Specific acoustic impedance measurements of an air-filled thermoacoustic prime mover

W. Pat Arnott,[a] Henry E. Bass, and Richard Raspet
Physical Acoustics Research Group, Department of Physics and Astronomy, University of Mississippi, University, Mississippi 38677

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Thermoacoustic heat engines can be used to produce sound from heat and to transport heat using sound. The air-filled prime mover studied is a quarter wavelength resonator that produces sound at nominally 115 Hz for a temperature difference of $\Delta T = 176$ K. Specific acoustic impedance at the mouth of the prime mover was measured as a function of the temperature difference between the hot and cold heat exchangers. The real part of the impedance changes sign for sufficiently large temperature differences, indicating the possibility of sound production. The theoretically predicted radiation impedance of an open pipe was compared to the measured impedance curves. The operating point was confirmed from the intersection of these experimental and theoretical impedance curves. These measurements allow for analysis of the prime mover as a sound source as discussed in a recent theoretical paper [T. B. Gabrielson, J. Acoust. Soc. Am. **90**, 2628–2636 (1991)].

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INTRODUCTION

Thermoacoustic engines are used to transport heat using sound and to produce sound from heat.[1] The latter application is the focus of this letter. Thermoacoustic sources are also known as prime movers by analogy to heat engines in thermodynamics.[1] Recent work has considered use of thermoacoustic prime movers as underwater sound sources.[2,3] Applications to both gas and liquid-filled prime movers were investigated. A schematic diagram of a prime mover is given in Fig. 1. Starting at the top in Fig. 1, the essential elements are a section of a resonator with a cap on the end to establish a velocity node; a hot heat exchanger, indicated by vertical lines, to inject heat; a heat ionsulating section, shown as the squares region, known as the stack which supports the temperature gradient between the heat exchangers; and a cold heat exchanger to remove excess heat. When the hot heat exchanger faces a velocity node (or pressure antinode), acoustic power can be produced in the stack for sufficiently large temperature gradients. The radiation impedance at the mouth and the length of the resonator determine the conditions for impedance matching the open resonator section and the prime mover.[3] The general utility of the impedance framework was recently developed[4] for the linear theory of thermoacoustics.[5] The utility of impedance measurements for analyzing prime movers is demonstrated experimentally in this letter.

I. PRIME MOVER IMPEDANCE MEASUREMENTS

The experimental arrangement used is shown in Fig. 1. A prime mover was mounted vertically and an impedance tube was attached at the bottom via a flange. The prime mover was built originally for use as a demonstration device and for practice in fabricating the elements. A section of resonator made of copper pipe of length 20.96 cm and inner radius 4.32 cm, and capped at the top, was the first element. High-temperature heat tape was wrapped around the entire section followed by a 1-in. layer of heat insulating material. Heat was transported by conduction to next element which was the hot heat exchanger. A type K thermocouple was placed inside of the hot exchanger to monitor the temperature. The stack (or thermoacoustic engine) that supports the temperature gradient was the next element and will be discussed below. Heat was removed at the cold exchanger by water from the lab sink. Another section of resonator, which was wrapped with water-circulating tubing, was the next element. This section was 44.45 cm long with a radius of 4.32 cm.

The heat exchangers were made by laminating with epoxy copper sheets spaced by aluminum sheets. The copper–aluminum laminate was then turned to a cylindrical shape using a lathe. The cylindrical boundary was cladded with a shell of copper about 2.5 mm thick by using an electroplating technique. The heat exchanger was then machined into a disk form and inserted into a flanged holder for attachment to the other elements. Then the aluminum was etched away using a diluted hydrochloric acid solution. The resulting heat exchanger was made entirely of copper with plate-to-plate spacing of the copper strips equal to 1.65 mm. The cold heat exchanger flange included an open tank for water circulation around the periphery of the plates. The hot and cold exchangers were 1.638 and 1.610 cm long, and had open-to-total volume ratios of $\Omega = 0.74$.

The stack was a ceramic cylindrical sample of a monolithic catalyst support.[6] Reference 7 describes the analysis of some acoustic properties of the catalyst supports. It is a section of a porous medium in which the open pores have square

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[a] Current address: Atmospheric Sciences Center, Desert Research Institute, P.O. Box 60220, Sage Bldg., Reno, NV 89506-0220.

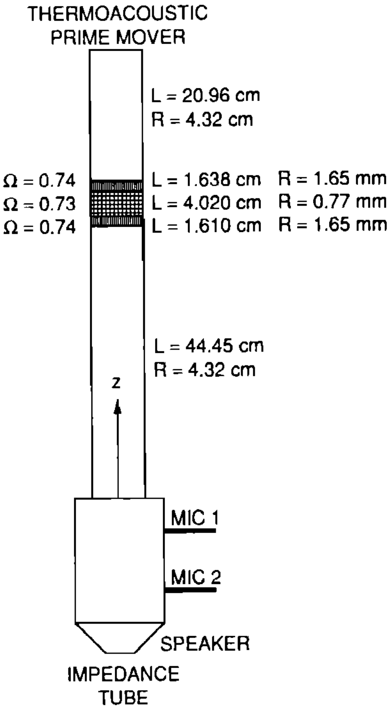

FIG. 1. Air-filled thermoacoustic prime mover ($z > 0$) and impedance tube. Lengths of each thermoacoustic element are given by L, R is the characteristic transverse dimension, and Ω is the porosity. Microphones were separated by 10 cm and were 5 cm from the speaker and prime mover. Impedance tube inner radius was 7.3 cm.

boundaries of semi-width 0.77 mm, and are straight tubes in the z direction (Fig. 1). The ceramic sample had a radius of 7.3 cm. To attach it to the heat exchangers, a ring of inner radius 4.32 cm, outer radius 7.3 cm, and depth 3.2 mm was removed from the ceramic sample. This left a protruding central portion. Copper rings of thickness 3.2 mm, inner radius 4.32 cm, and outer radius of 12 cm were supported between the ends of the ceramic piece using threaded rod stand-offs. Holes were drilled in the copper disks to match the heat exchanger flange holes.

An impedance tube with an inner radius of 7.3 cm was attached at the bottom of the prime mover. Microphones were placed 5 cm from the bottom of the prime mover and the speaker below (10-cm separation). The impedance tube[8] is generally used to determine the specific acoustic impedance (or pressure divided by particle velocity) at $z = 0$ in Fig. 1. Denote by P_1, V_1, and v_1 the pressure, volume velocity, and particle velocity at $z = 0$ in the impedance tube of cross-sectional area $A_1 = 167.5\ \text{cm}^2$. Denote by subscript 2 the corresponding quantities for the prime mover. Assuming conservation of pressure and volume velocity at the interface, $P_1/V_1 = P_2/V_2$. The quantity measured using the impedance tube was $Z_1 = P_1/v_1 = A_1P_1/V_1 = A_1P_2/V_2 = A_1 P_2/(v_2A_2)$. The desired quantity $Z_2 = P_2/v_2$ was thus determined from $Z_2 = Z_1A_2/A_1$. Neglect of interfacial effects of the impedance tube to prime mover radius discontinuity is a low-frequency approximation. The prime mover was evaluated using swept sine wave analysis at sufficiently low amplitudes that negligible heat was transported thermoacoustically. The measured impedance can become a function of the amplitude of driving pressure signal at high levels due to the alteration of the static temperature gradient by thermoacoustic streaming.[9]

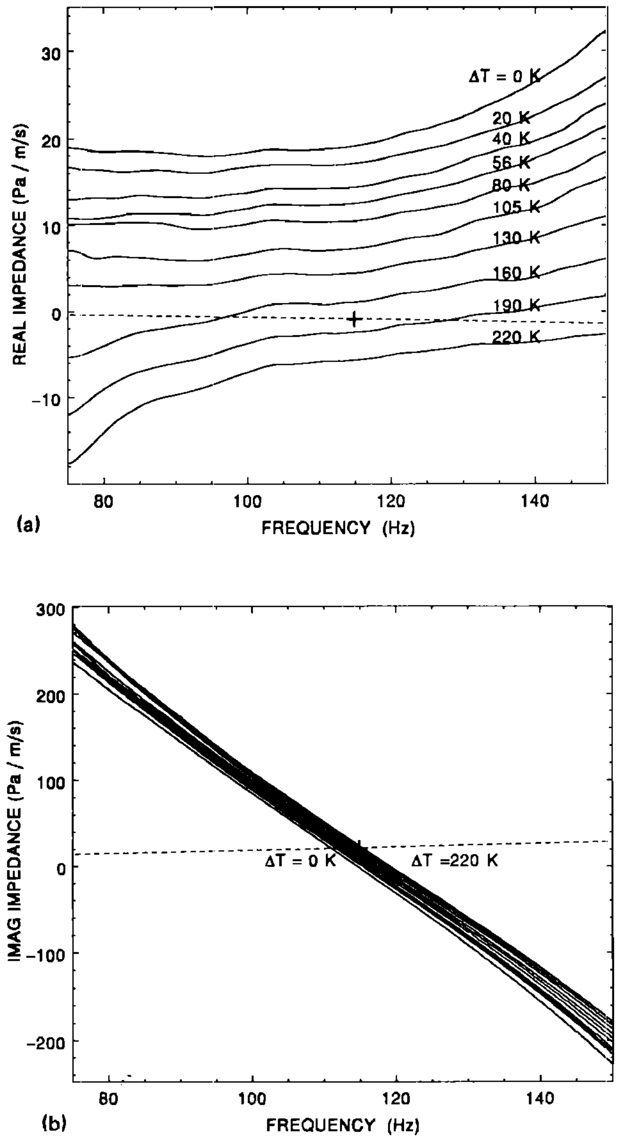

FIG. 2. (a) Real and (b) imaginary parts of the measured specific acoustic impedance (solid lines) as a function of the externally applied temperature difference. The dashed line is the theoretical radiation impedance for the prime mover mouth. The plus symbol locates the operating point of the prime mover at the onset of sound production at 115 Hz and $\Delta T = 176$ K. The ambient temperature was 296 K.

The real and imaginary parts of the measured specific acoustic impedance as a function of the temperature difference are shown in Fig. 2(a) and (b). The real part becomes negative at some frequencies, indicating the possibility of having an active system with reflection coefficients greater than one.[10] When the impedance tube is removed, which of course changes the prime mover termination impedance, sound at a nominal frequency of 115 Hz is produced for $\Delta T \geqslant 176$ K. The expression for the specific acoustic radiation impedance[11] at the mouth of the prime mover is $Z_{\text{rad}}(\omega) = -\rho_0 c[(k_0R/2)^2 - i\,0.6k_0R]$, where $k_0 = \omega/c$,

ω is the radian frequency, ρ_0 is the ambient air density, c is the adiabatic sound speed of air, and $R = 4.32$ cm is the tube radius. The minus sign occurs because of our choice for the positive direction of z in the coordinated system on Fig. 1. Radiation impedance is represented by the dashed lines in Fig. 2(a) and (b). One immediate check of the measurements is that the initial operating point (115 Hz, $\Delta T = 176$ K) given by the plus symbols occurs, for both the real and imaginary parts of the radiation impedance, at the intersection of the calculated and measured impedance values.

II. CONCLUSION

Specific acoustic impedance measurements were made as a function of the temperature gradient across the stack. Among other uses, these measurements are helpful for evaluating the possibility of using the prime mover as a sound source. Another interpretation of Fig. 2 is, for example, that the plane wave reflection coefficient at 80 Hz and $\Delta T = 160$ K is > 1 for waves incident in an infinite length tube of the same diameter as the prime mover but in the location of the impedance tube in Fig. 1. For prime movers far above the onset of sound production, or for strongly driven thermoacoustic refrigerators, the temperature distribution from hot to cold is not simply the static distribution established by the thermal conductivity of the gas and stack. The presence of the strong acoustic wave influences its thermal surroundings[9] by heat transport and in this sense the thermoacoustic oscillation is an example of a self-interacting wave process.

ACKNOWLEDGMENTS

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