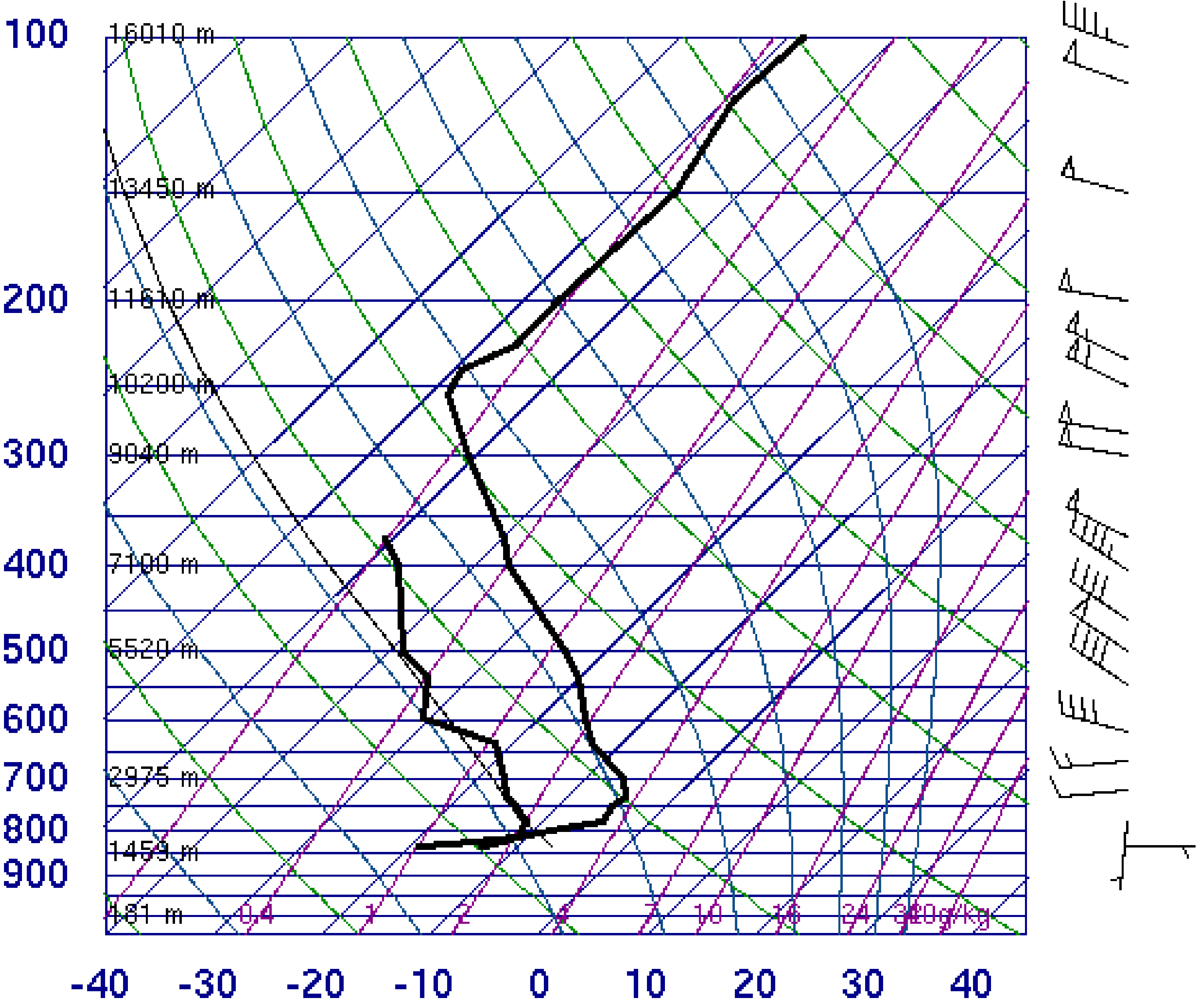
Tornadic Supercell Schematic



From Petty

Dynamic Nature of the Atmosphere

72469 DNR Denver



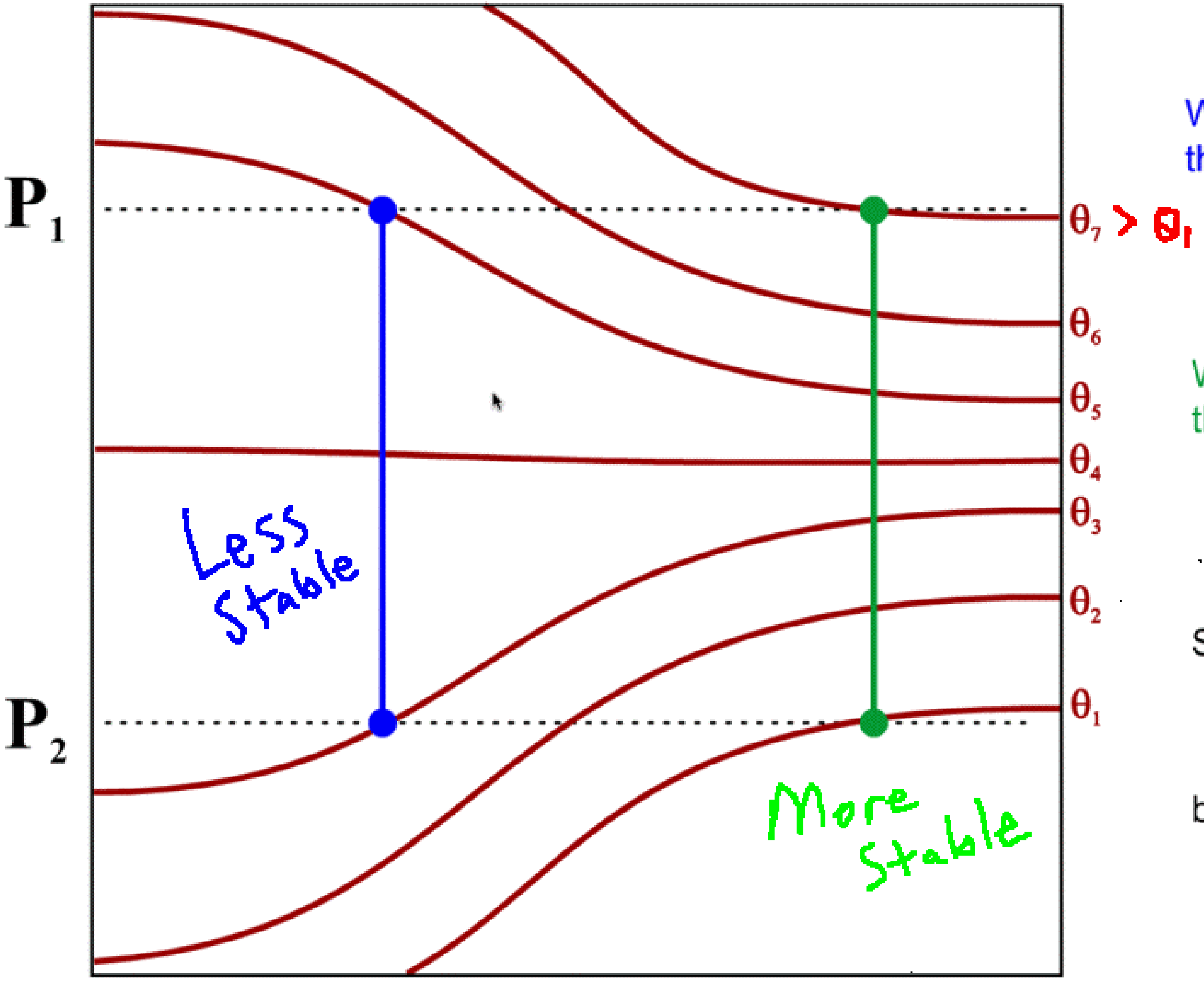
SLAT	39.75
SLON	-104.87
SELV	1625.
SHOW	-9999
LIFT	15.71
LFTV	15.72
SWET	-9999
KINX	-9999
CTOT	-9999
VTOT	-9999
TOTL	-9999
CAPE	0.00
CAPV	0.00
CINS	0.00
CINV	0.00
EQLV	-9999
EQTV	-9999
LFCT	-9999
LFCV	-9999
BRCH	0.00
BRCV	0.00
LCLT	260.0
LCLP	767.8
MLTH	280.4
MLMR	1.86
THCK	5339.
PWAT	4.13

Pat

12Z 24 Mar 1974

University of Wyoming

$$s = c_p \ln(\theta) + \text{constant}$$



When isentropes are loosely packed, the absolute value of $\Delta\theta/\Delta p$ is small.

$$\left|\frac{\Delta\theta}{\Delta p}\right| = \left|\frac{\theta_3-\theta_5}{p_2-p_1}\right|$$

When isentropes are tightly packed, the absolute value of $\Delta\theta/\Delta p$ is large.

$$\left|\frac{\Delta\theta}{\Delta p}\right| = \left|\frac{\theta_1-\theta_7}{p_2-p_1}\right|$$

Since the pressure difference is fixed,

$$\left|\frac{\theta_3-\theta_5}{p_2-p_1}\right| < \left|\frac{\theta_1-\theta_7}{p_2-p_1}\right|$$

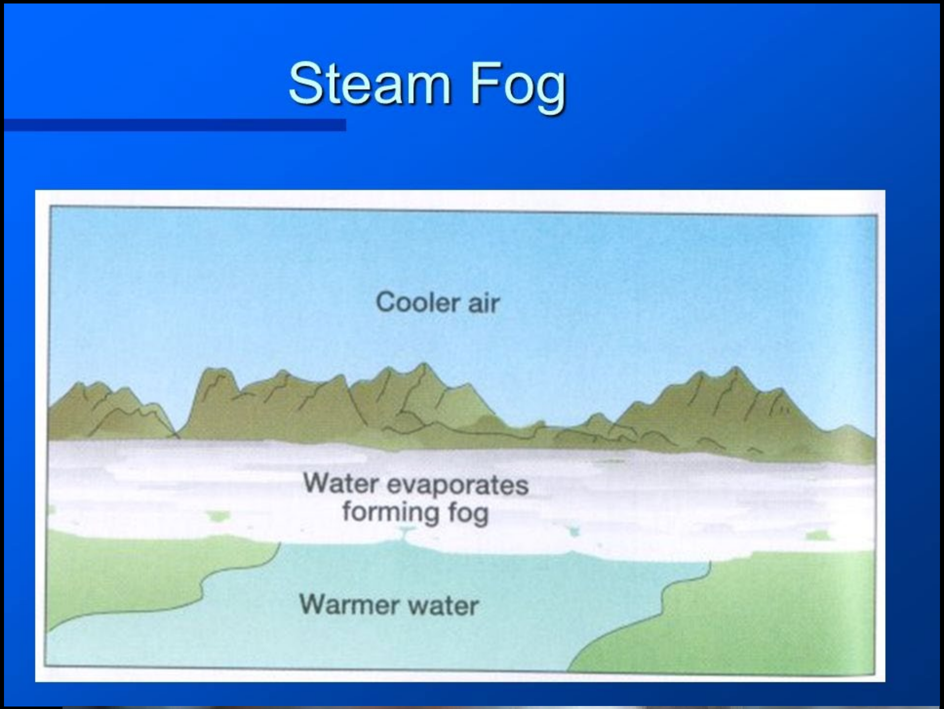
because...

$$|\theta_3-\theta_5| < |\theta_1-\theta_7|$$

Curves of constant potential temperature are isentropes, curves of constant entropy, s.

Pat Arnott

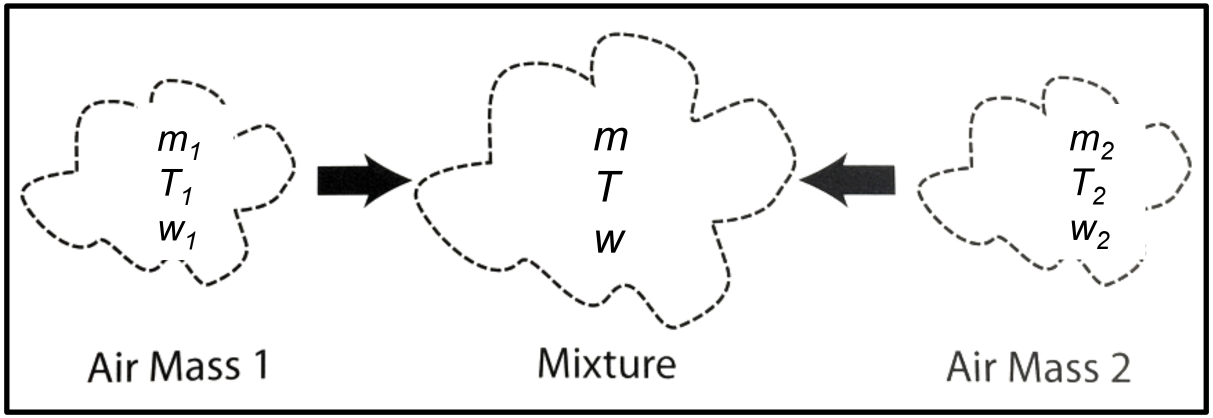
Observation Evidence of Mixing Air Masses



Steam Fog

Jet Contrail

Mixing Air Masses: Theory for isobaric mixing



m=mass
T=temperature
w=water vapor mixing ratio
e=vapor pressure

$$w \approx \frac{\epsilon\, e}{p}$$

$$m = m_1 + m_2$$ Mass of mixture

$$w = \frac{m_1 w_1 + m_2 w_2}{m_1 + m_2}$$ Water vapor mixing ratio of mixture

$$T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2}$$ From enthalpy of mixture

2 equations, two unknowns, get rid of m_1 and m_2

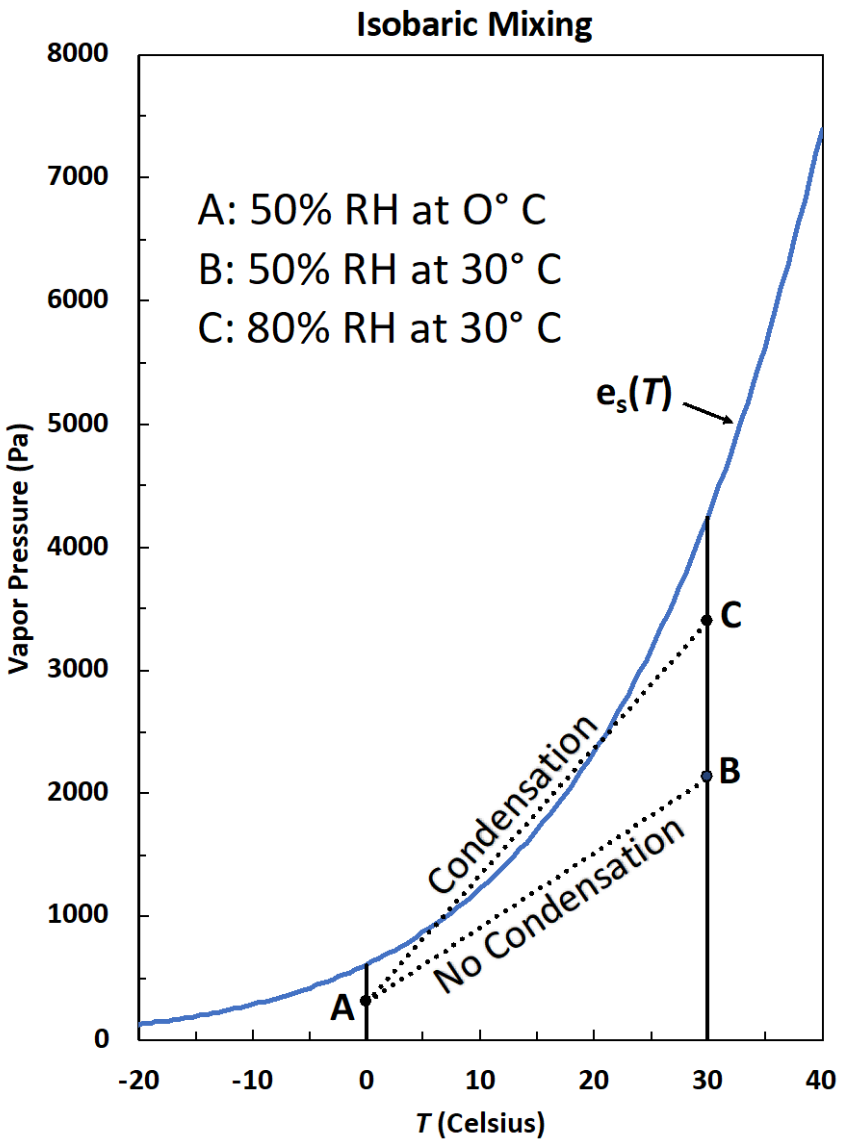
$$\boxed{e(T) = \frac{T - T_1}{T_2 - T_1}(e_2 - e_1) + e_1}$$

Compare $e(T)$ with $e_{sat}(T)$ to see if condensation happens.

Pat Arnott

From Petty

Example of Mixing Air Masses



$$e_s(T) = 611.2 \exp\left(\frac{17.67\ T_c}{T_c + 243.5}\right) \text{ Pa}$$

T_1=0 C

T_2=30 C

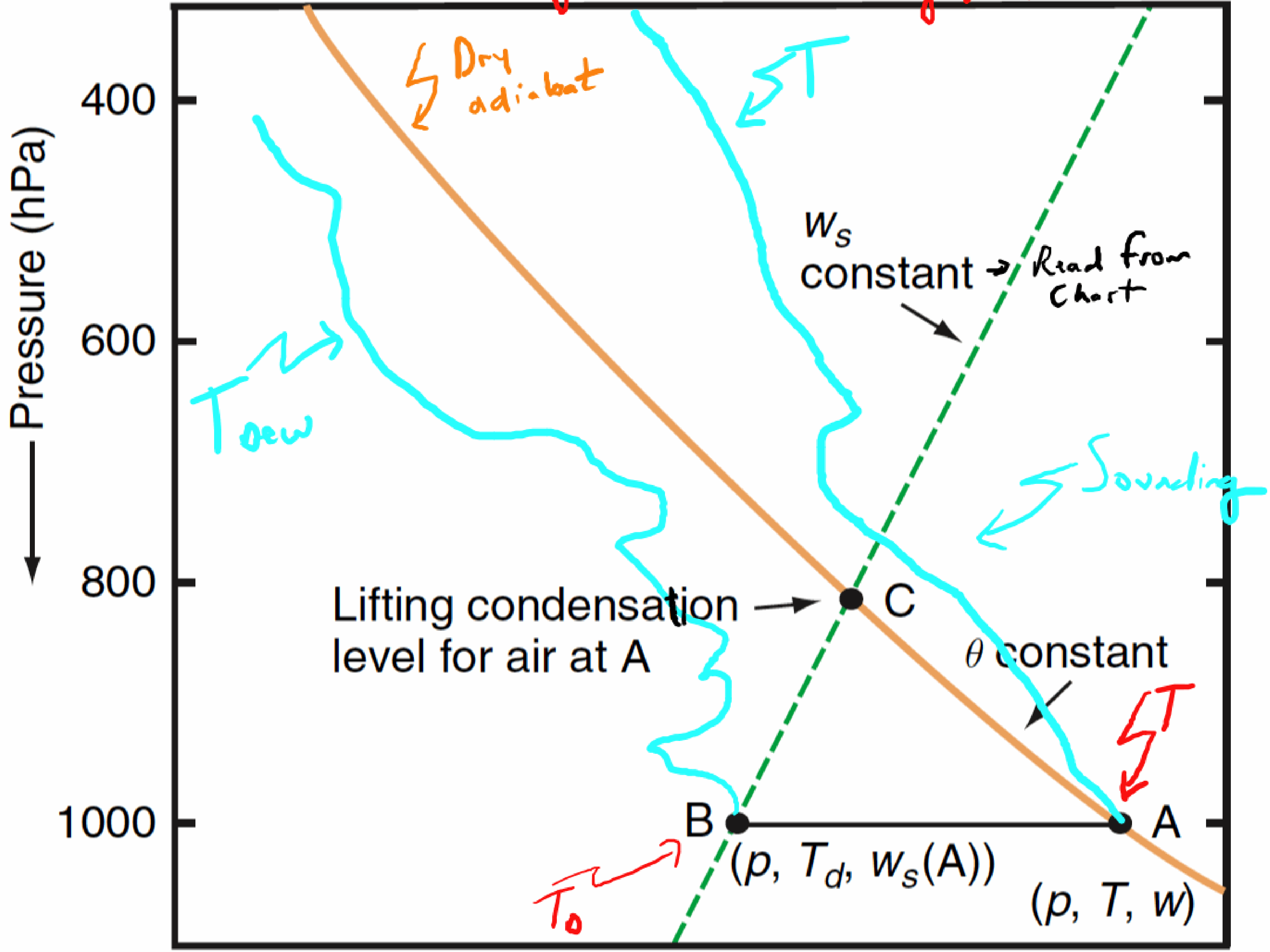
$$e(T) = \frac{T - T_1}{T_2 - T_1}(e_2 - e_1) + e_1$$

Air masses A and C are not saturated.

Certain combinations of them are super saturated.

from Petty

Small section of a skew T log p Plot



Example using skew T log P

Parcel @ 850 mb has $T = 13$ C, $T_{Dew} = 7$ C.

Find:

① Mixing ratio $w(T)$

② RH

③ LCL = lifting condensation level.

④ θ = Potential temperature.

Example using skew T log P

Parcel @ 850 mb has $T = 13$ C, $T_{Dew} = 7$ C.

Find:
① Mixing ratio $W(T)$
② RH
③ LCL = lifting condensation level.
④ Θ = Potential temperature.

Solution: ① Goto 850 mb, $T = 7$ C, read $W_s(T_D) = 8\,\frac{g}{kg} = W(T)$

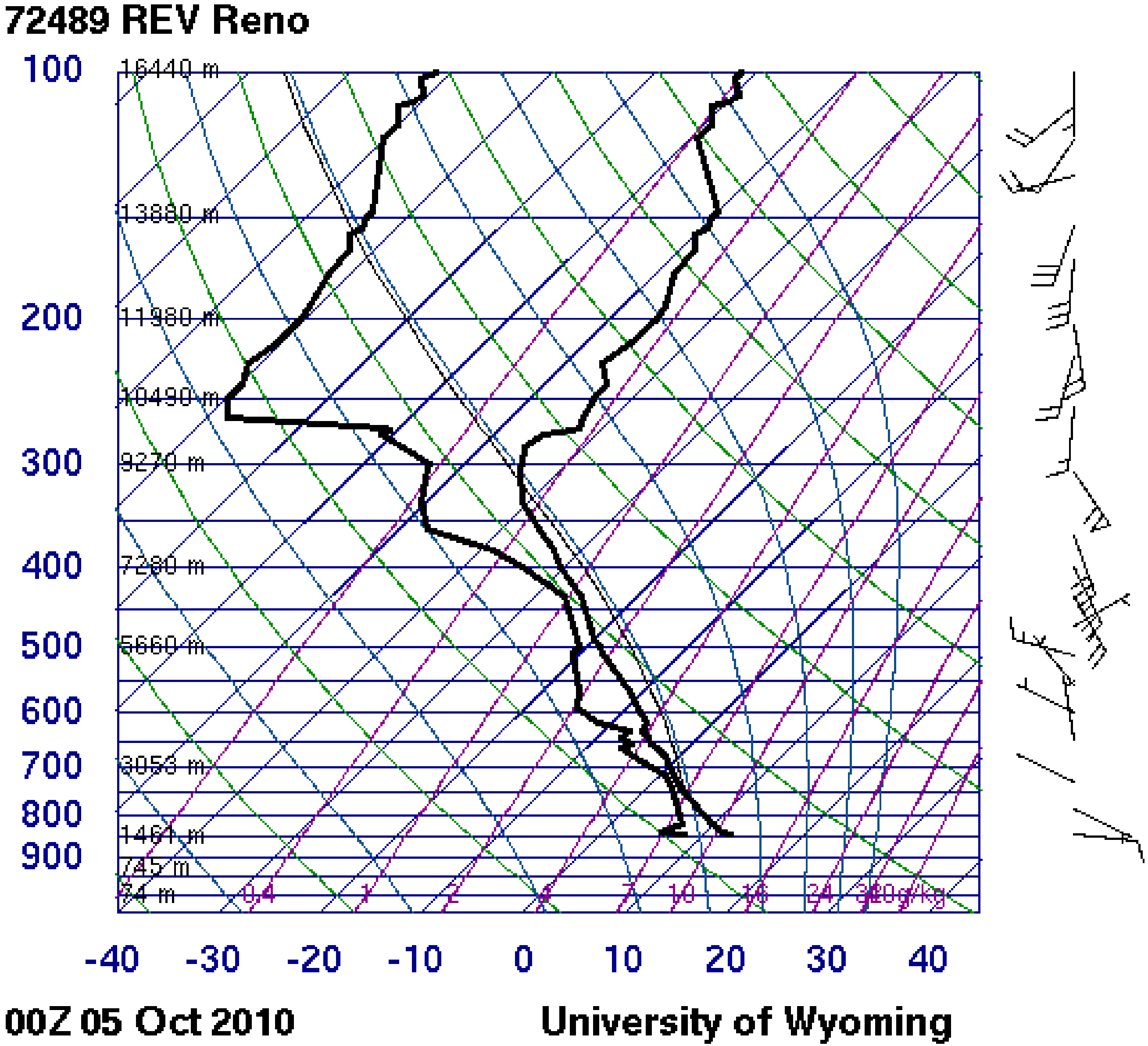
② Read $W_s(T) \approx 11.3$ g/kg. Then $RH = 100\,\frac{W_s(T_D)}{W_s(T)} = 100 \cdot \frac{8\,g/kg}{11.3\,g/kg}$

$RH = 71\%$

③ 8 g/kg line and 300 K = Θ dry adiabat meet at ≈ 780 mb = LCL.

④ Just look at nearby potential temperature lines.

Moist Adiabats: Trajectories of Saturated Parcels. Pseudoadiabats because products of condensation may fall out of the parcel as precipitation. Trace a few of them below.



SLAT	39.56
SLON	-119.80
SELV	1516.
SHOW	-9999
LIFT	-2.97
LFTV	-3.10
SWET	-9999
KINX	-9999
CTOT	-9999
VTOT	-9999
TOTL	-9999
CAPE	475.5
CAPV	508.5
CINS	-5.63
CINV	-4.56
EQLV	315.0
EQTV	314.6
LFCT	755.2
LFCV	758.1
BRCH	67.25
BRCV	71.91
LCLT	278.6
LCLP	774.7
MLTH	299.7
MLMR	7.38
THCK	5586.
PWAT	16.40

Pat Arnott

The Saturated Adiabatic Lapse Rate
Temperature Changes Inside Clouds

Two processes occur simultaneously inside clouds that affect the temperature.

(1) Rising air expands, does work and cools;

(2) Condensation releases latent energy which is then stored as internal energy and warms the air inside the cloud.

(3) Normally, the cooling due to the work of expansion is greater than the warming associated with the release of latent energy and its conversion to internal energy.

Thus, as air rises inside a cloud it still gets **colder**, but it does so at a slower rate than the Dry Adiabatic Lapse rate.

The rate at which rising air inside a cloud cools is called the **Saturated Adiabatic Lapse Rate (SALR)**.

The Saturated Adiabatic Lapse Rate (SALR)

The derivation of the equation for the SALR begins with a form of the First Law of Thermodynamics

$$\mathbf{dq = c_p dT - \alpha dp}$$

What would we do here to get the dry adiabatic lapse rate?

The Saturted Adiabatic Lapse Rate (Cont.)

In this case the energy gained, **dq**, is equal to the latent energy released when water vapor condenses inside the cloud.

$$\mathbf{dq = -L_v dw_s}$$

where

L_v is the latent heat of vaporization, and

dw_s is the change of specific humidity of the air parcel when water vapor condenses

The SALR (Cont.)

Substitute for **dq** in the First Law of thermodynamics to get

$$-L_v dw_s = c_p dT - \alpha dp$$

Now divide by dz and use the hydrostatic equation to get …

Aside: Can write the **Moist Static Energy** for a rising air parcel as

$$MSE = c_p T + g z + L_v w_s \approx \text{constant}$$

The Saturated Adiabatic Lapse Rate (Cont.)

Substitution results in

$$-\frac{dT}{dz} = \overbrace{\frac{g}{c_p}}^{\Gamma_{dry}} + \frac{L_v}{c_p}\frac{dw_s}{dz} \equiv \Gamma_s$$

Saturated Adiabatic Lapse Rate = **cooling due to work of expansion** + **warming due to latent energy released during condensation**

$$\Gamma_s = \Gamma_{dry} + \frac{L_v\, dw_s}{c_p\, dz}$$

Use chain rule: $\frac{dw_s}{dz} = \frac{dw_s}{dT}\frac{dT}{dz} + \frac{dw_s}{dP}\frac{dP}{dz}$

Can show: $\frac{dw_s}{dz} = -\frac{dw_s}{dT}\Gamma_s + \frac{w_s c_p}{R_{dry}T}\Gamma_{dry}$

Summary of Equations for Saturated Adiabatic Lapse Rate

$\Gamma = -\frac{dT}{dz}$ is the environmental lapse rate in general.

$\Gamma_{dry} = \frac{g}{c_p} \approx 10\frac{K}{km}$ is the dry adiabatic lapse rate.

$$\Gamma_{moist} = \Gamma_{dry}\frac{1+\frac{L_v}{R_D T}w_s}{1+\frac{L_v}{c_p}\frac{dw_s}{dT}} \approx \Gamma_{dry}\frac{1+\frac{L_v}{R_D T}w_s}{1+\frac{\varepsilon L_v^2}{R_D c_p T^2}w_s} = \text{moist adiabat}$$

where

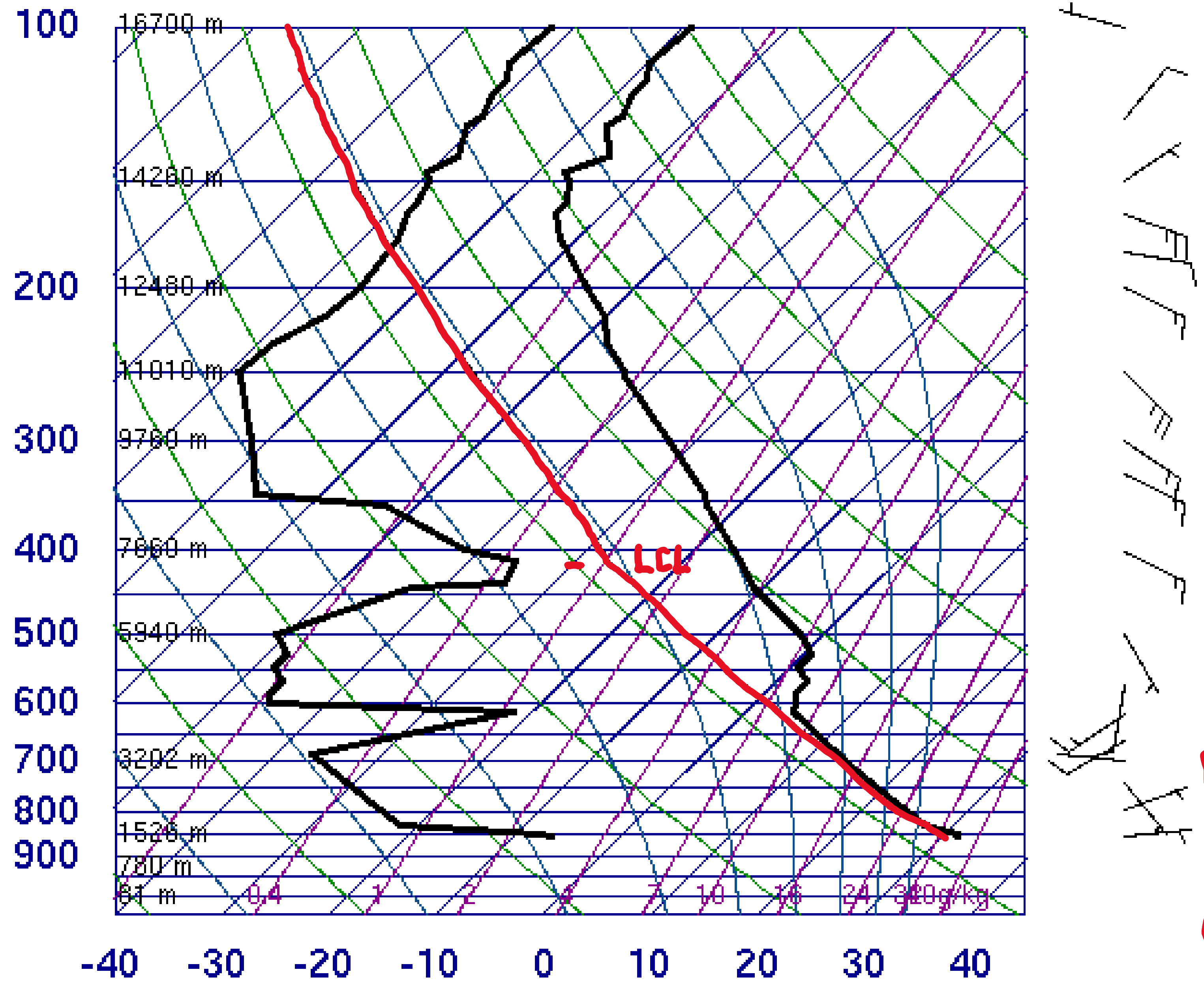
$$w_s = \varepsilon\frac{e_s(T)}{P}, \quad \varepsilon = 0.622$$

$$e_s(T) \approx 6.11\,mb\exp\left[\frac{L_v}{R_v}\left(\frac{1}{273}-\frac{1}{T}\right)\right]$$

Pat Arnott

72489 REV Reno

Find the LCL for a surface parcel: find Tw, the wet bulb temperature.



SLAT	39.56
SLON	-119.80
SELV	1516.
SHOW	10.51
LIFT	13.71
LFTV	13.37
SWET	20.98
KINX	-23.5
CTOT	-5.70
VTOT	33.30
TOTL	27.60
CAPE	0.00
CAPV	0.00
CINS	0.00
CINV	0.00
EQLV	-9999
EQTV	-9999
LFCT	-9999
LFCV	-9999
BRCH	0.00
BRCV	0.00
LCLT	246.3 K
LCLP	411.8 mb
MLTH	317.4
MLMR	1.21
THCK	5859.
PWAT	2.09

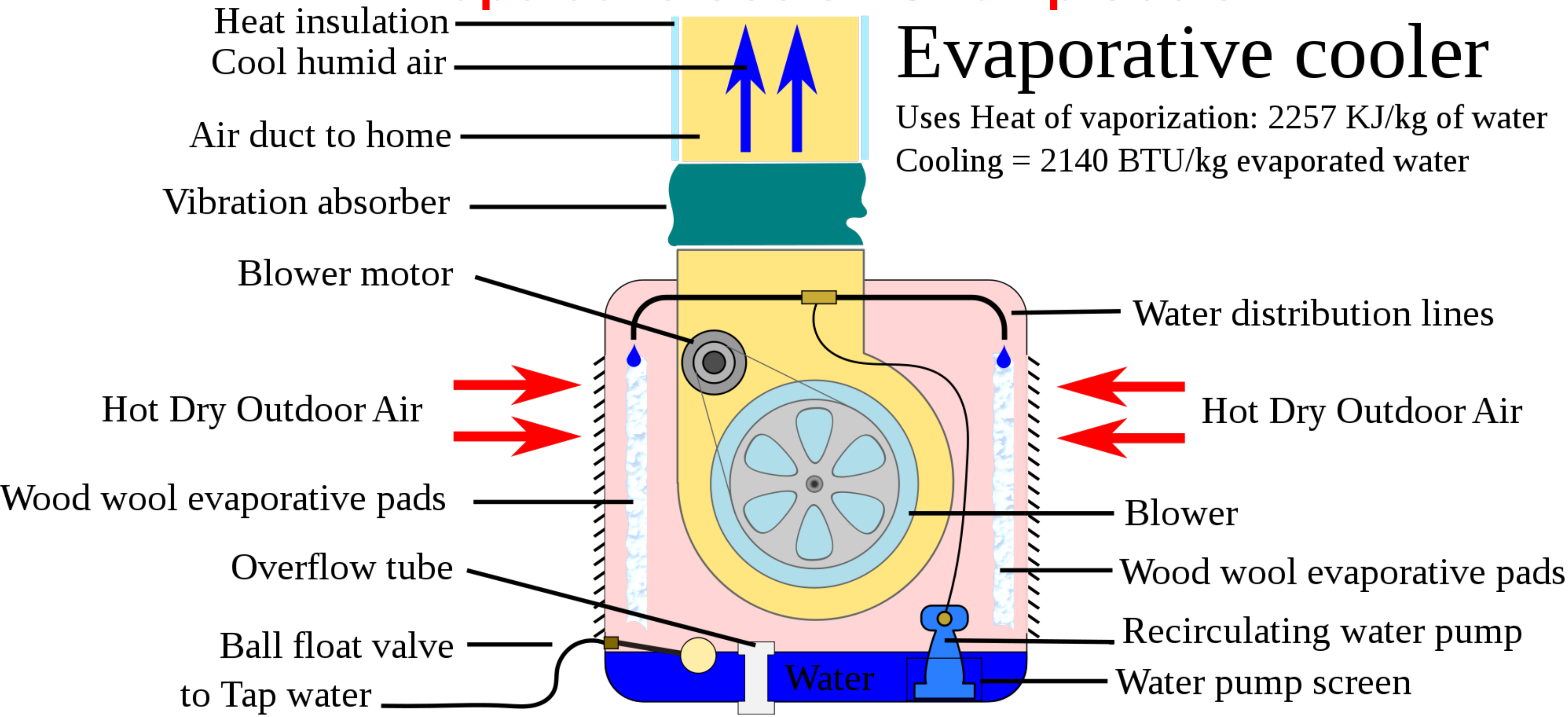
Precip H_2O in mm

'dry'

00Z 28 Sep 2010

University of Wyoming

Evaporative Cooler: Swamp Cooler.

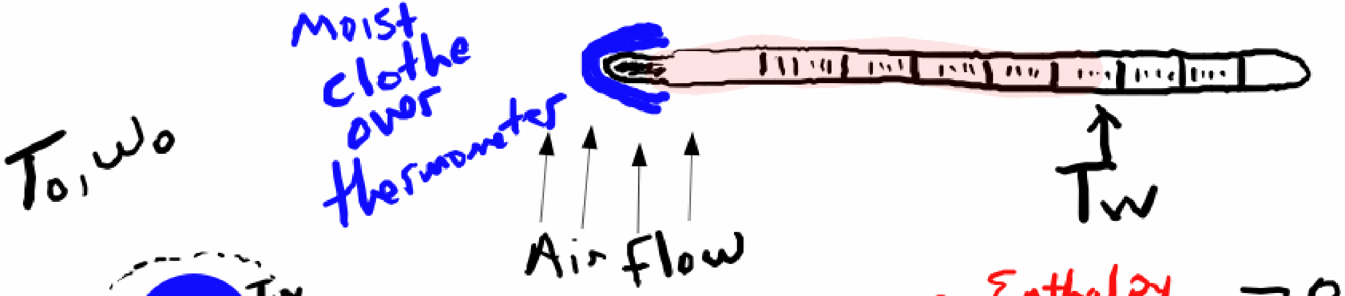


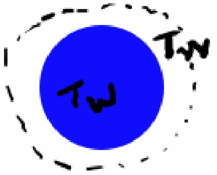
Evaporative cooler

Uses Heat of vaporization: 2257 KJ/kg of water

Cooling = 2140 BTU/kg evaporated water

Calculation of T_W, the Wet Bulb Temperature





Evaporating raindrop or cloud droplet

$$dq = C_p\,dT - \underset{\text{Enthalpy}}{}\cancel{dP/\rho} \xrightarrow{} 0 \text{ Constant Pressure Process}$$

dq: heat required to overcome the H_2O liquid state potential energy of attraction. dq comes from air surrounding the drop.

$$m_D\,dq = -L\left(\frac{\text{Joules}}{\text{Kg}}\right)dM_v \Rightarrow dq = -L\,dw$$

$$dw = \frac{dm_v}{m_D}$$

Integrating, $C_p(T_w - T_o) = -L\left(w_s(T_w) - w_o\right) \Rightarrow$ Solve for T_w

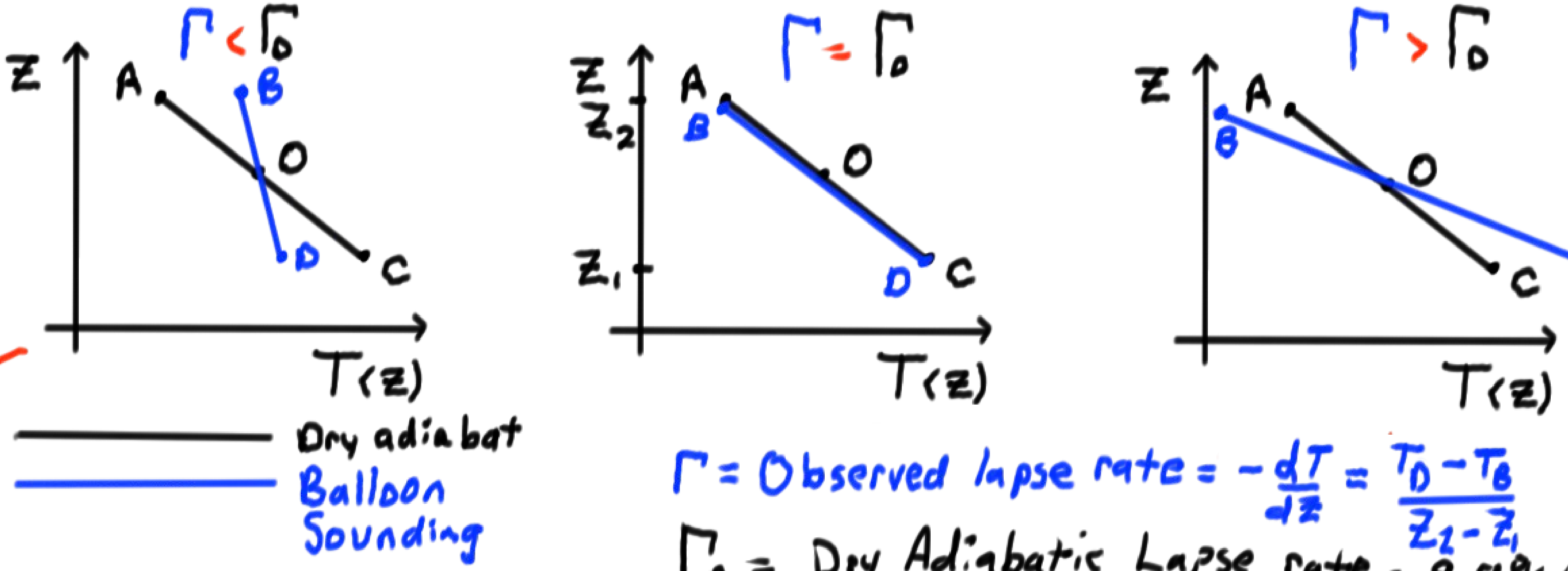
Recall: $w_s(T_w, P) = \dfrac{\varepsilon e_s(T_w)}{P}$, $e_s(T) \approx 6.11\,mb \exp\left[\dfrac{L}{R_v}\left(\dfrac{1}{273} - \dfrac{1}{T}\right)\right]$

Contrast to dew point: $e_{observed}(T_0) = e_s(T_{Dew})$

Solve for T_{Dew}.

Pat Arnott

Stability of An Unsaturated Air Parcel



$$\Gamma = \text{Observed lapse rate} = -\frac{dT}{dz} = \frac{T_D - T_B}{Z_2 - Z_1}$$

$$\Gamma_0 = \text{Dry Adiabatic Lapse rate} = 9.8\,^{\circ}K/km$$

$\Gamma < \Gamma_0$

Raise parcel from O to A. $T_A < T_B$, Parcel Sinks since $\rho_A > \rho_B$

Stable $\Gamma < \Gamma_0$

$\Gamma < \Gamma_0$ Stable

$\Gamma > \Gamma_0$ unstable

$\Gamma = \Gamma_0$ neutrally Stable

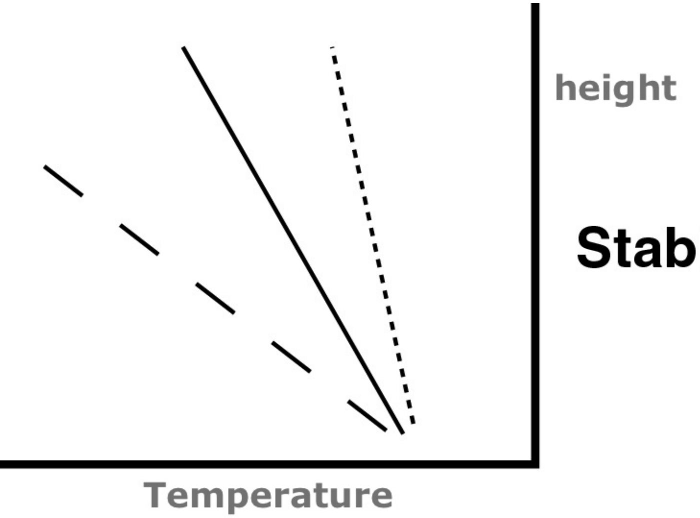
Raise Parcel from O to A. $T_A > T_B$. $\rho_A < \rho_B$.

Parcel Continues to rise!

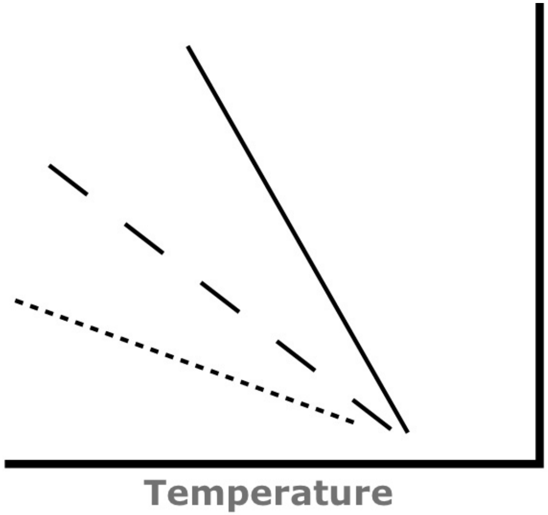
$\Gamma > \Gamma_0$ Unstable

Pat Arnott

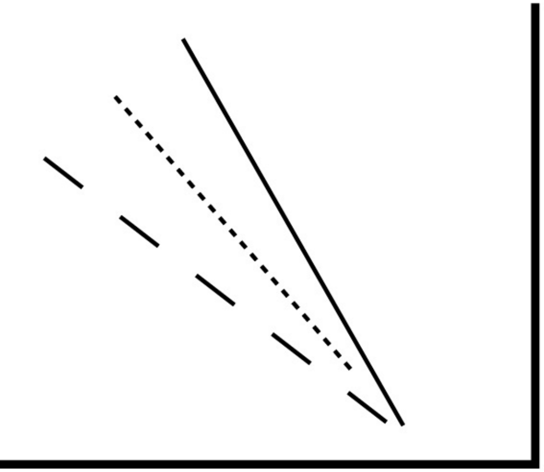
Summary of Stability at a Given Level



Stable



Unstable



Conditionally unstable

— — — — —	Dry adiabat
————————	Moist adiabat
------------------------	Environmental lapse rate

Pat Arnott

Downslope Windstorms And Gravity Wave Clouds

Downslope Windstorms

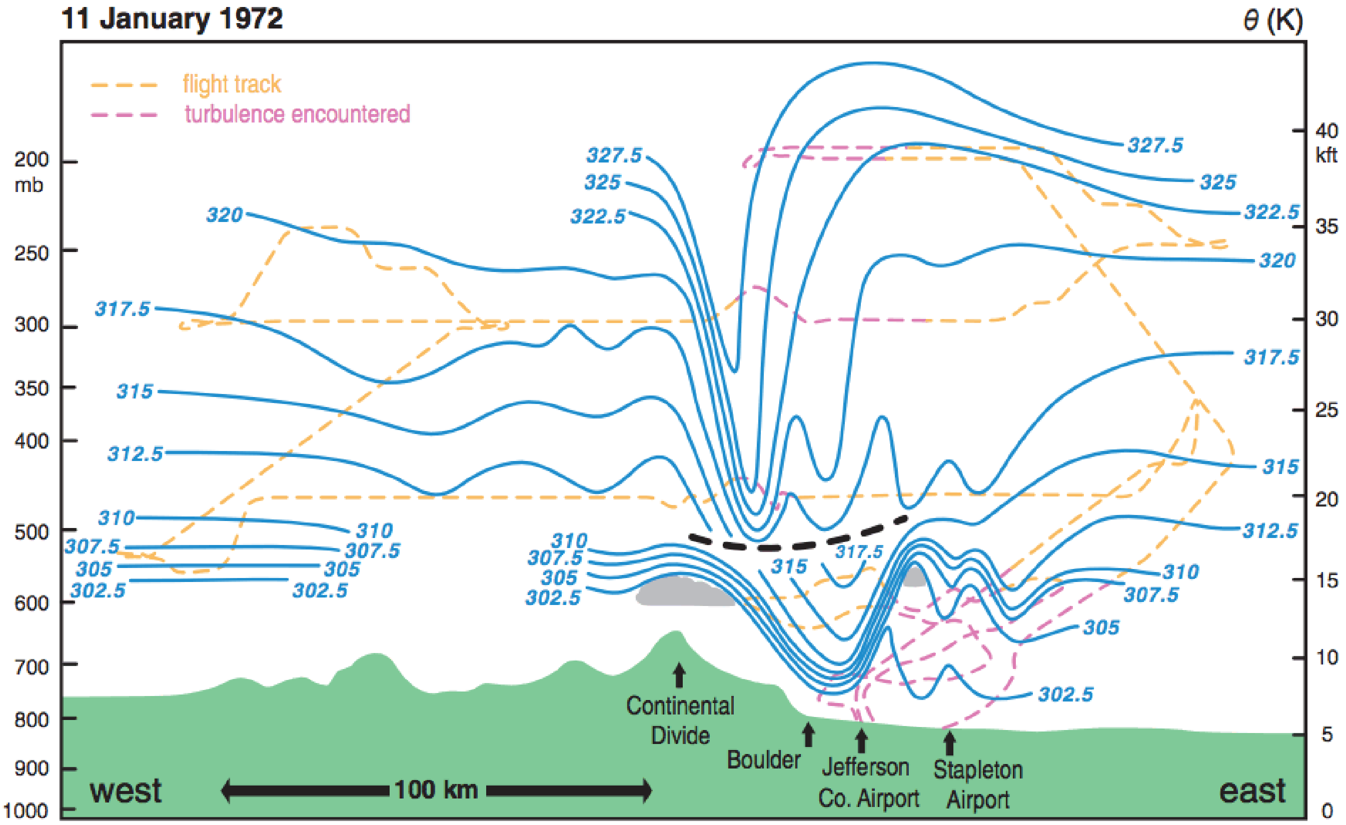
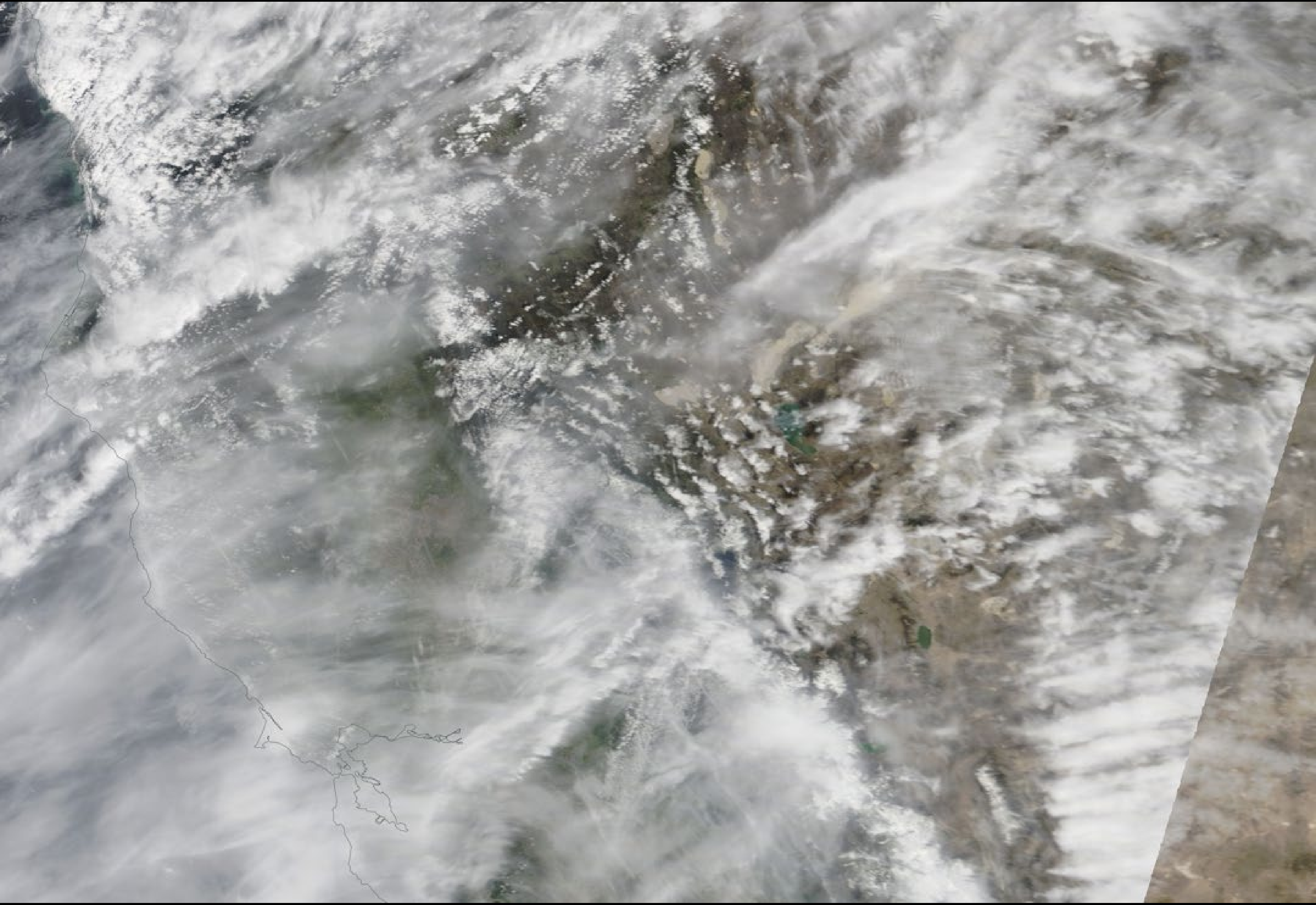
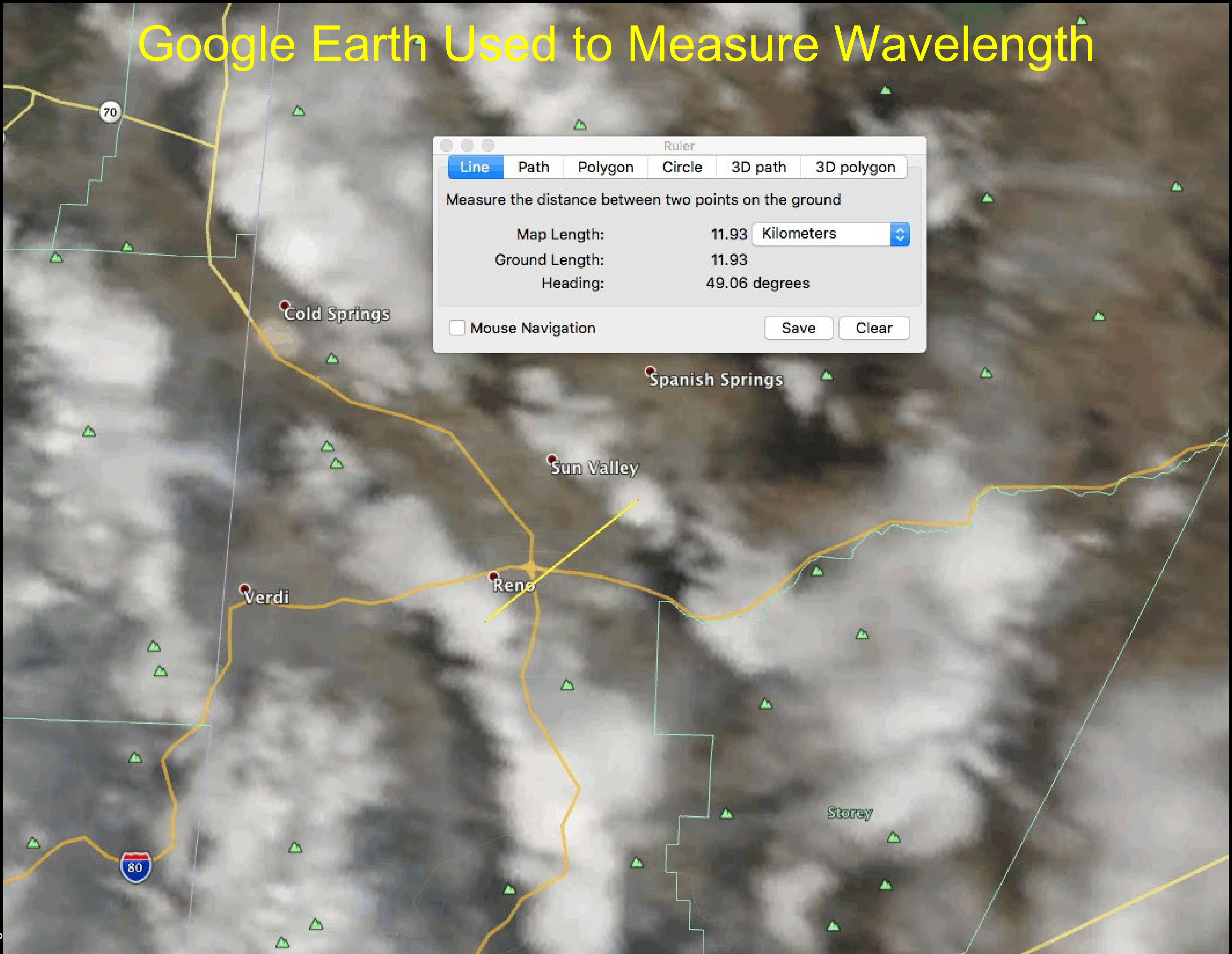


Figure 12.9 Analysis of potential temperatures (blue contours; K) from aircraft flight data (aircraft flight tracks are indicated with dashed lines) and rawinsondes on 11 January 1972 during a downslope windstorm near Boulder, CO. The heavy dashed line separates data taken by the Queen Air at lower levels before 2200 UTC from that taken by the Sabreliner aircraft in the middle and upper troposphere after 0000 GMT (12 January). The aircraft flight tracks were made along an approximate 130° –310° azimuth, but the distances shown are along the east–west projection of these tracks. (Adapted from Lilly [1978].)

22 April 2017 1:37 LST MODIS Satellite Image



Google Earth Used to Measure Wavelength



Measure the distance between two points on the ground

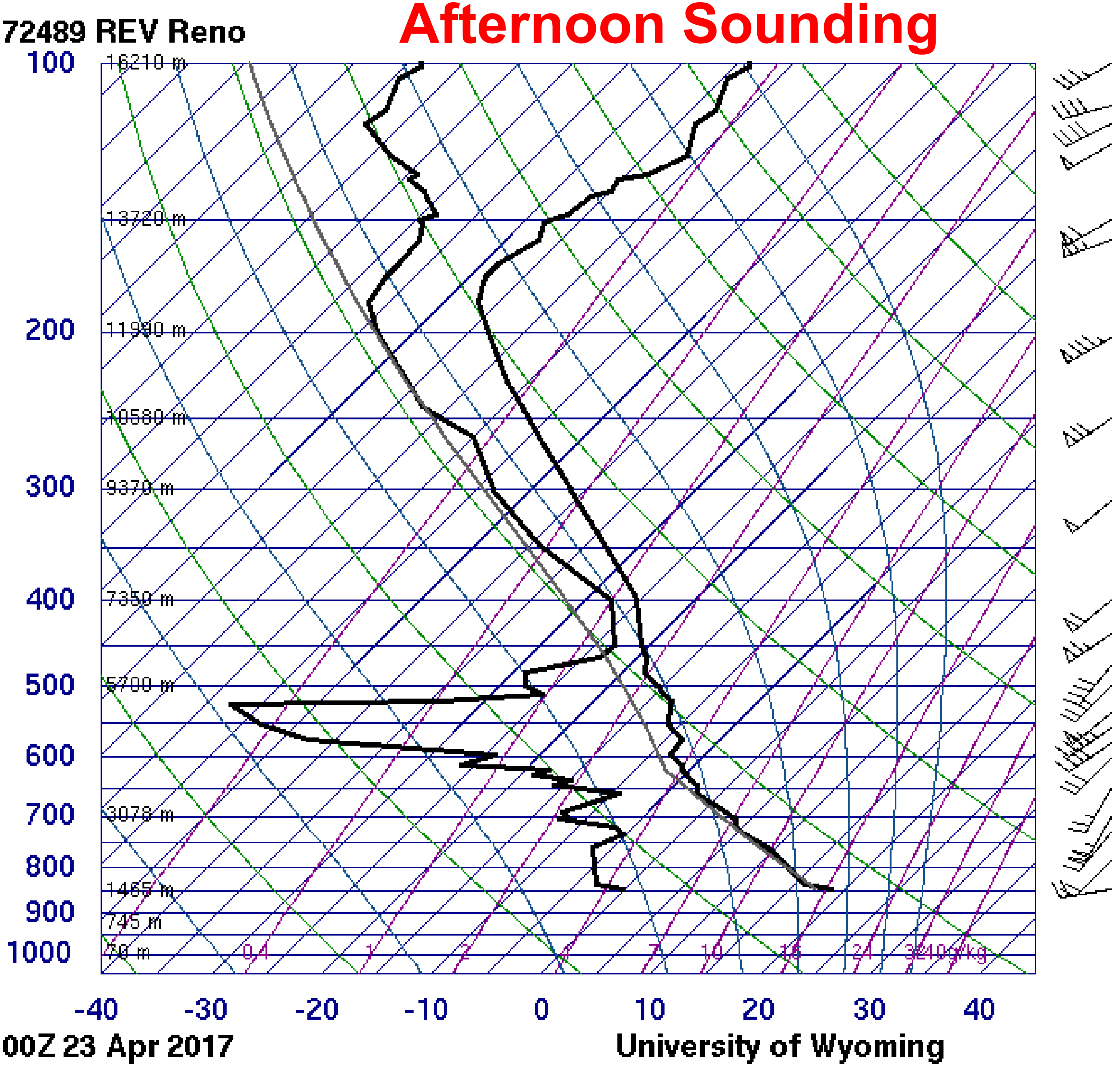
Map Length: 11.93 Kilometers

Ground Length: 11.93

Heading: 49.06 degrees

☐ Mouse Navigation

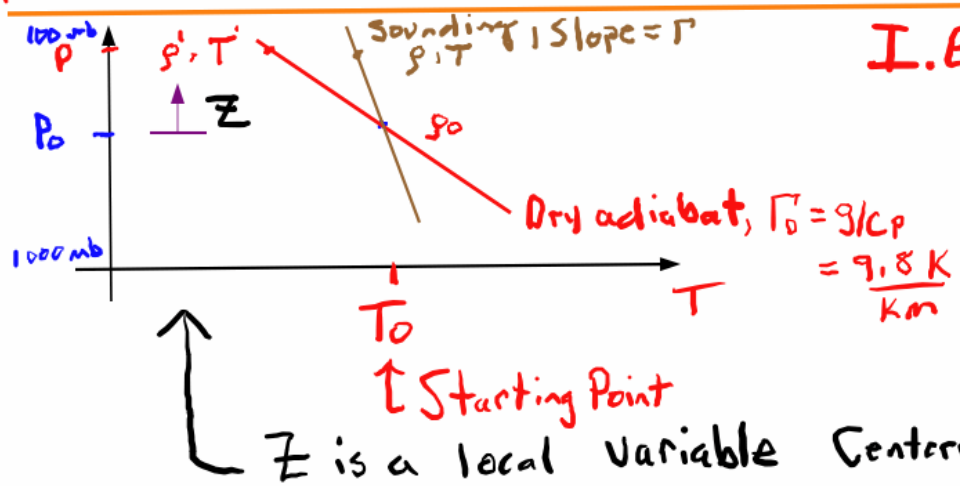
Afternoon Sounding



SLAT	39.56
SLON	-119.80
SELV	1516.
SHOW	-9999
LIFT	3.12
LFTV	2.97
SWET	-9999
KINX	-9999
CTOT	-9999
VTOT	-9999
TOTL	-9999
CAPE	0.00
CAPV	0.00
CINS	0.00
CINV	0.00
EQLV	-9999
EQTV	-9999
LFCT	-9999
LFCV	-9999
BRCH	0.00
BRCV	0.00
LCLT	265.9
LCLP	620.5
MLTH	304.8
MLMR	3.62
THCK	5630.
PWAT	8.82

Pat

Brunt Vaisalla Frequency: Oscillation of air about its equilibrium point.



Z is a local variable Centered at the Starting Point ρo, Po, To.

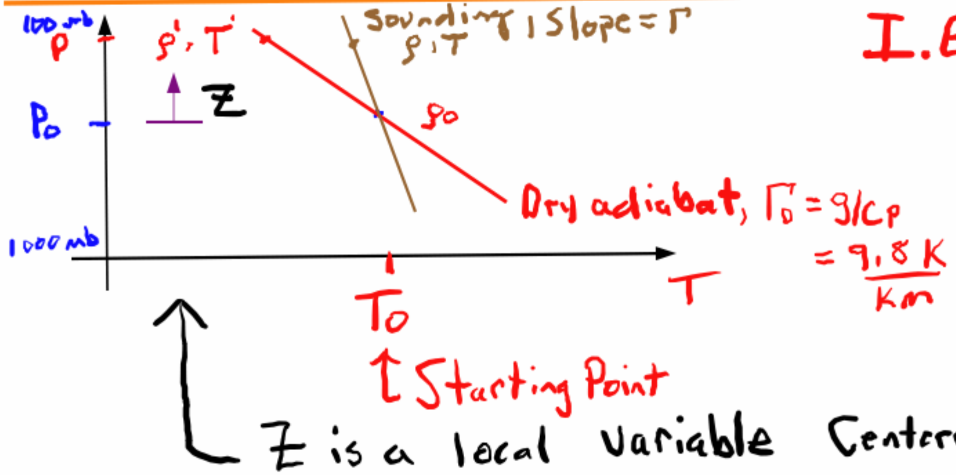
I.G.L. $P = \rho R_D T$, ρ = density $= \frac{P}{R_D T}$

Questions: Let $\Gamma \equiv -\frac{dT}{dz}$.

1). If $\Gamma < \Gamma_D$, is the point ρ_o, T_o, P_o stable?

2). If we disturb the parcel from ρ_o, T_o, P_o, what is the force on it?

Brunt Vaisalla Frequency: Oscillation of air about its equilibrium point.



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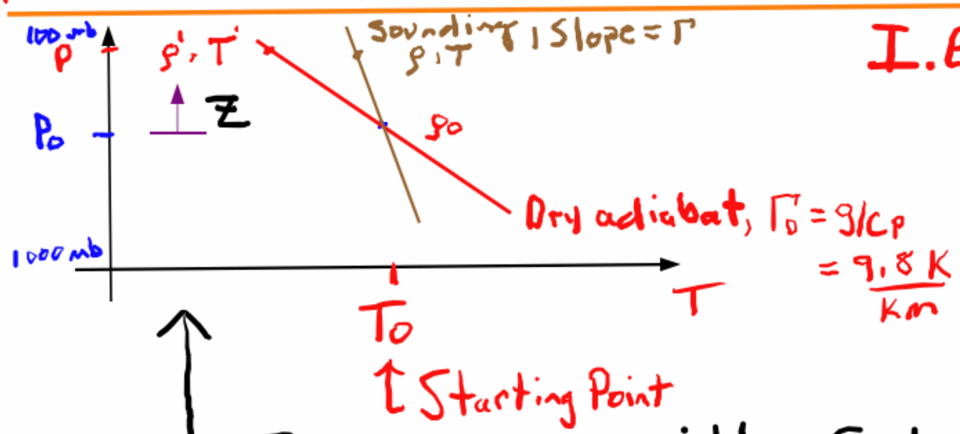
2). If we disturb the parcel from ρ_o, T_o, P_o, what is the force on it?

Analysis: $\Gamma_D = \frac{T_o - T'}{z}$. $\Gamma = \frac{T_o - T}{z}$. Can you see that $\Gamma < \Gamma_D$?

Brunt Vaisalla Frequency: Oscillation of air about its equilibrium point.



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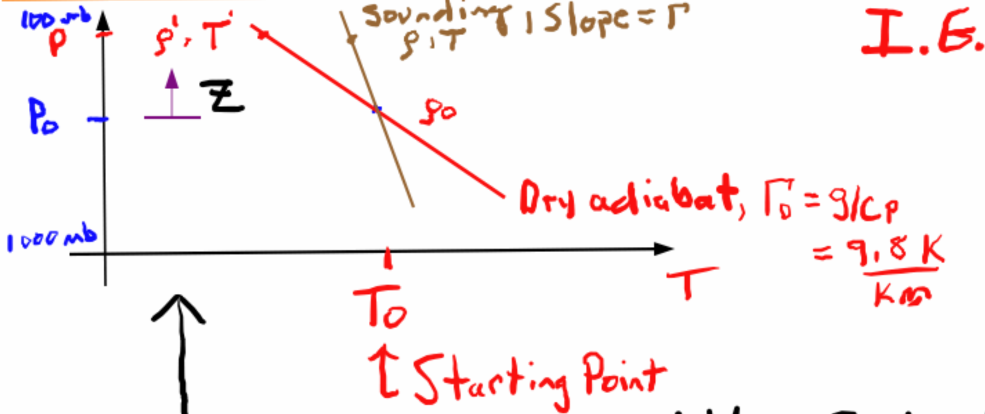
1). If $\Gamma < \Gamma_D$, is the point ρ_0, T_0, P_0 stable?

2). If we disturb the parcel from ρ_0, T_0, P_0, what is the force on it?

Analysis: $\Gamma_D = \frac{T_0 - T'}{Z}$. $\Gamma = \frac{T_0 - T}{Z}$. Can you see that $\Gamma < \Gamma_0$?

① Raise Parcel from T_0, P_0 along Γ_D. Can you see it is cooler than Environmental air at the same level?

Brunt Vaisalla Frequency: Oscillation of air about its equilibrium point.



Z is a local variable Centered at the Starting Point ρ_0, P_0, T_0.

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① Raise Parcel from T_0, P_0 along Γ_D. Can you see it is cooler than Environmental air at the same level?

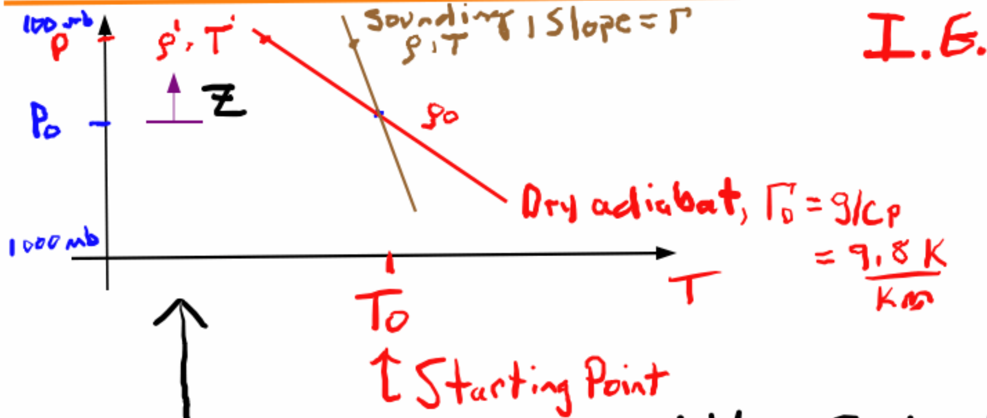
② $\frac{\text{Boyant Force}}{\text{Volume}}$ on Parcel $= -(\rho' - \rho) g \hat{z}$ ← unit vector in the z direction.

Can you see that if $\rho' < \rho$ the parcel goes up?
if $\rho' > \rho$ the Parcel goes down?

ρ ← density of environmental air at z.

ρ' ← density of air lifted hypothetically along the dry adiabat.

Brunt Vaisalla Frequency: Oscillation of air about its equilibrium point.



I.G.L. $P = \rho R_D T$, ρ = density = $\frac{P}{R_D T}$

Questions: Let $\Gamma \equiv -\frac{dT}{dZ}$.

1). If $\Gamma < \Gamma_D$, is the point ρ_0, T_0, P_0 stable?

2). If we disturb the parcel from ρ_0, T_0, P_0, what is the force on it?

Z is a local variable Centered at the Starting Point ρ_0, P_0, T_0.

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ρ → density of environmental air at Z.

ρ' → density of air lifted hypothetically along the dry adiabat.

③ Newton says...

$$\ddot{Z} = \frac{\text{Force}}{\text{mass}} = \text{Parcel Acceleration} = \frac{-(\rho' - \rho) g}{\rho'} = \frac{d^2 Z}{dt^2}$$

Brunt Vaisalla Frequency Continued ...

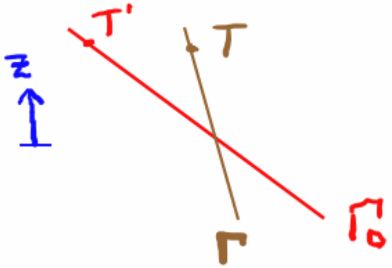
③ Newton says ...

$$\ddot{z} = \frac{\text{Force}}{\text{mass}} = \text{Parcel Acceleration} = \frac{-(\rho'-\rho)g}{\rho'} = \frac{d^2 z}{dt^2}$$

From the ideal gas equation, $\rho = \frac{p}{RT}$.

So $$\frac{\rho'-\rho}{\rho'} = \frac{\frac{p}{RT'} - \frac{p}{RT}}{\frac{p}{RT'}} = \frac{\frac{1}{T'} - \frac{1}{T}}{\frac{1}{T'}} = \frac{T-T'}{T}.$$

Brunt Vaisalla Frequency Continued ...



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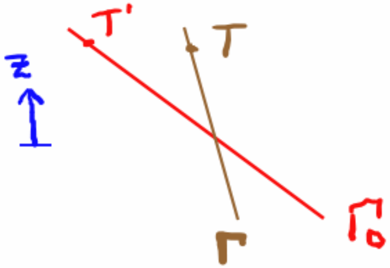
Then $$\ddot{z} = \frac{d^2z}{dt^2} = -\left(\frac{T-T'}{T}\right)g.$$



Brunt Vaisalla Frequency Continued ...

③ Newton says...

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From the ideal gas equation, $\rho = \frac{P}{RT}$.

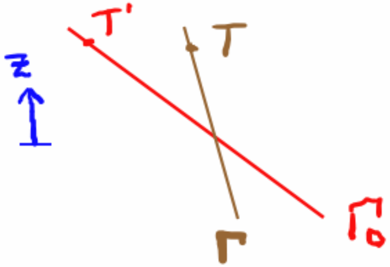
So $$\frac{\rho'-\rho}{\rho'} = \frac{\frac{P}{RT'} - \frac{P}{RT}}{\frac{P}{RT'}} = \frac{\frac{1}{T'} - \frac{1}{T}}{\frac{1}{T'}} = \frac{T-T'}{T}.$$

Then $$\ddot{z} = \frac{d^2z}{dt^2} = -\left(\frac{T-T'}{T}\right)g.$$

Now manipulate:

$$\ddot{z} = -\left(\frac{(T-T_0)}{z} - \frac{(T'-T_0)}{z}\right)\frac{g}{T_0}z$$

Add and subtract T_0.
Multiply by $\frac{z}{z} = 1$.



Brunt Vaisalla Frequency Continued ...

③ Newton says ...

$$\ddot{z} = \frac{\text{Force}}{\text{mass}} = \text{Parcel Acceleration} = \frac{-(\rho'-\rho)g}{\rho'} = \frac{d^2z}{dt^2}$$

From the ideal gas equation, $\rho = \frac{P}{RT}$.

So $$\frac{\rho'-\rho}{\rho'} = \frac{\frac{P}{RT'} - \frac{P}{RT}}{\frac{P}{RT'}} = \frac{\frac{1}{T'} - \frac{1}{T}}{\frac{1}{T'}} = \frac{T-T'}{T}.$$

Then $$\ddot{z} = \frac{d^2z}{dt^2} = -\left(\frac{T-T'}{T}\right)g.$$

Now manipulate:

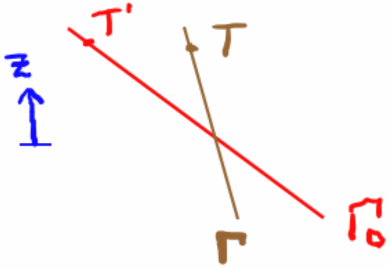
$$\ddot{z} = -\left(\underbrace{\frac{(T-T_0)}{z}}_{-\Gamma} - \underbrace{\frac{(T'-T_0)}{z}}_{-\Gamma_0}\right)\frac{g}{T_0}z$$

Add and subtract T_0. Multiply by $\frac{z}{z} = 1$.

$$\boxed{\ddot{z} = -(\Gamma_0 - \Gamma)\frac{g}{T_0}z}$$

Simple harmonic motion equation, Hooke's Law. Gravity g is the restoring force.

As long as $\Gamma < \Gamma_0$, Parcel is stable, because if z increases, $\ddot{z}$ decreases. Force opposite displacement



Brunt Vaisalla Frequency Continued ...

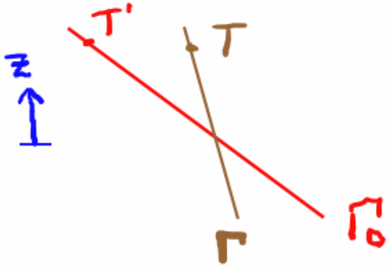
$$\boxed{\ddot{z} = -(\Gamma_0 - \Gamma)\frac{g}{T_0} z}$$

Trial Solution: $z(t) = \underbrace{A}_{\text{Amplitude.}} \sin \underbrace{\omega_0}_{\text{Radian Frequency.}} t$

Then $\dot{z} = \frac{dz}{dt} = A\omega_0 \cos \omega_0 t$,

$$\ddot{z} = \frac{d^2 z}{dt^2} = -A\omega_0^2 \sin \omega_0 t$$

Brunt Vaisalla Frequency Continued ...



$$\boxed{\ddot{z} = -(\Gamma_0 - \Gamma)\frac{g}{T_0} z}$$

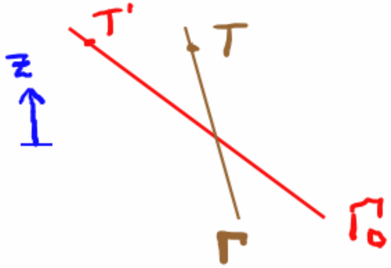
Put z here

Trial Solution: $z(t) = \underbrace{A}_{\text{Amplitude}} \sin \underbrace{\omega_0}_{\text{Radian Frequency.}} t$

Put $\ddot{z}$ here

Then $\dot{z} = \frac{dz}{dt} = A\omega_0 \cos \omega_0 t$,

$$\ddot{z} = \frac{d^2 z}{dt^2} = -A\omega_0^2 \sin \omega_0 t$$



Brunt Vaisalla Frequency Continued ...

$$\boxed{\ddot{z} = -(\Gamma_0 - \Gamma)\frac{g}{T_0} z}$$

Put z here

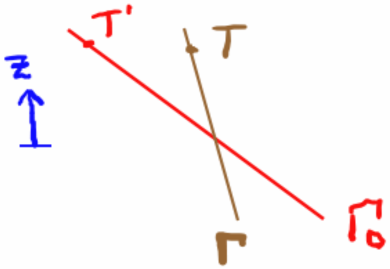
Trial Solution: $z(t) = \underbrace{A}_{\text{Amplitude.}} \sin \underbrace{\omega_0}_{\text{Radian Frequency.}} t$

Then $\dot{z} = \dfrac{dz}{dt} = A\omega_0 \cos\omega_0 t$,

$\ddot{z} = \dfrac{d^2z}{dt^2} = -A\omega_0^2 \sin\omega_0 t$

Put $\ddot{z}$ here

$$-A\omega_0^2 \sin\omega_0 t = -(\Gamma_0 - \Gamma)\frac{g}{T_0} A \sin\omega_0 t$$



Brunt Vaisalla Frequency Continued ...

$$\boxed{\ddot{z} = -(\Gamma_0 - \Gamma)\frac{g}{T_0} z}$$

Put Z here

Put $\ddot{Z}$ here

Trial Solution: $Z(t) = A \sin \omega_0 t$

Radian Frequency.

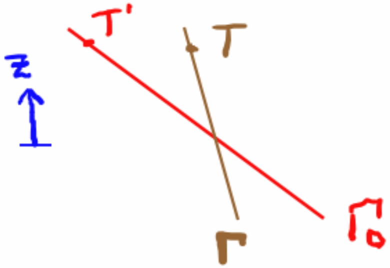
Amplitude.

Then $\dot{z} = \frac{dz}{dt} = A\omega_0 \cos \omega_0 t$,

$$\ddot{z} = \frac{d^2 z}{dt^2} = -A\omega_0^2 \sin \omega_0 t$$

$$\cancel{-}\cancel{A}\omega_0^2 \cancel{\sin \omega_0 t} = \cancel{-}(\Gamma_0 - \Gamma)\frac{g}{T_0}\cancel{A}\cancel{\sin \omega_0 t}$$

Brunt Vaisalla Frequency Continued ...



$$\boxed{\ddot{z} = -(\Gamma_0 - \Gamma)\frac{g}{T_0} z}$$

Put z here

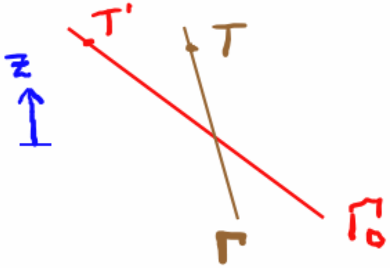
Trial Solution: $z(t) = \underbrace{A}_{\text{Amplitude.}} \sin \underbrace{\omega_0}_{\text{Radian Frequency.}} t$

Put $\ddot{z}$ here

Then $\dot{z} = \frac{dz}{dt} = A\omega_0 \cos \omega_0 t$,

$$\ddot{z} = \frac{d^2 z}{dt^2} = -A\omega_0^2 \sin \omega_0 t$$

$$\omega_0^2 = (\Gamma_0 - \Gamma)\frac{g}{T_0}$$



Brunt Vaisalla Frequency Continued ...

$$\boxed{\ddot{z} = -(\Gamma_0-\Gamma)\frac{g}{T_0}\,z}$$

Put z here

Trial Solution: $z(t) = A \sin\omega_0 t$

Amplitude.

Radian Frequency.

Put $\ddot{z}$ here

Then $\dot{z} = \frac{dz}{dt} = A\omega_0 \cos\omega_0 t$,

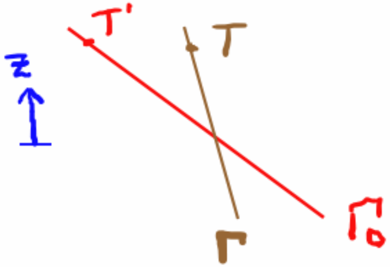
$\ddot{z} = \frac{d^2z}{dt^2} = -A\omega_0^2 \sin\omega_0 t$

$$\boxed{\omega_0^2 = (\Gamma_0-\Gamma)\frac{g}{T_0}}$$

$\omega_0 = \left[(\Gamma_0-\Gamma)\frac{g}{T_0}\right]^{1/2}$ = Brunt-Vaisalla Frequency of oscillation of a gravity wave.

τ ≡ Period of oscillation.

$\tau = 2\pi/\omega_0$.



Brunt Vaisalla Frequency Continued ...

Example: $\Gamma_0 = 9.8$ K/km. $\Gamma = 5$ K/km.

$T_0 = 250$ K. $\Rightarrow$ $\tau = 7.6$ minutes.

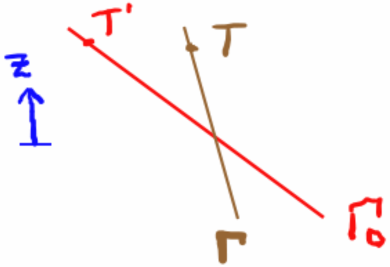
$$\omega_0 = \left[(\Gamma_0 - \Gamma) \frac{g}{T_0} \right]^{1/2}$$

= Brunt-Vaisalla Frequency of oscillation of a gravity wave.

$\tau \equiv$ Period of oscillation.

$\tau = 2\pi/\omega_0$.

Brunt Vaisalla Frequency Continued ...



Example: $\Gamma_0 = 9.8\,K/km$. $\Gamma = 5\,K/km$.

$T_0 = 250\,K$. ⇒ $\tau = 7.6$ minutes.

$$\omega_0 = \left[(\Gamma_0 - \Gamma)\frac{g}{T_0}\right]^{1/2}$$ = Brunt-Vaisalla Frequency of oscillation of a gravity wave.

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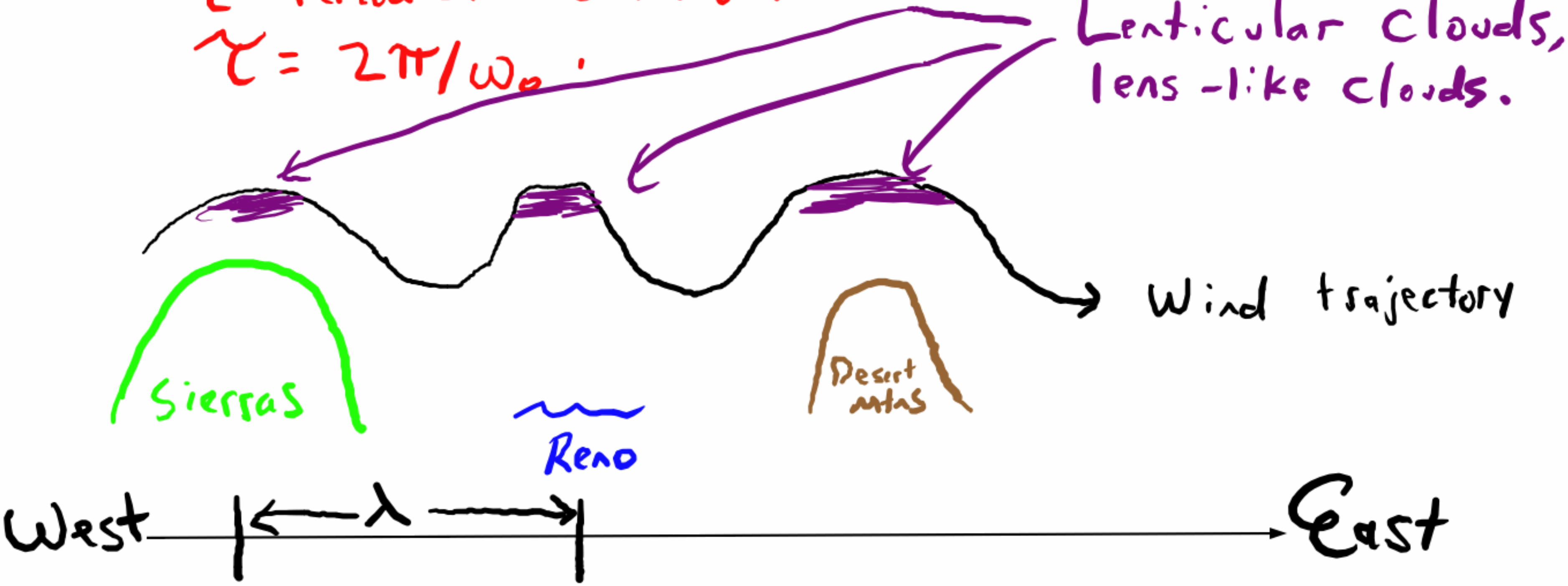
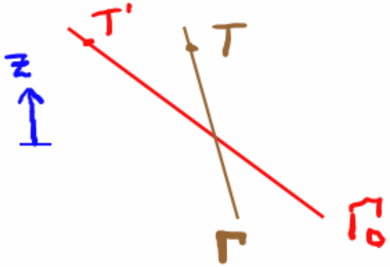


Brunt Vaisalla Frequency Continued ...



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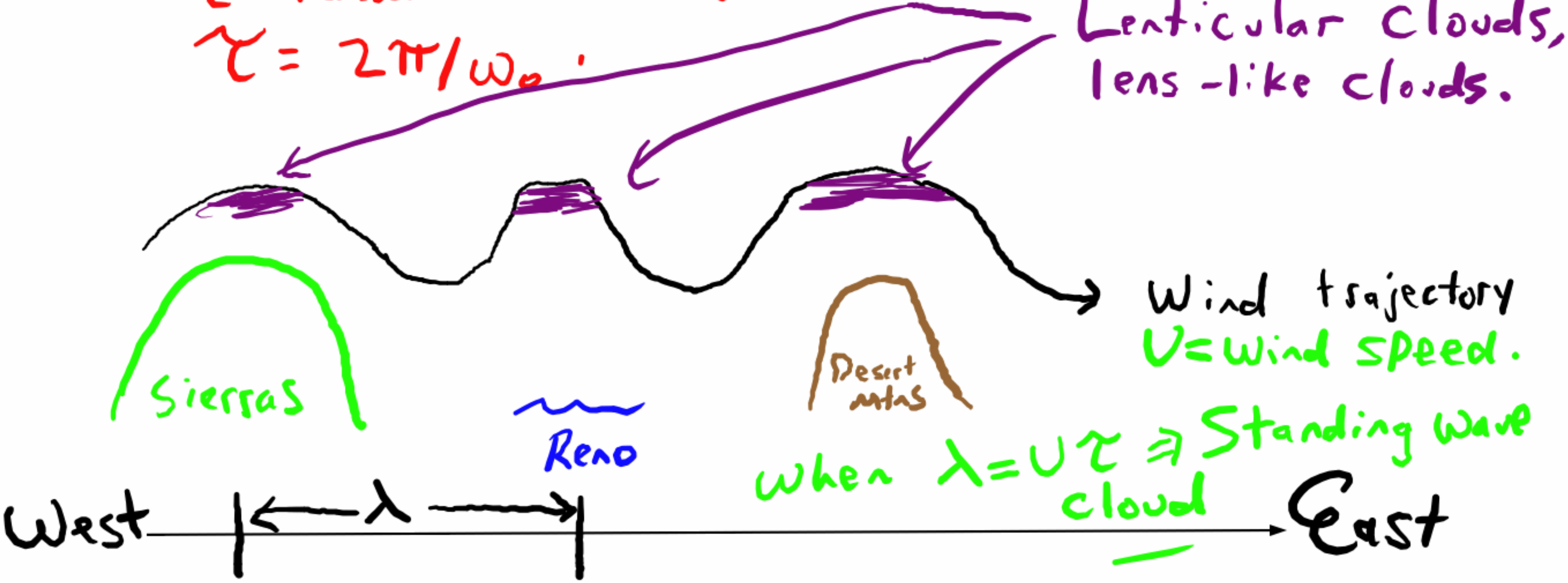
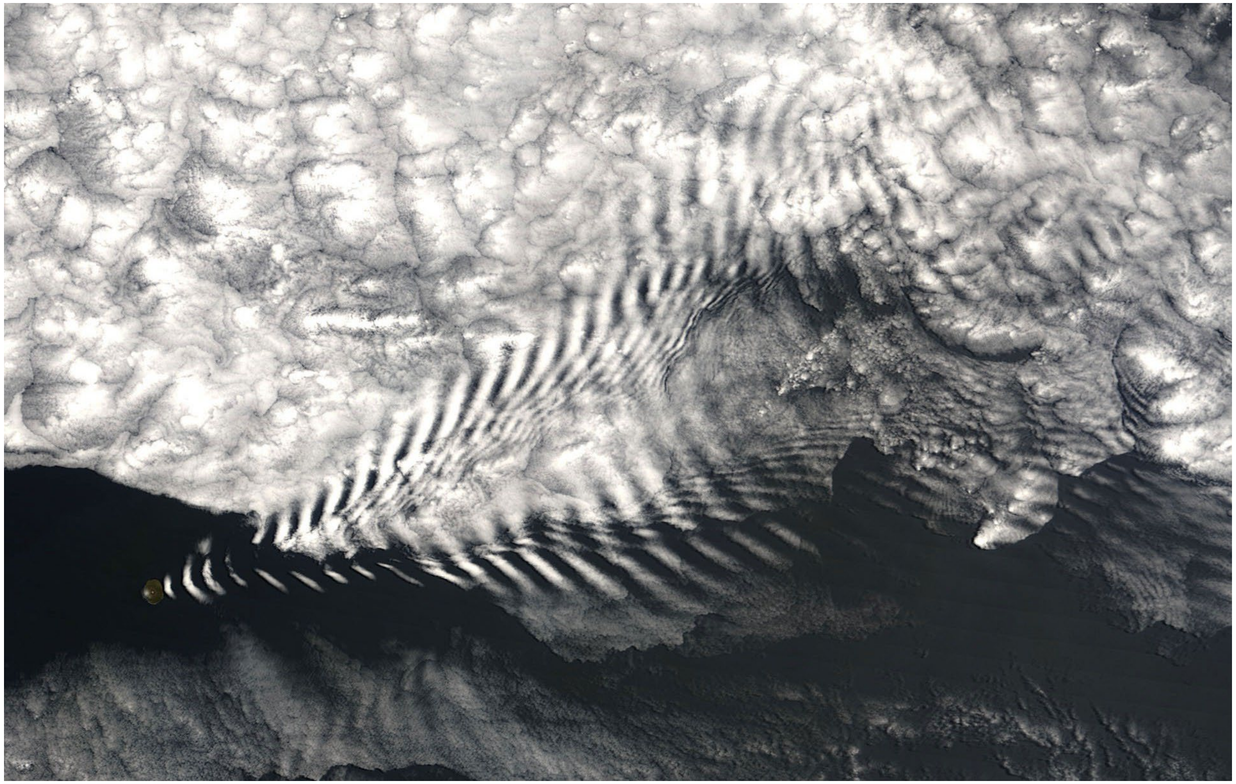


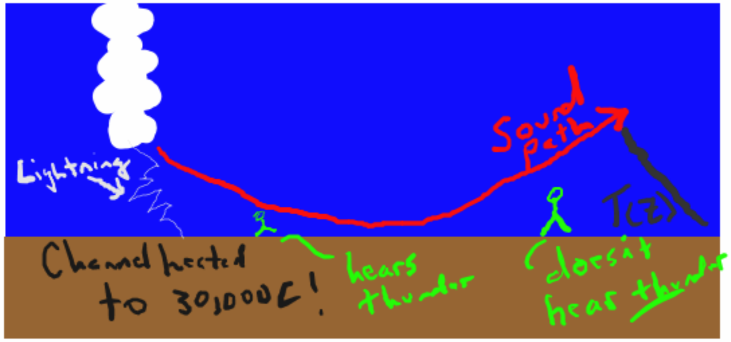
Lenticular clouds, lens-like clouds.

U=Wind speed.

when $\lambda = U\tau$ ⇒ Standing wave cloud



NASA satellite image (MODIS imager on board the Terra satellite) of a wave cloud forming off of Amsterdam Island in the far southern Indian Ocean. Image taken on December 19, 2005.



Sound in the atmosphere.

Snell's Law of refraction.

V_1 = Wave speed in medium 1

medium 1

medium 2

V_2 = Wave speed in medium 2

Snell's law:

$$\frac{C}{V_1}\sin\theta_1 = \frac{C}{V_2}\sin\theta_2$$

or

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

C ≡ reference speed

i.e. $C = 3 \times 10^8$ m/s (light)

$C = 345$ m/s, dry air at 23 C = 73.4F.

So if $V_1 < V_2$, $\theta_1 < \theta_2$ since $\sin\theta_1 = \frac{V_1}{V_2}\sin\theta_2$.

"Slow is more normal"

(rays are closer to the surface normal on the side with the lowest wave velocity).

Speed of Sound in most gases:

$$V = (\gamma R T)^{1/2} + U + V_{dry} * h$$

γ: $C_P/C_V = 1.4$ for air

$R = \frac{R^*}{MW} = \frac{8314.3 \text{ J/Kmol °K}}{29 \text{ Kg/Kmole}}$

T = Absolute Temperature (Kelvin)

U = Wind Speed component in Sound direction.

$h = \frac{\text{\# water vapor molecules / Volume}}{\text{total \# Molecules / volume.}}$

$$V_{DRY\,air} = (\gamma R T)^{1/2}$$

Sound Speed and Sound Path depends on meteorology!

V is a function of Temperature, Wind Speed, and relative humidity

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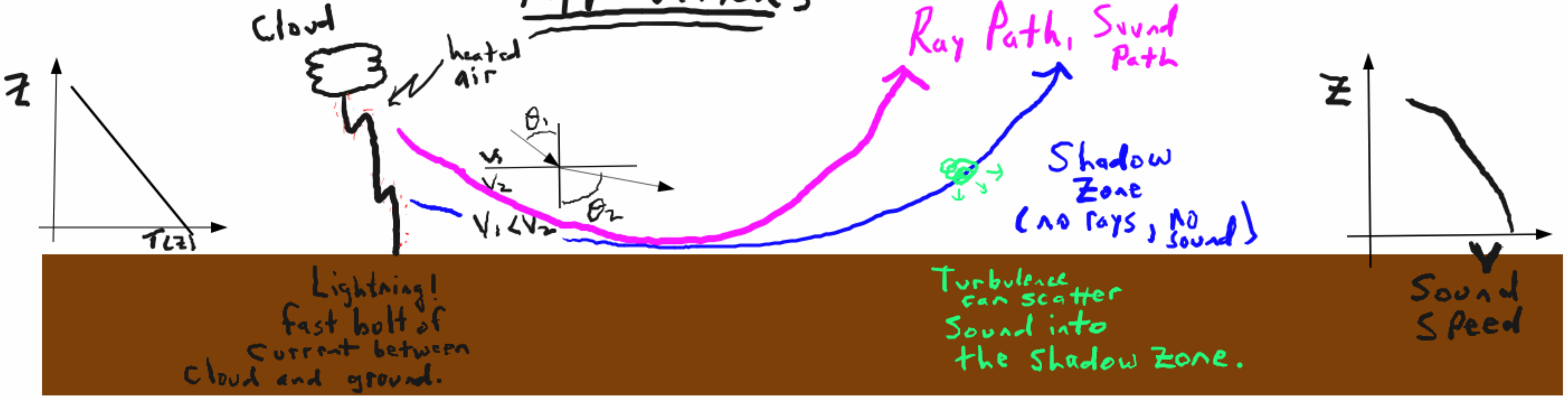
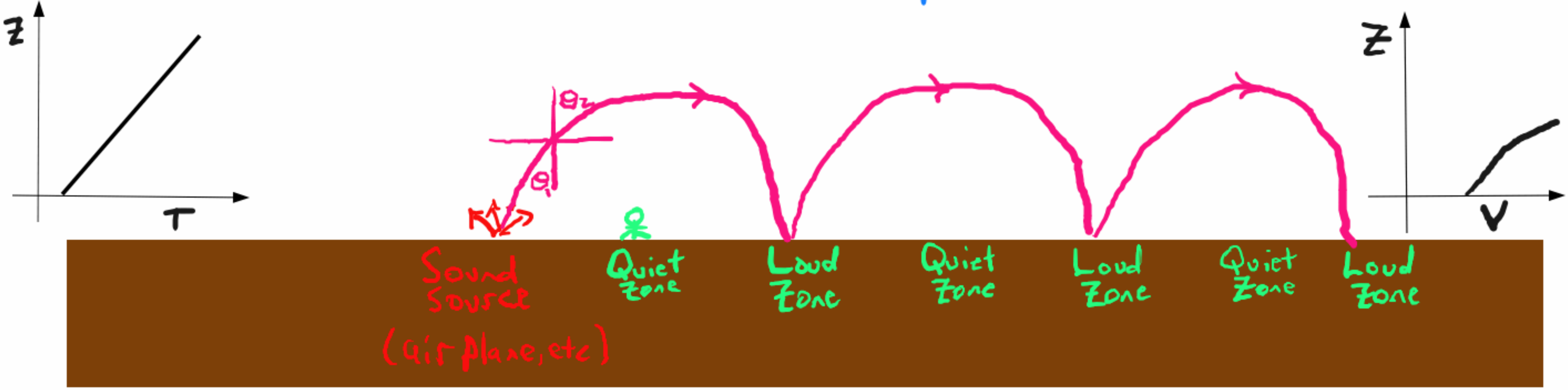
Applications

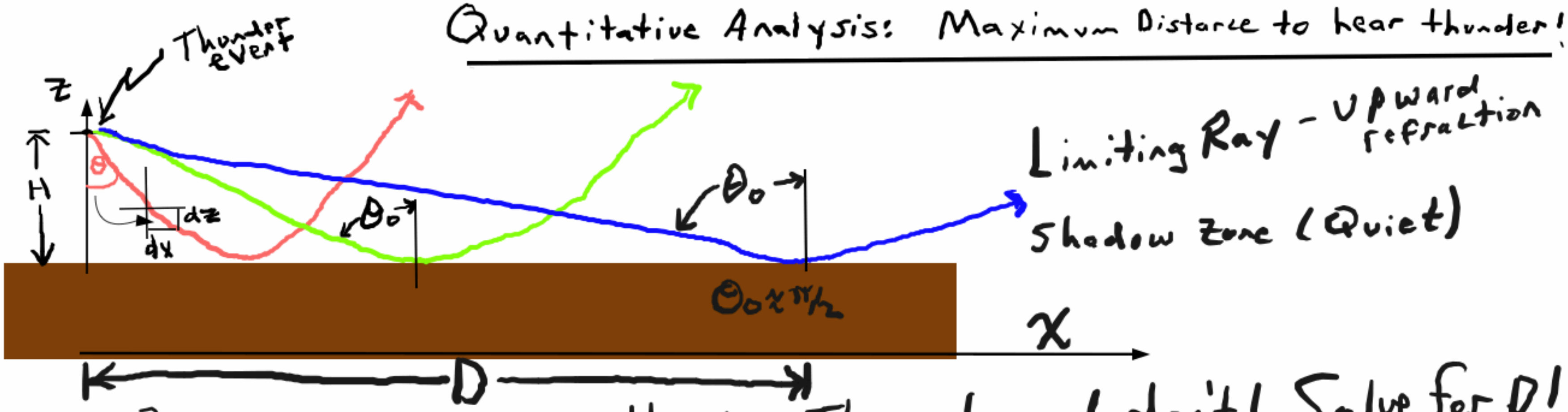


Inverted Atmosphere

Temperature inversions tend to trap sound, just as they do air pollution.



Quantitative Analysis: Maximum Distance to hear thunder!



People out to D hear thunder: Those beyond don't! Solve for D!

Givens: 1. Snell's law gives the ray paths. $\frac{V(z)}{\sin\theta(z)} = \text{Constant}$

2. V(z) = Sound speed at height z.

$V(z) = V_0 \left(\frac{T(z)}{T_0}\right)^{1/2}$ $\quad T_0 \equiv T(z=0)$. $\quad V_0 = V(z=0)$.

3. Temperature Structure; $T(z) = T_0 - \Gamma(z)\, z$ $\quad \Gamma(z)$ = lapse rate $\equiv -dT/dz$.

Determine: Ray Paths From Geometry and trig,

$$\frac{dx}{dz} = -\tan\theta = -\frac{\sin\theta}{\cos\theta} = \frac{-\sin\theta}{(1-\sin^2\theta)^{1/2}} = \frac{-1}{\left(\underline{\underline{1/\sin^2\theta}} - 1\right)^{1/2}}.$$

From Snell's Law, $\frac{V(z)}{\sin\theta(z)} = \frac{V_0}{\sin\theta_0} = \frac{V(z=H)}{\sin\theta(z=H)}$, So $\underline{\underline{\frac{1}{\sin^2\theta}}} = \left(\frac{V_0}{V(z)}\right)^2 \frac{1}{\sin^2\theta_0} = \underline{\underline{\frac{T_0}{T(z)}\frac{1}{\sin^2\theta_0}}}$

Substitute where indicated...

Shadow Zone location, Continued...

General Relation for Ray Paths:

$$\frac{dx}{dz} = \frac{-1}{\left(\frac{T_o}{T(z)\sin^2\theta_o} - 1\right)^{1/2}}.$$

Choose $\theta_o = \pi/2$ = Limiting Ray.

Use $T(z) = T_o - \Gamma z$

Now integrate to find D, shadow zone distance from lightning.

$$\int_0^D dx = -\int_H^0 \left(\frac{T_o - \Gamma z}{\Gamma z}\right)^{1/2} dz \approx -\int_H^0 \left(\frac{T_o}{\Gamma z}\right)^{1/2} dz$$

Since $\Gamma z \ll T_o$

$$\boxed{D = \left(\frac{T_o}{\Gamma}\right)^{1/2} \int_0^H z^{-1/2}\,dz = 2\left(\frac{H T_o}{\Gamma}\right)^{1/2}}$$

Estimate: $T_o \approx 300\,K$, $\Gamma \approx 7.5\,K/km$, $H \approx 4\,km$, $\Gamma H \approx 30\,K \ll 300\,K$

$$D = 2\left(\frac{T_o H}{\Gamma}\right)^{1/2} = 2\left(\frac{300K \cdot 4km}{7.5\,K/km}\right)^{1/2} \approx 25\,km.$$

Beyond $D = 25$ km, a person sees lightning but does not hear thunder because sound suffers upward refraction due to the temperature $\overset{z\uparrow}{}\searrow T(z)$.

Exercises

3.18 Answer or explain the following in light of the principles discussed in this chapter.

- (a) To carry a given payload, a hot air balloon cruising at a high altitude needs to be bigger or hotter than a balloon cruising at a lower altitude.
- (b) More fuel is required to lift a hot air balloon through an inversion than to lift it through a layer of the same depth that exhibits a steep temperature lapse rate. Other conditions being the same, more fuel is required to operate a hot air balloon on a hot day than on a cold day.
- (c) Runways are longer at high altitude airports such as Denver and stricter weight limits are imposed on aircraft taking off on hot summer days.
- (d) The gas constant for moist air is greater than that for dry air.
- (e) Pressure in the atmosphere increases approximately exponentially with depth, whereas the pressure in the ocean increases approximately linearly with depth.
- (f) Describe a procedure for converting station pressure to sea-level pressure.
- (g) Under what condition(s) does the hypsometric equation predict an exponential decrease of pressure with height?
- (h) If a low pressure system is colder than its surroundings, the amplitude of the depression in the geopotential height field increases with height.
- (i) On some occasions low surface temperatures are recorded when the 1000- to 500-hPa thickness is well above normal. Explain this apparent paradox.
- (j) Air released from a tire is cooler than its surroundings.
- (k) Under what conditions can an ideal gas undergo a change of state without doing external work?
- (l) A parcel of air cools when it is lifted. Dry parcels cool more rapidly than moist parcels.
- (m) If a layer of the atmosphere is well mixed in the vertical, how would you expect the potential temperature within it to change with height?
- (n) In cold climates the air indoors tends to be extremely dry.
- (o) Summertime dew points tend to be higher over eastern Asia and the eastern United States than over Europe and the western United States.
- (p) If someone claims to have experienced hot, humid weather with a temperature in excess of 90 °F and a relative humidity of 90%, it is likely that he/she is exaggerating or inadvertently juxtaposing an afternoon temperature with an early morning relative humidity.
- (q) Hot weather causes more human discomfort when the air is humid than when it is dry.
- (r) Which of the following pairs of quantities are conserved when unsaturated air is lifted: potential temperature and mixing ratio, potential temperature and saturation mixing ratio, equivalent potential temperature and saturation mixing ratio?
- (s) Which of the following quantities are conserved during the lifting of saturated air: potential temperature, equivalent potential temperature, mixing ratio, saturation mixing ratio?
- (t) The frost point temperature is higher than the dew point temperature.
- (u) You are climbing in the mountains and come across a very cold spring of water. If you had a glass tumbler and a thermometer, how might you determine the dew point of the air?
- (v) Leaving the door of a refrigerator open warms the kitchen. (How would the refrigerator need to be reconfigured to make it have the reverse effect?)
- (w) A liquid boils when its saturation vapor pressure is equal to the atmospheric pressure.

3.19 Determine the apparent molecular weight of the Venusian atmosphere, assuming that it consists of 95% of CO_2 and 5% N_2 by volume. What is the gas constant for 1 kg of such an atmosphere? (Atomic weights of C, O, and N are 12, 16, and 14, respectively.)

Pat Arnott


S = k. log W
LVDWIG
BOLTZMANN
1844 – 1906
DR PHIL. PAULA
BOLTZMANN
GEB. CHIARI
1891 – 1977
ARTHUR
BOLTZMANN
DIPL. ING. DR PHIL. HOFRAT
1881 – 1952
LVDWIG
BOLTZMANN
1923 – 1943
TZTER MÄNNLICHER NACHKOMME.
GEFALLEN BEI SMOLENSK
HENRIETTE
BOLTZMANN
EB. EDLE von AIGENTLER
1854 – 1938

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Entropy

$$du = \underline{\underline{dq}} - dw = \underline{\underline{T\,dS}} - PdV$$

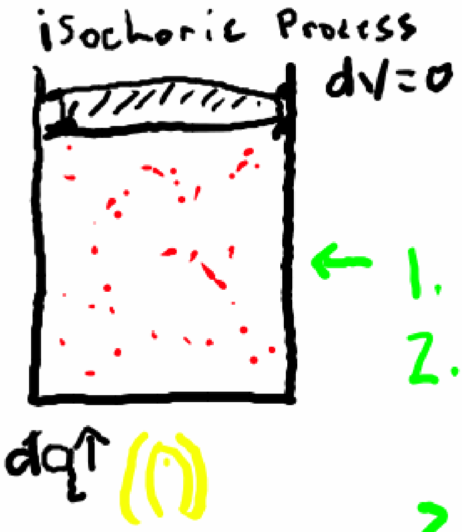
Entropy

Entropy - Sort of sounds like energy - For good reason!

$$S = k \ln \Omega$$

# of places or cubby holes to put energy.

Boltzmann's constant ⇒ why he invented it.

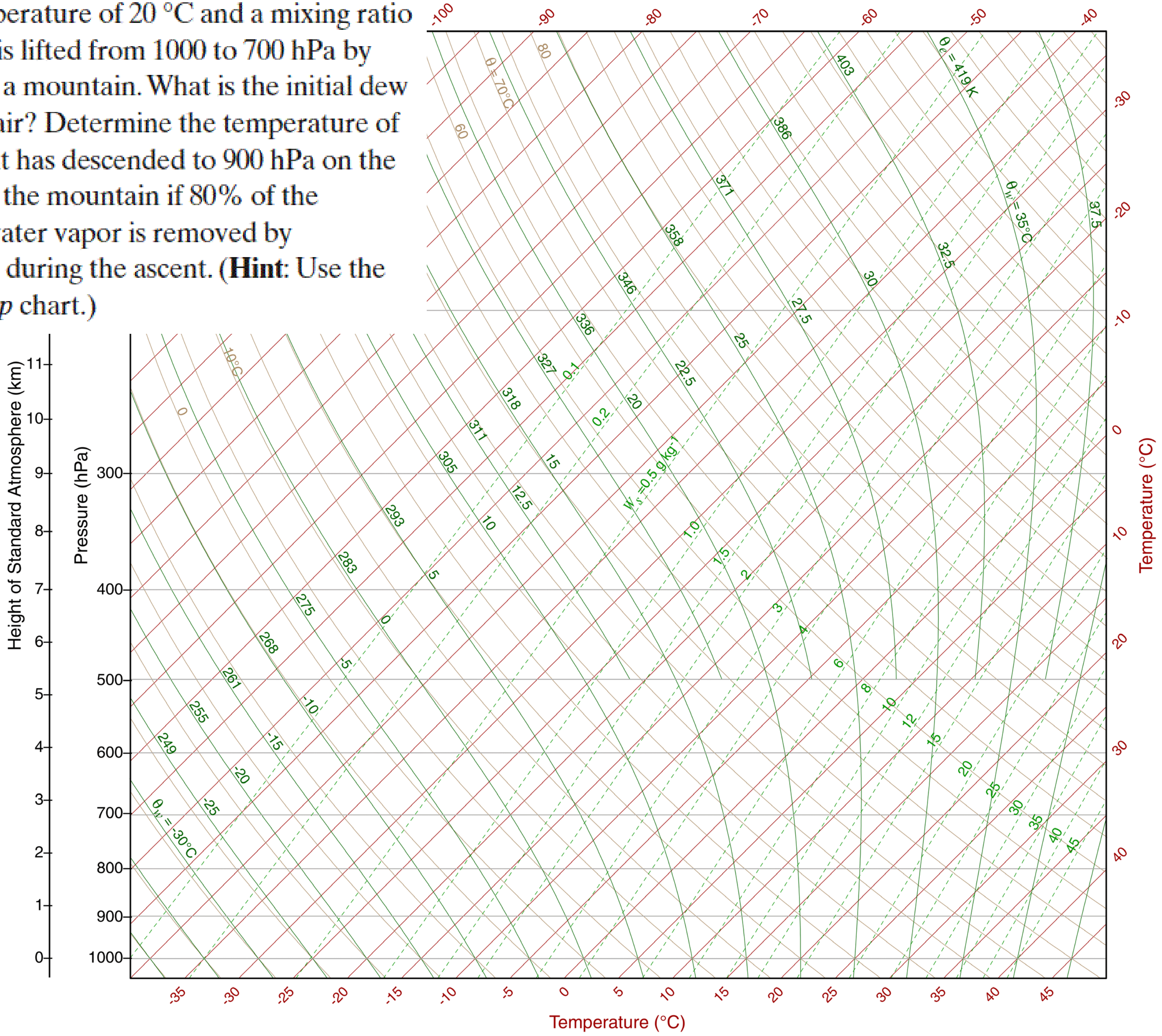


Equilibrium ≡ S = maximum for given U.
(nature abhors slacker molecules).

1. Dump in heat (random, chaotic energy).
2. Raises temperature ⇒ creates more states to hold energy.
3. $du = dq = C_v\,dT = T\,dS \rightarrow S_f = S_i + \int_{T_i}^{T_f} C_v \frac{dT}{T}$

$$S_f = S_i + C_v \ln(T_f/T_i)$$

3.48 Air at a temperature of 20 °C and a mixing ratio of 10 g kg^{-1} is lifted from 1000 to 700 hPa by moving over a mountain. What is the initial dew point of the air? Determine the temperature of the air after it has descended to 900 hPa on the other side of the mountain if 80% of the condensed water vapor is removed by precipitation during the ascent. (**Hint**: Use the skew $T - \ln p$ chart.)



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