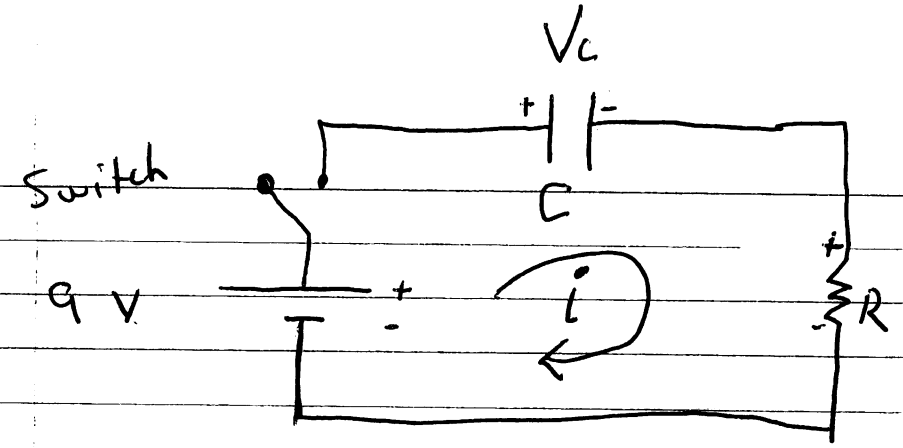
Time Constants



$C \equiv \frac{q}{V_C}$

$V_R = iR$

or

$V_R = \frac{dq}{dt} R$

$q \equiv$ Charge on capacitor

$i \equiv dq/dt$ = current $\quad V_R = iR$ (Ohm's Law)

Suppose at $t=0$ we close the switch.

What are $V_C(t) + V_R(t)$?

Kirchoff's Law: $V = V_C + V_R$

(1) $$V = \frac{q}{C} + R\frac{dq}{dt}$$

Guess a solution:

(2) $$q = q_0(1 - e^{-t/\tau})$$

(3) Note: $dq/dt = q_0/\tau \, e^{-t/\tau}$

Use (2) and (3) in (1).

$$V = \frac{q_0}{C}(1 - e^{-t/\tau}) + \frac{R q_0}{\tau} e^{-t/\tau}$$

Boundary Conditions: $t \to \infty$, $q_0 = CV$ (Capacitor is Charged)

$t \to 0$ $V = \frac{RCV}{\tau}$ $\therefore$ $\tau \equiv RC$ Time Constant

$$V_C(t) = V(1 - e^{-t/\tau})$$

$$V_R(t) = Ve^{-t/\tau}$$