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Fast quasi-adiabatic gas cooling: an experiment revisited

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Abstract
The well-known experiment of the rapid expansion and cooling of the air contained in a bottle is performed with a rapidly responsive, yet very cheap thermometer. The adiabatic, low temperature limit is approached quite closely and measured with our apparatus. A straightforward theoretical model for this process is also presented and discussed. Both the experimental setup and the associated theoretical interpretation of the cooling phenomenon are suited for a standard general physics course at undergraduate level.

(Some figures may appear in colour only in the online journal)

Introduction

The opening of a soda or beer produces a small cloud of condensation which is correctly used as experimental evidence for the low temperature reached by the gas exiting the bottle. It is also possible to adopt this experiment as a small-scale version of the main mechanism responsible for atmospheric cloud formation, as several textbooks and papers remark [1]. Two points, however, are of relevance here: (i) the gas cooling occurs very rapidly and (ii) the temperature can reach very low values. These aspects make an actual measure of this phenomenon a non-trivial one. At the high-school laboratory level, one should point out that a sensor (of temperature, but not only that) should satisfy three main characteristics in order to function properly. Two such aspects are always taken into consideration: the instrument has to be accurate enough and capable of providing values in the range at issue. There is a third requirement, however, which is less often stressed: the instrument needs a time responsiveness better than the fastest time change of the observed phenomenon. For example, if the measurement of ambient air temperature is of concern, since its time drift is typically in the min/h range, the thermometer used could have a time response of several minutes without affecting the measurement of the actual temperature in a significant way. Quite differently, the cooling of air expanding from a suddenly opened can or bottle occurs quite expectedly

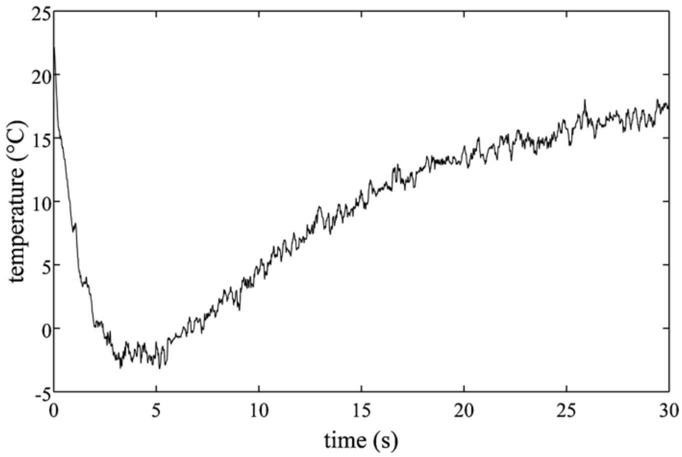

Figure 1. Temperature change during air expansion as measured by a conventional thermocouple.

and, as said, very rapidly. We thus need a correspondingly faster thermometer, i.e. with a time response shorter than the typical time evolution of the experiment. The faster, the better.

Rapid temperature measurements are often needed in research laboratories where fast gas dynamics phenomena are considered, such as in time evolution of turbulent flows like those observed in the gas exhausts of turbine engines [2] or in specific meteorological studies [3]. Fast thermometers in standard temperature ranges of interest in thermodynamics are based on thermocouples made up of very thin (a few microns) wires. Their time response can be as fast as a few milliseconds; yet they are quite expensive and difficult to use, at least in high-school or introductory college level laboratories. In a previous paper [4], we introduced, as an alternative to commercial thermocouples, a very cheap, fast device for measuring temperature in the range at issue (from about −200 to 150 °C). Our thermometer is made with the filament of a small lamp; we can obtain the environment temperature by measuring its electrical resistance after proper calibration and through a Wheatstone bridge connection. This simple instrument has a time response, in still air, of about 10 ms. Its commercial cost is basically zero (a few cents for the small lamp).

We report in the following the setup of the apparatus and the results of our measurements along with a simplified theoretical description of the most relevant experimentally observed features.

Experiment

As is well known, it is possible to give a qualitative demonstration of air cooling after its expansion. A bicycle pump can be used to raise the air pressure in a PET bottle closed with a valve. In order to make the cooling of air more evident when opening the bottle, it is convenient to add some condensation centres inside the bottle [5], such as those made available by a small quantity of smoke (an extinguishing match will serve). The quick opening of the cork will produce a pressure/temperature drop and the visible formation of a little condensation cloud from the water vapour. One might be tempted to measure the temperature by inserting a commercial thermocouple inside the bottle. We show in figure 1 the resulting temperature change. The thermocouple reads a temperature drop of about 25 °C (from 22 to −3°C) in about 3 s. As shown in our previous paper [4], the commercial thermocouple

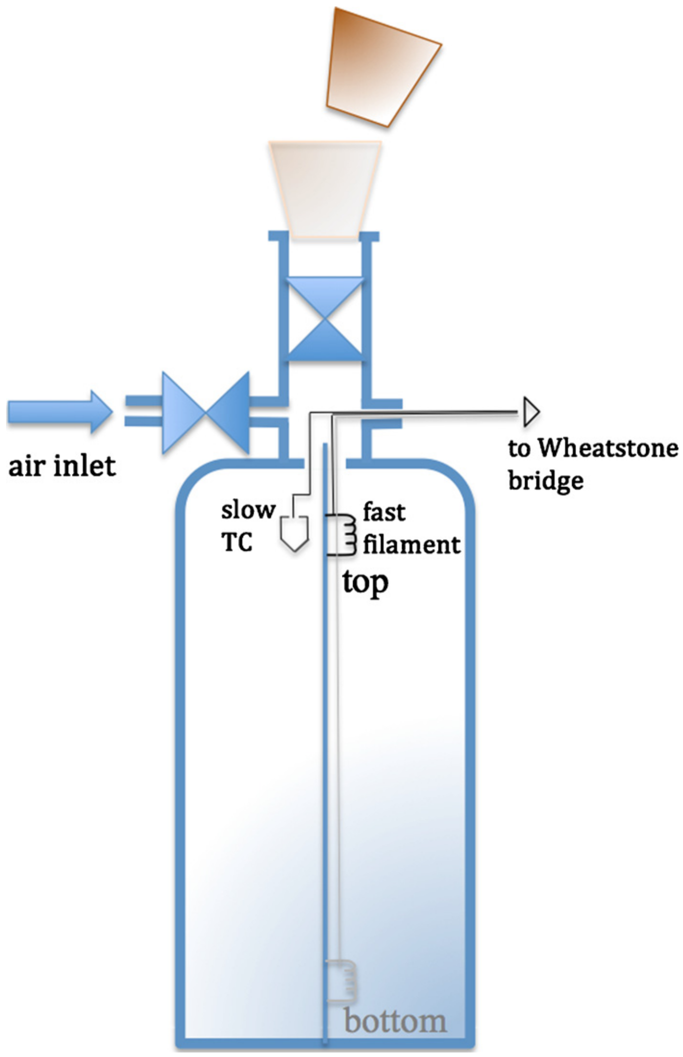

Figure 2. Schematic view of the experimental apparatus.

(K-type, chromel–alumel) has a time response of about 0.3 s in still air (we can expect some improvement—a shorter time response—by applying ventilation to the sensor [6]). In any case, phenomena much faster than these characteristic times (a few tenths of seconds) are not properly reproduced or are simply skipped. In particular, we cannot deduce from the plot of figure 1 the actual, lowest temperature of the air in the bottle, nor the time needed to reach it.

With the aim of obtaining more precise measurements, we have modified our experimental setup as shown in figures 2 and 3. The bottle (still a regular PET soda bottle) can be pressurized and evacuated through a vacuum feedthrough and a manual valve. The most important part of our experiment is based on the adoption of a filament as a temperature sensor. Details about this device can be found in our previous paper [4]. Here we recall that its time response (obtained after its heating with an electrical square impulse) is as low as 3 ms in a ventilated environment. The filament was mounted inside the bottle on a rigid, adjustable rod along with the standard thermocouple. The filament could be placed at different distances from the hole

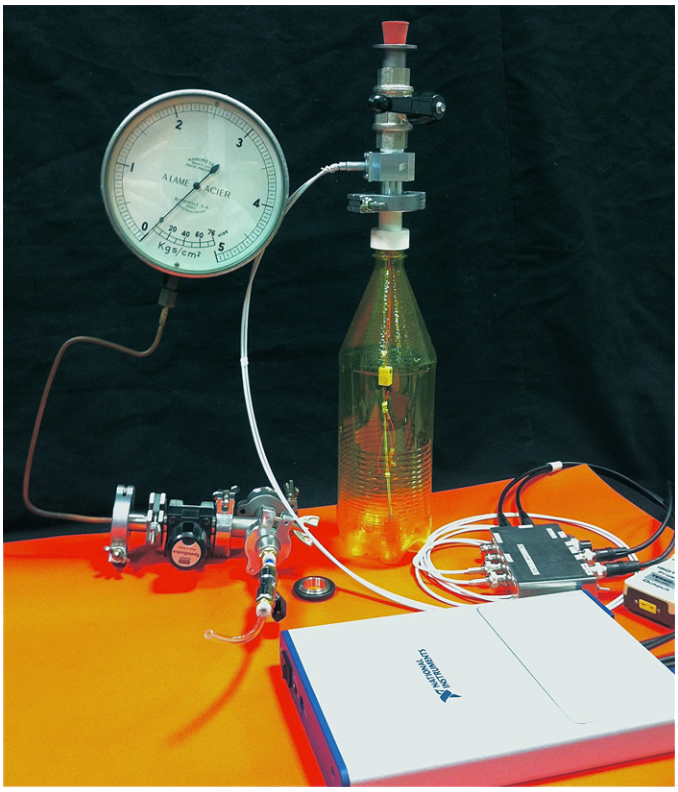

Figure 3. Experimental setup.

of the bottle. The filament was electrically connected to a Wheatstone bridge whose output was measured with an ADC card (16 bit, data taken at 100 kHz sampling rate).

This experiment, as related above, is usually exploited to describe the fast cooling of air and the condensation of water vapour included in it. Yet, in our setup we use dry air (N_2), i.e. we avoid from the very beginning the phenomenon of energy exchange between air and water vapour. This has been done since our theoretical, didactical model presented and discussed in the following section would have required a much more complex onset for including a two-phase component (gaseous and liquid) with its energy balance. Moreover, the main aim of this work is related to the study of the fast time response of our sensing unit and not to thermodynamic and gas dynamics details that are beyond levels appropriate to high-school or introductory college physics.

We show in figure 4 (blue line) a typical output of temperature versus time as obtained with the filament close to the bottle hole. The first, relevant point is that the temperature drop is very fast (less than 1/10 s) and quite deep (from 23 °C to about −40 °C, i.e. a 63 °C jump, to be compared with the 25 °C value measured with the commercial thermocouple). In figure 4, we also report a second temperature measurement obtained with the filament placed as close as possible to the bottle bottom, i.e. far from its hole (red line). We observe that the time required to reach the lowest temperature is again a very short one, but the corresponding temperature value is definitely higher than that in the previous case. We will comment more on this in the following section. Figure 4 also shows a complex behaviour of temperature as time goes by to larger and larger values. Quite close to the minimum temperature, and within a

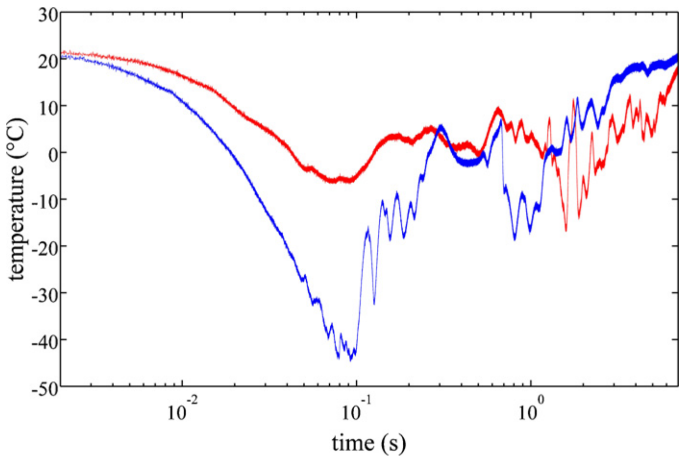

Figure 4. Temperature versus time of dry air expansion (filament in the top position, blue/black, bottom, red/grey). Note the logarithmic timescale.

few seconds, we systematically observe very fast, non-random temperature oscillations (in the range of a few hundreds of seconds) as well as slower changes (in the tenths of seconds range). We also note a much slower, average temperature rising as time reaches the several seconds range. We believe that the very fast oscillations close to the minimum temperature are caused by turbulent flow motions and gas shock-waves inside the bottle, while the overall, slower temperature increase is associated with the thermal adjustment caused by several, complex heat exchange processes between the gas inside the bottle and its walls as well as by the mixing with external air entering the bottle. In the following section, we will present a theoretical model capable of justifying the rapid temperature drop to the observed minimum value by combining two basic aspects of the gas transformation: its quick expansion (and cooling) and its simultaneous, continuous heating. We will not attempt to describe the above-mentioned, fine structures at a longer timescale for our experiment, since such treatment is far beyond the purpose of this work (and of standard undergraduate physics courses).

The notable result here is that we have experimentally obtained a clear figure of the characteristic time for the gas to cool down to its lowest temperature as a consequence of its fast expansion. Fine observable details of its irreversible, turbulent behaviour are also accessible in this experiment despite its very simple setup.

Theory

If it is true that the detailed behaviour of the air inside the bottle is an extremely complex event based on non-reversible, turbulent flows of a real gas, it is, however, possible to obtain some general information in terms of relatively simple theoretical considerations. Since these ideas make use of classical thermodynamics concepts at the introductory college physics level, we think it is worth spending some time on this.

In our experiment, as already discussed, we observe the fast, continuous temperature drop of the air in the bottle after its opening and a subsequent, much less regular temperature increase during its exposure to the surrounding media (walls of the bottle and external atmosphere).

Here we show that the first of these two trends can be described fairly well in terms of an adiabatic gas expansion through a convergent nozzle (the bottle hole). In the following discussions and computations, temperatures are obviously in kelvin. However, to allow a more intuitive understanding of the results, values are always converted into Celsius.

In a simple, stationary (and reversible) representation of the adiabatic cooling/expansion of an ideal gas, one obtains the final temperature associated with a pressure ratio (p_1/p_2) according to

$$T_1/T_2 = (p_1/p_2)^{\frac{\gamma-1}{\gamma}}. \tag{1}$$

In the case of our experiment, we started with air in the bottle pressurized to the value $p_1 = p_0 = 4 \times 10^5$ Pa at the initial temperature $T_1 = T_0 = 23$ °C. So, we readily obtain, for the depressurization to $p_2 = p_{\text{fin}} = p_{\text{atm}} = 10^5$ Pa, $T_2 = T_{\text{fin}} = T_0(p_{\text{atm}}/p_0)^{2/7} = -74$ °C, in which we use $\gamma = c_P/c_V = 7/5$ for a bi-atomic ideal gas. In other words, the gas, being adiabatically expanded to the atmospheric pressure, undergoes a temperature drop of 97 °C (from about 23 to −74 °C). This result, which is the ideal limit for a full adiabatic, reversible transformation, does not explain the time evolution of our experiment, i.e. does not provide any information concerning the rapidity with which the temperature drops to a new value after the gas expansion to the atmospheric pressure.

So, we consider the specific way in which the gas leaves the bottle, i.e. through the hole (nozzle) which is almost instantaneously opened in our experiment. We show in the following that one can adopt the model of an isoentropic flow (i.e. adiabatic, stationary, non-dissipative) of a compressible, ideal fluid [7]. Energy dissipation can be left out if the flow is characterized by relatively large Reynolds numbers, so that viscous forces are negligible in comparison to other interactions. Under these conditions, one can adopt, as a starting, pilot equation, the Bernoulli relation for unit mass,

$$\frac{u^2}{2} + h = \text{const}, \tag{2}$$

in which we omit volume forces (such as gravitational effects) and introduce the specific enthalpy, h. Equation (2) is computed along a given streamline of the fluid with speed u.

Before considering the problem at issue, i.e. the efflux of gas through the nozzle, we modify the Bernoulli equation by choosing, as a specific coordinate for our computation, the stagnation point of the gas, which is characterized by the condition $u = u_0 = 0$. If one writes the specific enthalpy h as $h = c_P T$, and by referring to stagnation point values with the subscript ‘0’, equation (2) can be manipulated to give

$$u = \left[2c_P T_0\left(1 - \frac{T}{T_0}\right)\right]^{1/2}. \tag{3}$$

Introducing the speed of sound in the gas $a = \sqrt{\gamma RT}$ (here R is the gas constant referred to the molar unit) and the adiabatic relation between temperature and pressure, equation (1), one obtains

$$u = a_0\left\{\frac{2}{\gamma - 1}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}, \tag{4}$$

which is also known as the Saint-Venant and Wentzel equation [8]. This expression allows us to compute the speed of gas at pressure p starting from the speed of sound $a_0 = \sqrt{\gamma RT_0}$ and the gas pressure both under stagnation conditions $(u = u_0 = 0, p_0)$.

It is possible to exploit equation (4) to compute the mass flow through the nozzle with area A by simply developing the basic definition:

$$\frac{dm}{dt} = \rho u A, \tag{5}$$

in which we manipulate the density $\rho = \rho_0(\rho/\rho_0) = \frac{\gamma p_0}{a_0^2}\left(\frac{p}{p_0}\right)^{1/\gamma}$ to write

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{p_0}{a_0}A_{\mathrm{eff}}. \tag{6}$$

In this expression, we introduce the effective nozzle area A_{eff}, which is given by

$$A_{\mathrm{eff}} = A\left\{\frac{2\gamma^2}{\gamma-1}\left(\frac{p}{p_0}\right)^{2/\gamma}\left[1-\left(\frac{p}{p_0}\right)^{(\gamma-1)/\gamma}\right]\right\}^{1/2}. \tag{7}$$

It is customary to introduce the ‘efflux coefficient’ defined as

$$\psi = \psi(p, p_0) = \left\{\frac{2\gamma^2}{\gamma-1}\left(\frac{p}{p_0}\right)^{2/\gamma}\left[1-\left(\frac{p}{p_0}\right)^{(\gamma-1)/\gamma}\right]\right\}^{1/2} \tag{8}$$

in such a way that equation (7) becomes $A_{\mathrm{eff}} = A\psi$ [9]. A very important result of gas dynamics studies is that the efflux coefficient, despite its variability as a function of local thermodynamic properties of the fluid, attains its maximum value for the so-called critical conditions of speed, temperature and pressure (denoted as u^*, p^* and T^*). Such conditions correspond to the sonic speed of gas, i.e. when $u = u^* = a$ (the speed of gas equals the speed of sound under the same conditions), that is, $M = M^* = 1$, where $M = u/a$ is the Mach number. We thus obtain that under critical conditions the efflux coefficient is simply given by

$$\psi^* = \psi_{\mathrm{max}} = \gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}, \tag{9}$$

which, for an ideal bi-atomic gas, is $\psi^* = 0.82$.

We will now make use of this result in considering the outflow of air from the bottle. This can be done since, for pressure ranges similar to those of our experiment, the speed of gas in the convergent nozzle is a fraction (about 0.6–0.8) of the speed of sound. So, the efflux coefficient is close to its maximum value and we write, in place of equation (7), $A_{\mathrm{eff}} = A\psi^*$.

In order to apply the above results to our case study, we consider a bottle whose volume V_B is much larger than the volume V_N of the exit nozzle. In our model, we take the air in the bottle under stationary stagnation conditions, i.e. its pressure, density and temperature change with time, but the speed of fluid is negligibly small. (This can be justified since the fluid acquires speed almost entirely in the exit nozzle and not in the bottle volume.) So, the varying, stagnation coordinates of the air in the bottle are $p_0(t)$, $\rho_0(t)$ and $T_0(t)$.

The conservation of air mass in the bottle/nozzle system ($m_B + m_N =$ constant) can now be explicitly written as

$$0 = \frac{\mathrm{d}m}{\mathrm{d}t} = \frac{\mathrm{d}m_B}{\mathrm{d}t} + \frac{\mathrm{d}m_N}{\mathrm{d}t} = V_B\frac{\mathrm{d}\rho_0}{\mathrm{d}t} + \frac{p_0}{a_0}A_{\mathrm{eff}}. \tag{10}$$

We modify this last equation by explicitly introducing p_0 and T_0. We thus obtain

$$\frac{V_B}{\sqrt{R}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_0}{T_0}\right) + \frac{A_{\mathrm{eff}}}{\sqrt{\gamma}}\frac{p_0}{\sqrt{T_0}} = 0. \tag{11}$$

If we describe the air outflow according to an adiabatic transformation, once again p_0 and T_0 are related to each other by means of the equation $T_0/T_{0i} = (p_0/p_{0i})^{\frac{\gamma-1}{\gamma}}$, where we introduce the initial ($t = 0$) stagnation values for temperature/pressure, $T_{0i} = T_0(t = 0)$, $p_{0i} = p_0(t = 0)$.

The differential equation (11), in the adiabatic hypothesis, is analytically solved to provide the following expressions for the stagnation pressure and temperature time dependences:

$$p_0(t) = p_{0i}\left(1 + \frac{\gamma-1}{2}\frac{t}{\tau}\right)^{\frac{2\gamma}{1-\gamma}}, \tag{12}$$

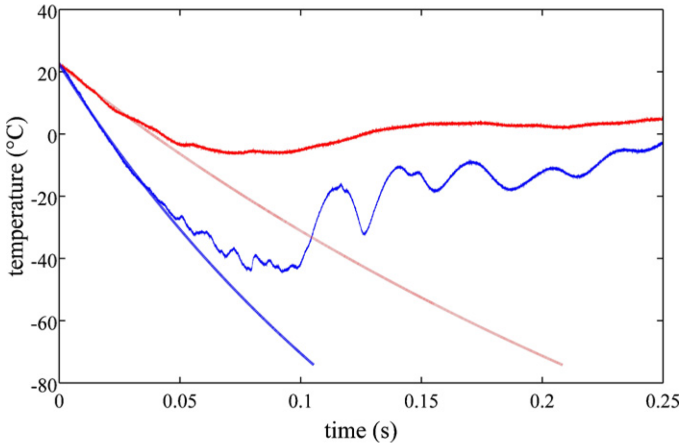

Figure 5. Comparison between computed adiabatic temperature drop of dry air (thin lines) and observed data (thick lines) for the two filament positions (top, blue/black; bottom, red/grey). Note the linear timescale.

$$T_0(t) = T_{0i}\left(1 + \frac{\gamma - 1}{2}\frac{t}{\tau}\right)^{-2} \tag{13}$$

in which we introduce the characteristic time

$$\tau = \frac{V_B}{A_{\mathrm{eff}}}\sqrt{\frac{\gamma}{RT_{0i}}}. \tag{14}$$

Since the final value of the gas stagnation pressure is p_{atm}, a known value, we can use equation (12) to determine the time needed for the gas to reach such a final state (whose temperature, as already discussed, is $T_{\mathrm{fin}} = T_0(p_{\mathrm{atm}}/p_0)^{2/7} = -74\ ^\circ\mathrm{C}$).

By direct inversion of equation (12), we simply obtain for the time of adiabatic cooling:

$$t_{\mathrm{cool}} = \frac{2\tau}{\gamma - 1}\left[\left(\frac{p_{0i}}{p_{\mathrm{atm}}}\right)^{\frac{\gamma-1}{2\gamma}} - 1\right], \tag{15}$$

which, for our experiment ($p_{0i}/p_{\mathrm{atm}} = 4$), is equal to 1.09 τ. It is easy to obtain that, for $t = t_{\mathrm{cool}}$, equation (13) gives back the expected final temperature, T_{fin}.

The first, direct comparison between the theory and experiment is shown in figure 5 in which the adiabatic cooling curve given by equation (13) is plotted along with the measured temperature in the first few tenths of seconds after the opening of the bottle. In this same figure, we report the two cases for the sensing unit close to and far from the exit nozzle of the bottle. The theoretical curves have been adjusted by changing the effective nozzle area (A_{eff}) to determine τ in equation (14). Using a smaller, effective area of the exit hole is equivalent to assuming that, in order to reach the bottom of the bottle, the information associated with its uncorking must travel a longer, higher impedance path. The adjustment of the parameter has been done by comparing the initial slope of the cooling curves. Starting from equations (13) and (14), one obtains

$$\frac{\mathrm{d}T_0}{\mathrm{d}t}(t = 0) = \frac{1 - \gamma}{V_B}\sqrt{\frac{RT_{0i}^3}{\gamma}}A_{\mathrm{eff}}. \tag{16}$$

By inserting the proper numerical values in the expression above, and the fitted slopes of the cooling curves ($\mathrm{d}T_0/\mathrm{d}t\ (t = 0) = -646\ ^\circ\mathrm{C\ s^{-1}}$ and $-1226\ ^\circ\mathrm{C\ s^{-1}}$, at the bottom and top filament positions, respectively), we obtain the corresponding $A_{\rm eff}$ values as well as the effective, reduced nozzle radius according to

$$r_{\rm eff} = \sqrt{A/\pi} = \sqrt{A_{\rm eff}/(\pi\psi^*)}. \tag{17}$$

Its values, going again from bottom to top, are 3.2 and 4.5 mm, respectively. These values should be compared with the actual, geometric radius of the exit nozzle of the bottle used in the present experiment, which is 8 mm. These differences, as already hinted, are due to the actual behaviour of the nozzle in comparison with the ideal hole. An important point is that in the two reported measurements the minimum value of the temperature is definitely higher than the adiabatic computed value. This effect is more pronounced in the case of 'bottom' filament positioning: this is expected since here the gas has a longer time to exchange energy with the warm bottle's walls. However, our computation is in good agreement with the observed temperature drop in the first, quick phase of the gas expansion (0–0.1 s). Equations (14) and (15) provide for the cooling time 10.4 and 20.7 ms for the filament at the top and bottom positions, respectively.

The adiabatic minimum is not reached since the air in the bottle is continuously heated from the very beginning, mainly because of its contact with the warmer bottle's walls. We account for this fact with a very simple conduction/convection model for energy exchange by means of which the adiabatic behaviour will be corrected. We consider the external air as an infinite thermal capacity, energy source at fixed temperature $T_{\rm air}$ (23 °C in our experiment). The walls of the bottle are also at this same temperature and we suppose that they do not change this status as a consequence of their contact with the cold internal air. This assumption is supported by the quickness of the gas cooling and by the fact that the thermal inertia of the wall material (PET plastic) is large enough to ensure a much slower temperature adjustment. So, the only way for the internal air to acquire heat from outside is by means of convective energy transfer from the walls at constant temperature.

We consider, for the system constituted by the bottle and the nozzle, the following equation for the energy conservation:

$$\mathrm{d}(m_B U_B) + h_N dm_N = \delta Q, \tag{18}$$

in which the first term on the left-hand side represents the change of internal energy of the air in the bottle, while the second term is related to the flow of enthalpy through the nozzle. Being that air is flowing from the bottle to the nozzle, one has that $\mathrm{d}m_B = -\mathrm{d}m_N = \mathrm{d}m$. We thus can write, putting $U = U_B$ and $h = h_N = U + pV$:

$$m\mathrm{d}U + U\mathrm{d}m - h\mathrm{d}m = \mathrm{d}m(U - h) + m\mathrm{d}U = m\mathrm{d}U - pV\mathrm{d}m. \tag{19}$$

By replacing mass values with molar quantities and referring the above equations to the number of moles $n(t)$, we obtain

$$nc_V\mathrm{d}T - RT\mathrm{d}n = \delta Q. \tag{20}$$

As a final step, we consider stagnation values for the number of moles and of temperature, and instantaneous values:

$$n_0 c_V \frac{\mathrm{d}T_0}{\mathrm{d}t} - RT_0\frac{\mathrm{d}n_0}{\mathrm{d}t} = \dot{Q}. \tag{21}$$

The thermal flux of energy $\dot{Q}$ is expressed by means of the convection/conduction law for a fluid at temperature T_0 in contact with a wall (surface S) at temperature T_W, $\dot{Q} = k_c S(T_W - T_0)^z$. In this formula, k_c is the effective convection constant for the gas–wall heat transmission and the exponent z is empirically settled in the function of the (complex) characteristics of the convection geometry and dynamics [10]. The value of k_c is usually obtained from a class of typical experimental configurations. In our case, the walls are those of the bottle, with the

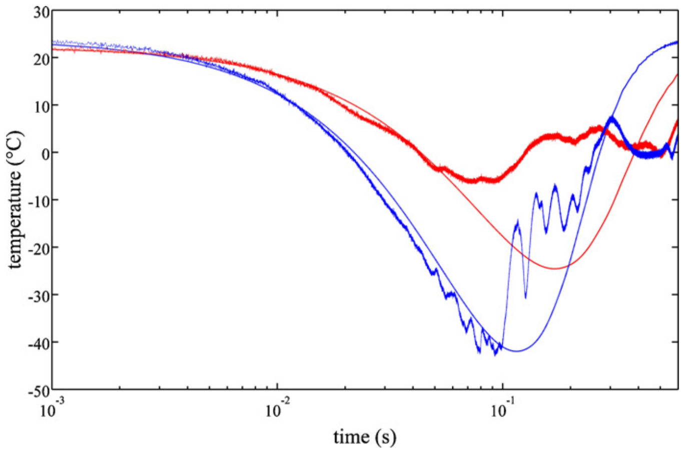

Figure 6. Comparison between observed data (thick lines) and theoretical model (equations (22) and (23), thin lines) for the two different filament locations (top, blue/black; bottom, red/grey). Note the logarithmic timescale.

surface S_B, and the temperature will be that of the outside air, as explained above. We are thus left with the energy balance differential equation relating density (number of moles) and temperature of the internal air at stagnation points:

$$n_0 c_V \frac{\mathrm{d}T_0}{\mathrm{d}t} - RT_0 \frac{\mathrm{d}n_0}{\mathrm{d}t} = k_c S_B (T_{\mathrm{air}} - T_0)^z. \tag{22}$$

The differential equation for the gas outflow, equation (11), can be written in terms of the number of moles under the stagnation conditions, n_0, in place of the associated gas pressure, p_0, with the result being

$$\frac{\mathrm{d}n_0}{\mathrm{d}t} + \frac{A_{\mathrm{eff}}}{V_B}\sqrt{\frac{R}{\gamma}} n_0 \sqrt{T_0} = 0. \tag{23}$$

This last equation, along with equation (22), constitutes the system of differential equations leading to the time dependences of relevance here, i.e. $n_0(t)$ and $p_0(t)$. A numerical approach to this problem is convenient since the nonlinear form of these equations strongly discourages any attempt to seek for analytical solutions.

So, we proceed with the numerical integration of equations (22) and (23). We show in figure 6 the results of this computation along with the observed data for the two different filament positions. The convective parameter k_c of equation (22) has been fixed to the value 70 W m^{-2} K^{-1} (according to 'recommended' figures for air convection phenomena [10]), but one could slightly change such values without affecting the overall aspect of the computed curve too much. The exponent z, according to the literature, is in the 1–1.5 range [11]. We chose, quite arbitrarily, $z = 1.3$. We observe that the overall agreement between experiment and our model is fair. There is evident discrepancy in the case of the filament located at the bottom of the bottle where, as already discussed, we expect a more complex and less predictable behaviour because of the (basically unknown) detailed dynamics of the expanding air. However, the lowest temperature reached as well as the time required to obtain this value are well described, with our model, in the 'top' configuration. The gas heating has been modelled in such a way that only the minimum reached temperature and the associated cooling time have been reproduced fairly well: the subsequent, longer timescale behaviour of temperature (along with its fine, shorter timescale details) are definitely beyond the predictive possibilities of this model.

Conclusion

Since the main aim of this work was to show the fast time responsiveness of our temperature sensor, we think that the filament is a very well-suited device in this respect. We have observed very complex, still repeatable oscillations in the few-millisecond scale of the temperature change of dry air expansion within our bottle. Moreover, the theoretical model presented here is capable of reproducing the first phases of the experiment with quite good accuracy, i.e. the dry gas cooling, with a cooling time of about 0.1 s for our setup. The minimum temperature is in the -40 to $-60\,^{\circ}$C range, showing that the pure adiabatic limit is quite far from being reached. A fair adjustment of this ideal limit is achieved by allowing energy exchange via effective convective processes with the bottle walls. The overall solution of the resulting differential equations is compatible with the observed values of minimum temperature and the associated cooling time. It could also be that exchanges of electromagnetic radiation because of the finite temperature difference between the filament and the surrounding environment would play a certain role in our experiment. We have modelled a classical $\varepsilon\sigma T^4$ energy exchange term in our computation. The result is that there is a net energy flow of electromagnetic origin such that a difference of temperature of up to 3–4 $^{\circ}$C is accounted for between the filament and the expanding gas. Such a phenomenon, however, does not modify our main conclusions, since the time within which the filament starts to warm up (following a thermal radiation process as a consequence of its lowered temperature because of the gas cooling) is of the order of 0.5 s, i.e. too long to be relevant in the first, rapid phase of our experiment.

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References

[1] Güeméz J, Fiolhais C and Fiolhais M 2007 Physics of the fire piston and the fog bottle *Eur. J. Phys.* **28** 1199–205
Graham M T 2004 Cloud formation *Phys. Teach.* **42** 301–4
Bohren C F 1987 *Clouds in a Glass of Beer* (New York: Wiley)
[2] Zukrow M J 1948 *Principles of Jet Propulsion and Gas Turbines* (New York: Wiley)
[3] Bohren C F and Albrecht B A 1998 *Atmospheric Thermodynamics* (Oxford: Oxford University Press)
[4] Calzà G, Gratton L M, López-Arias T and Oss S 2012 Very fast temperature measurement with a thin lamp filament *Phys. Educ.* **47** 334–7
[5] Coop J J 1941 A demonstration of fog production *Am. J. Phys.* **9** 242
[6] Haman K E, Makulski A and Malinowski S P 1997 A new ultrafast thermometer for airborn measurements in clouds *J. Atmos. Ocean. Technol.* **14** 217–27
[7] Batchelor G K 2005 *An Introduction to Fluid Dynamics* (Cambridge: Cambridge University Press)
[8] Sedov L I 1997 *Mechanics of Continous Media* vol 2 (Singapore: World Scientific)
[9] Ziegler F 1995 *Mechanics of Solids and Fluids* (New York: Springer)
[10] Bohren C F 2011 Why do objects cool more rapidly in water than in still air? *Phys. Teach.* **49** 550–3
[11] Çengel Y A 2008 *Introduction to Thermodynamics and Heat Transfer* (New York: McGraw-Hill)