Estimation of temperature gradient effects on the normalized surface impedance of soils

Richard Raspet, James M. Sabatier, and W. Patrick Arnott[a)]
Department of Physics and Astronomy and National Center for Physical Acoustics, University of Mississippi, University, Mississippi 38677

(Received 13 November 1995; revised 14 August 1996; accepted 26 August 1996)

Ground impedance measurements are used for sound propagation predictions and to determine soil properties. Solar heating of the ground leads to significant temperature swings and gradients in the near surface soil. The equations of thermoacoustics are applied to estimate the magnitude of the temperature effects on the impedance and develop an approximate equation for the adjustment of measured impedance. Ambient temperature effects are shown to be significant; temperature gradient effects in soils are negligible. The theory is applicable to noise control applications where larger gradients may occur in sound absorbing materials. © *1997 Acoustical Society of America.* [S0001-4966(97)03912-X]

PACS numbers: 43.28.Fp, 43.55.Ev [LCS]

INTRODUCTION

Measurement of the surface impedance of the ground is important for the prediction of sound propagation over the ground.[1] Surface impedance measurements are also used to determine soil properties of agricultural grounds.[2,3]

Soils outdoors undergo wide temperature variations. The temperature profiles in the ground are governed by the heat input to the surface and the thermal properties of the soil. In many cases the temperature profiles are approximately linear down to the damping depth d in the soil (the $1/e$ length for the daily cycle). Surface temperatures can vary by as much as 30 K in a day.[4] Damping depths depend on the soil type and moisture content but generally range[5] from 7 to 15 cm.

Temperature gradients can have large effects on sound propagation in the boundary layer of solid surfaces. In resonance tubes, large temperature gradients can produce amplification of sound waves. The study of this effect is called ''thermoacoustics.''[6] Arnott *et al.*[7] explicitly demonstrated the connection between the literature and notation of sound propagation through porous media to thermoacoustics. In a subsequent paper, Arnott *et al.* measured the changes in impedance of a thermoacoustic stack as the temperature gradient was varied.[8] Significant, measurable changes in the impedance occurred for a 20-K change in temperature across a 4.0-cm-long porous stack.

In this paper, the theory of thermoacoustics is employed to investigate the possible effects of temperature and temperature gradients on the normalized surface impedance of soils. Section I presents the thermoacoustics equations and describes the adaptation of these equations to calculate the surface impedance. Section II develops an ambient temperature normalization factor for the fitted flow resistivity, then develops a calculation of the effect of temperature gradients on the surface impedance of ordinary soils. Section III develops an approximation for the effect of temperature gradients on ground impedance. Section IV contains a discussion of the results and conclusions.

I. APPLICATION OF THE THERMOACOUSTIC EQUATIONS TO GROUND IMPEDANCE

A thorough review of thermoacoustic research is contained in a paper by Swift.[6] Arnott *et al.*[7] demonstrated that thermoacoustics could be formulated in terms of previous porous media research. The notation of Ref. 7 will be followed in this paper.

Temperature gradients in porous materials have two principal effects; the gas properties are functions of temperature and the complex compressibility is modified due to the change in temperature of the porous media with respect to the gas as the gas is displaced by the sound wave. The relevant equations from Ref. 7 give a second-order differential equation for the complex acoustic pressure amplitude in a pore:

$$\frac{\rho(z)}{F(\lambda)}\frac{d}{dz}\left(\frac{F(\lambda)}{\rho(z)}\frac{d\hat{p}(z)}{dz}\right)+2\alpha(\lambda,\lambda_T)\frac{d\hat{p}(z)}{dz}+k_0^2(\lambda,\lambda_T)\hat{p}(z)=0, \tag{1}$$

where

$$\alpha(\lambda,\lambda_T)=\beta\frac{dT}{dz}\frac{1}{2}\left(\frac{F(\lambda_T)/F(\lambda)-1}{1-N_{\mathrm{pr}}}\right), \tag{2}$$

and

$$k_0^2(\lambda,\lambda_T)=\frac{\omega^2}{c^2}\frac{1}{F(\lambda)}[\gamma-(\gamma-1)F(\lambda_T)]; \tag{3}$$

and the complex average velocity amplitude in the pore in terms of the pressure amplitude gradient is

$$\hat{v}(z)=\frac{F(\lambda)}{i\omega\rho(z)}\frac{d\hat{p}(z)}{dz}. \tag{4}$$

Here, $\alpha(\lambda,\lambda_T)$ is a complex damping or gain parameter and $k_0(\lambda,\lambda_T)$ is the complex acoustic wave number. For an ideal gas $\beta=1/T$; this is appropriate for air filled pores. The pores

[a)]W. Patrick Arnott is at the Desert Research Institute, Atmospheric Sciences Center, P.O. Box 60220, Reno, NV 89506, and is an Adjunct Assistant Professor in the Department of Physics and Astronomy, University of Mississippi.

are assumed to have a circular cross section of radius R so that the shear wave number (λ) and thermal wave number (λ_T) are given by

$$\lambda = R\sqrt{\rho\omega/\eta}, \quad \lambda_T = R\sqrt{\rho\omega c_p/\kappa}, \tag{5}$$

where $\rho(z)$ is the gas density, ω the radial frequency of the sound wave, η the viscosity, κ the thermal conductivity, and c_p the heat capacity at constant pressure. Here, λ_T and λ are related by the square root of the Prandtl number $N_{\rm pr}$, $\lambda_T = \sqrt{N_{\rm pr}}\lambda$. The thermoviscous function for circular pores is

$$F(\lambda) = 1 - (2/\sqrt{i}\lambda)[J_1(\sqrt{i}\lambda)/J_0(\sqrt{i}\lambda)]. \tag{6}$$

Figure 5 of Ref. 7 shows that the choice of pore shape makes little gross difference in the value of $F(\lambda)$. These small differences are significant when optimizing the performance of thermoacoustic devices, but should be negligible in estimates of temperature effects on ground impedance. The relative independence of the wave number and impedance of uniform porous materials on pore shape factor is discussed in detail by Stinson and Champoux.[9]

The temperature dependence of ρ, λ, λ_T, and speed of sound, c, in terms of reference values at T_0 are[10]

$$\rho = \rho_0 T_0/T, \quad c = c_0(T/T_0)^{0.5}, \tag{7a}$$

$$\lambda = \lambda_0(T_0/T)^{0.9245}, \quad \lambda_T = \lambda_{T_0}(T_0/T)^{0.9245}. \tag{7b}$$

Values (7a) are directly from Ref. 10, pages 29–30, and (7b) are from a power law fit to Eq. (10-1.16a) of Ref. 10. The fact that the Prandtl number for air is approximately independent of temperature was also utilized.

In thermoacoustic studies where the temperature gradients can be as large as 2000 K/m, Eqs. (1) and (4) are usually solved by numerical integration. Atchley *et al.*[11] achieved good agreement with data by solving Eq. (1) as a wave equation with constant $F(\lambda)/\rho$, $\alpha(\lambda,\lambda_T)$, and $k_0(\lambda,\lambda_T)$ evaluated at the center of the stack. The change in the material property $F(\lambda)/\rho(z)$ with depth is large for the small pores typical of the ground, so the method of Ref. 11 is extended. Equation (1) can be rewritten for ideal gases as

$$\frac{d^2\hat{p}'(z)}{dz^2} + 2\alpha'(\lambda,\lambda_T)\frac{d\hat{p}(z)}{dz} + k_0^2(\lambda,\lambda_T)\hat{p}(z) = 0, \tag{8}$$

where

$$\alpha'(\lambda,\lambda_T) = \frac{1}{F(\lambda)}\left\{\frac{dF(\lambda)}{d\lambda}\frac{d\lambda}{dT} + \frac{1}{T}\left[\frac{F(\lambda_T) - N_{\rm pr}F(\lambda)}{(1-N_{\rm pr})F(\lambda)}\right]\right\}\frac{dT}{dz}, \tag{9}$$

includes the effect of the derivative of $F(\lambda)/\rho(z)$.

In this paper, soil is modeled as a homogenous semi-infinite half-space with an imposed linear temperature gradient to the damping depth d. Although the daily and yearly soil temperature variation is quite complicated, the temperature profiles can often be approximated by a linear gradient to fixed depth d. The solution to Eq. (8) is then approximated by

$$\hat{p}(z) = \hat{A}e^{ikz}, \tag{10}$$

where $\hat{A}$ is a complex constant and k is a complex constant evaluated at the surface. Substitution of this form into Eq. (8) yields

$$-k^2 + ik2\alpha'(\lambda,\lambda_T) + k_0^2(\lambda,\lambda_T) = 0 \tag{11}$$

with solutions

$$k^+ = i\alpha' + \sqrt{k_0^2 - \alpha'^2}, \quad k^- = i\alpha' - \sqrt{k_0^2 - \alpha'^2}. \tag{12}$$

Here, α' and k_0 are the gain parameter and complex wave number evaluated at the surface.

The treatment herein differs from Ref. 11 in three ways:

(i) The complex roots are evaluated from Eq. (11) directly yielding unequal wave numbers, k^+ and k^-.

(ii) The wave numbers are evaluated at the surface, not at $d/2$, since the surface impedance is most affected by surface properties for sound absorbing soils.

(iii) The effect of the changing material property $F(\lambda)/\rho(z)$ is included in Eqs. (8) and (9).

If the temperature gradient is zero, the treatment above recovers the usual porous media wave numbers. The velocity contributions corresponding to the two wave numbers can be calculated from Eq. (4):

$$\hat{v}^{\pm}(z) = [F(\lambda)/\omega\rho(z)]k^{\pm}\hat{A}e^{ik^{\pm}z}. \tag{13}$$

The boundary condition at depth d is that the surface impedance is that of a uniform semi-infinite porous media. To account for additional pore length due to the pores not being normal to the surface or not being straight, we introduce the tortuosity factor q, which yields a pore length of qd at depth d. This reduces the temperature and pressure gradients by $1/q$. Impedance matching then determines the ratios of amplitudes $\hat{A}^-/\hat{A}^+$ within the gradient layer:

$$\frac{i\omega\rho(d)}{F(\lambda)}\frac{[e^{ik^+qd} + \hat{A}^-/\hat{A}^+e^{ik^-qd}]}{[ik^+e^{ik^+qd} + \hat{A}^-/\hat{A}^+ik^-e^{ik^-qd}]} = \frac{\omega\rho(d)}{F(\lambda)k(d)} \equiv Z(d), \tag{14}$$

where both $F(\lambda)$ are evaluated at damping depth d. The solution to Eq. (14) is

$$\frac{\hat{A}^-}{\hat{A}^+} = e^{i(k^+-k^-)qd}\frac{[1-k^+/k(d)]}{[1-k^-/k(d)]}. \tag{15}$$

The surface impedance of the media is then the ratio of $\hat{p}(0)/\hat{v}(0)$ modified to account for the porosity and tortuosity of the pores. Porosity, Ω, is the ratio of open pore area to total area. The tortuosity introduces an additional factor of q to the surface impedance. With these average medium modifications, the normalized surface impedance is given by

$$Z(0) = \frac{\omega q}{c(0)F(\lambda)\Omega}\frac{(1+\hat{A}^-/\hat{A}^+)}{[k^+ + (\hat{A}^-/\hat{A}^+)k^-]}. \tag{16}$$

Here, $F(\lambda)$ is evaluated at the surface.

Equation (16) is derived for cylindrical pores with tortuosity q and porosity Ω. In ground impedance studies[9,12,13] it is usual to approximate $F(\lambda)$ and to evaluate the impedance in terms of the dc flow resistivity:

$$\sigma = 8q^2\omega\rho/\Omega\lambda^2. \tag{17}$$

TABLE I. Porous media properties.

	Pore radius (R)	Porosity (Ω)	Tortuosity factor (q)	dc flow resistivity at 293 K
Low	9×10^{-5} m	0.5	1.4	72×10^{3} N s m^{-4}
Medium	6×10^{-5} m	0.3	1.4	270×10^{3} N s m^{-4}
High	4×10^{-5} m	0.3	1.4	600×10^{3} N s m^{-4}

II. PREDICTION OF THE EFFECT OF TEMPERATURE GRADIENTS ON NORMALIZED GROUND IMPEDANCE

First, the effect of a uniform temperature change on the normalized surface impedance should be understood. For a uniform temperature the gradient terms in Eq. (1) drop out. The only temperature variation in the formula for the normalized surface impedance is in the gas properties contained in λ and λ_T. Note that the measured flow resistance must be adjusted by $\sigma=\sigma(T_0)(T/T_0)^{0.849}$ as developed from Eq. (17) with Eqs. (7a) and (7b), and c by Eq. (7a) to predict the temperature dependence of Eq. (16). A temperature increase leads to an increase in flow resistivity. A 30-K temperature difference over a year leads to a 10% change in flow resistivity. Surface impedance data of homogeneous soils taken over a wide range of surface temperatures should be normalized to a standard surface temperature.

Next, the dependence of impedance on the temperature gradient in the soil is investigated. The depth dependence of the temperature in the ground as the surface is heated by the sun is well understood.[5] Although the form of the variation is quite complex, the temperature profile near the surface can be approximated by a linear gradient down to the daily damping depth d which is on the order of 7–15 cm. The total daily temperature variation may be as large as 20 K over this distance for cultivated soils. As an estimate of the maximum effect, the impedance for three media corresponding to low, medium, and high impedance soils for a temperature change of 20 K in 10 cm was calculated using Eq. (16). The properties of the soil models are listed in Table I.

Figure 1 displays the dependence of the normalized ground impedance on the temperature gradient for low flow resistivity ground. The imaginary part of the impedance in Fig. 1 is multiplied by −1 for display purposes. The surface temperature is 300 K for all cases. A 200-K/m gradient corresponds to 300 K at the surface and 320 K at 10-cm depth. The largest effect is on the imaginary part of the impedance at very low frequencies. The maximum change in impedance due to the temperature gradient is on the order of 6%. The effects of realistic temperature gradients on the impedance of ordinary soils is negligible. The effect for the medium and high impedance soils is smaller than that displayed in Fig. 1.

Also shown in Fig. 1 is the effect of depth of the gradient layer on the impedance. Doubling the layer depth to 20 cm with the same gradient leads to a small increase in the imaginary part at very low frequencies, but it should be noted that a 40-K total change between the surface and the bottom of the layer is unrealistic. The small change in impedance when the depth is doubled indicates that a semi-infinite approximation may be used for analytic analysis of temperature gradient effects on impedance. In addition, the similarity of the impedance change to the effect of exponentially varying porosity[14] leads one to suspect that an equivalence between the temperature gradient and exponentially varying porosity can be established and an analytic approximation developed to determine if temperature gradient effects on impedance measurements are significant. This calculation is developed in Sec. III below.

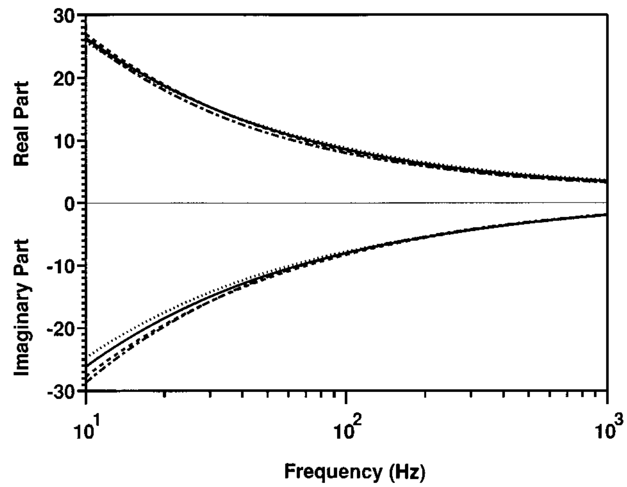

FIG. 1. Normalized surface impedance of the ground with no temperature gradient ———, a temperature gradient of 200 K/m for 10 cm – – –, a temperature gradient of −200 K/m for 10 cm ······, a temperature gradient of 200 K/m for 20 cm — - —.

III. APPROXIMATE FORMULATION FOR LOW FREQUENCIES

In this section an approximate expression is developed for the effect of a temperature gradient on the surface impedance of an homogeneous soil layer for low frequencies and for a semi-infinite temperature gradient. Approximations for $F(\lambda)$ have been used to develop expressions for the ground impedance of homogenous media for small thermal and viscous wave numbers.[12] For cylindrical pores

$$F(\lambda)=\left[\frac{4}{3}+\frac{i8}{\lambda^2}\right]^{-1} \tag{18}$$

and

$$k_0^2=\gamma\left(\frac{\omega}{c}\right)^2\left[\frac{4}{3}+\frac{\gamma-1}{\gamma}\,N_{\rm pr}+i\,\frac{8}{\lambda^2}\right]. \tag{19}$$

Further, $F(\lambda_T)$ can be written in terms of $F(\lambda)$ by noting $1-\sqrt{N_{\rm pr}}$ is small for air filled pores;

$$F(\lambda_T)=F(\sqrt{N_{\rm pr}}\lambda)$$
$$\cong F(\lambda)-\frac{dF(\lambda)}{d\lambda}\,(1-\sqrt{N_{\rm pr}})\lambda. \tag{20}$$

Substituting Eqs. (9), (19), and (20) in Eq. (8), using $\lambda=\lambda_0(T_0/T)^{\epsilon}$, and evaluating $dF(\lambda)/d\lambda$ in terms of $F(\lambda)$ results in

$$\frac{d^2\hat{p}}{dz^2}+\left[1-\left(\epsilon+\frac{1}{1+\sqrt{N_{\rm pr}}}\right)\frac{2}{[1-i(\lambda^2/6)]}\right]\frac{1}{T}\frac{dT}{dz}\frac{d\hat{p}}{dz}$$

$$+\gamma\left(\frac{\omega}{c_0}\right)^2\frac{T_0}{T}\left[\left(\frac{4}{3}-\frac{\gamma-1}{\gamma}N_{\rm pr}\right)+i\frac{8}{\lambda_0{}^2}\left(\frac{T}{T_0}\right)^{2\epsilon}\right]\hat{p}=0. \tag{21}$$

For low frequencies, λ is small and the terms $i\lambda^2/6$ and $\frac{4}{3}-(\gamma-1/\gamma)N_{\rm pr}$ can be dropped leaving:

$$\frac{d^2\hat{p}}{dz^2}+\left[1-2\left(\epsilon+\frac{1}{1+\sqrt{N_{\rm pr}}}\right)\right]\frac{1}{T}\frac{dT}{dz}\frac{d\hat{p}}{dz}$$

$$+i\gamma\left(\frac{\omega}{c_0}\right)^2\frac{8}{\lambda_0{}^2}\left(\frac{T}{T_0}\right)^{2\epsilon-1}\hat{p}=0. \tag{22}$$

Using Eq. (7), $1-2[\epsilon+1/(1+\sqrt{N_{\rm pr}})]=-1.925$ and $2\epsilon-1=0.849$. Note that the term containing $N_{\rm pr}$ is due to the thermoacoustic modification of the complex compressibility.

The substitutions $T=T_0(1+az)$ and $\zeta=(1+az)/a$, where a is the normalized temperature gradient in m^{-1}, lead to the differential equation and corresponding solution

$$\frac{d^2\hat{p}}{d\zeta^2}-\frac{1.925}{\zeta}\frac{d\hat{p}}{d\zeta}+k^2(0)a^{0.849}\zeta^{0.849}\hat{p}=0, \tag{23}$$

$$\hat{p}(z)=C\zeta^{1.46}H^{(1)}_{1.025}(\mathscr{K}\zeta^{1.425}), \tag{24}$$

where $\mathscr{K}=k(0)a^{0.4245}/1.425$ and C is a complex constant. Here, $H^{(1)}_{1.025}$ is chosen since we have assumed a semi-infinite gradient and radiation boundary conditions apply, i.e., $H^{(2)}_{1.025}$ becomes infinite as the depth increases. With the substitution $y=\mathscr{K}\zeta^{1.425}$, Eq. (24) is similar to Eq. (15) of Raspet *et al.*[14] and the normalized surface impedance can be derived from $\hat{p}(z)$ by following the procedure outlined in Ref. 14 yielding

$$z(0)=z_0\left[1+\frac{i1.5}{2k(0)}\frac{a}{q}\right]\cong z_0+\frac{i}{\gamma\Omega}\frac{3}{4}\frac{c_0}{\omega}\frac{a}{q}. \tag{25}$$

The reduction in gradient along the pore due to tortuosity is introduced in Eq. (25). z_0 is the surface impedance of an isothermal, uniform medium at the surface temperature. Equation (25) employs the low-frequency approximation that $z_0/k(0)=c_0/\gamma\Omega\omega$. Comparison with the result for exponential pores [Eq. (22), Ref. 14] shows that the semi-infinite linear normalized temperature gradient along the pores a/q is equivalent to a normal exponential porosity gradient α with a grain shape factor $n'=1$. Comparison of Eq. (25) with the exact results for a 40-K change in 20 cm from Eq. (16) showed good agreement.

The equivalence of the normalized temperature gradient and the exponential pore gradient is useful in establishing the minimum significant gradient at audio frequencies. Reference 14 demonstrates that the exponential porosity gradient corrections are significant at 100 Hz only when $\alpha\geqslant 10$, a factor of 20 larger than the gradient assumed in this paper. The equations above apply to any porous material (including those used for noise control) with viscous wave number, λ, less than one [see Eq. (5)]. Equation (25) shows that the critical variable for significance of the temperature gradients in such materials is the fractional temperature change per wavelength $c_0/\omega(a)$ compared to the normalized impedance z_0.

IV. CONCLUSIONS

The effect of uniform temperature changes in the ground can be calculated as modifying the dc flow resistivity and density [see Eq. (7) and Sec. II]. Realistic temperature gradients in homogeneous grounds lead to negligibly small changes in only the imaginary part of surface impedance. Temperature increases in the ground are analogous to decreasing exponential porosity grounds, while temperature decreases are analogous to increasing porosity grounds. An approximate formula for the effect of a linear temperature gradient on the surface impedance of the ground has been developed and is used to demonstrate that the effects are small for soils heated by the sun. Temperature gradient effects are measurable for low impedance surfaces as indicated by Eq. (25) and by the results of Reference 8. However, agricultural soils have too high a surface impedance for temperature gradient effects to be measurable.

ACKNOWLEDGMENTS

This work is supported in part by the U.S. Department of Agriculture National Sedimentation Laboratory. This work is an extension of research on thermoacoustic refrigeration supported by the Office of Naval Research.

[1] J. E. Piercy, T. F. W. Embleton, and L. C. Sutherland, ''Review of noise propagation in the atmosphere,'' J. Acoust. Soc. Am. **61**, 1403–1418 (1977).

[2] J. M. Sabatier, H. Hess, W. P. Arnott, K. Attenborough, M. J. M. Römkens, and E. H. Grissinger, ''*In situ* measurements of soil physical properties by acoustical techniques,'' Soil Sci. Soc. Am. J. **54**, 658–672 (1990).

[3] H. M. Hess, K. Attenborough, and N. W. Heap, ''Ground characterization by short-range propagation measurements,'' J. Acoust. Soc. Am. **87**, 1975–1986 (1990).

[4] D. Kirkham and W. L. Powers, *Advanced Soil Physics* (Wiley–Interscience, New York, 1972), Chap. 10.

[5] D. Hillel, *Fundamentals of Soil Physics* (Academic, San Diego, CA, 1980), Chap. 12.

[6] G. W. Swift, ''Thermoacoustic engines,'' J. Acoust. Soc. Am. **84**, 1145–1180 (1988).

[7] W. P. Arnott, H. E. Bass, and R. Raspet, ''General formulation of thermoacoustics for stacks having arbitrarily shaped pore cross sections,'' J. Acoust. Soc. Am. **90**, 3228–3237 (1991).

[8] W. P. Arnott, H. E. Bass, and R. Raspet, ''Specific acoustic impedance measurements of an air-filled thermoacoustic prime mover,'' J. Acoust. Soc. Am. **92**, 3432–3434 (1992).

[9] M. R. Stinson and Y. Champoux, ''Propagation of sound and the assignment of shape factors in model porous materials having simple pore geometries,'' J. Acoust. Soc. Am. **91**, 685–695 (1992).

[10] A. D. Pierce, *Acoustics, An Introduction to Its Physical Principles and Applications* (Acoustical Society of America, Woodbury, NY, 1989).

[11] A. A. Atchley, H. E. Bass, T. J. Hofler, and H. T. Lin, ''Study of a thermoacoustic prime mover below onset of self-oscillation,'' J. Acoust. Soc. Am. **91**, 734–743 (1992).

[12] K. Attenborough, ''Acoustical characteristics of rigid fibrous absorbents and granular materials,'' J. Acoust. Soc. Am. **73**, 785–789 (1983).

[13] K. Attenborough, ''Ground parameter information for propagation modeling,'' J. Acoust. Soc. Am. **92**, 418–427 (1992).

[14] R. Raspet and J. M. Sabatier, ''The surface impedance of grounds with exponential porosity profiles,'' J. Acoust. Soc. Am. **99**, 147–152 (1996).