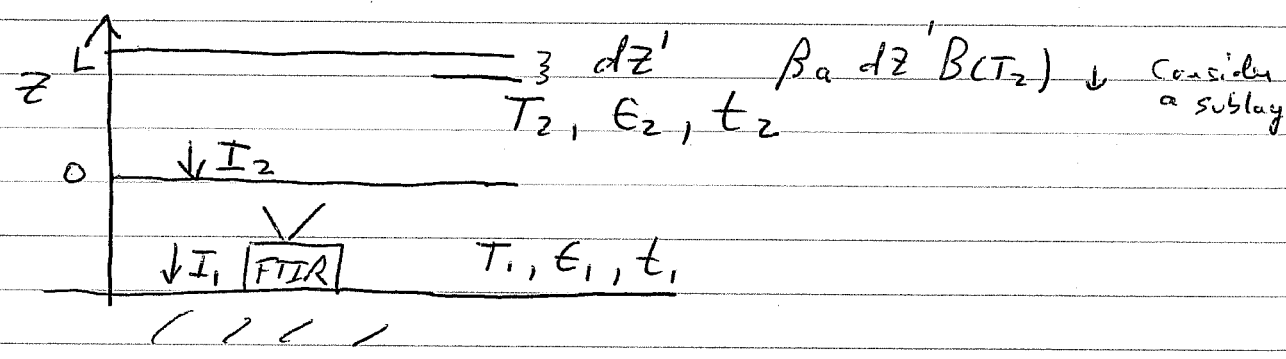


2 layer Idea for FTIR Spectra

(How to understand spectra for an atmosphere with a temperature inversion)

uninverted
layer 2

layer 1



From the Schwarzschild ϵ_L ,

$$t_2 = e^{-\int_0^L \beta_a(z') dz'} = e^{-\beta_{a,2} z'}$$

$$I_2^\downarrow(\nu) = \int_0^L t(z') \beta_a(z') B[T(z')] dz'$$

For layer 2, parameters are constant, so

$$I_2^\downarrow = \beta_a B(T_2) \int_0^L e^{-\beta_a z'} dz'$$

$$\underbrace{-\frac{1}{\beta_a} e^{-\beta_a z'}}_0^L$$

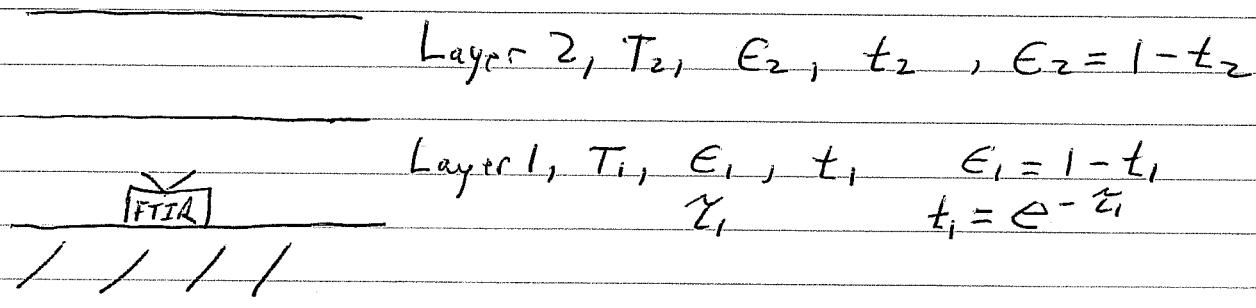
or

$$I_2^\downarrow = B(T_2) \underbrace{[1 - e^{-\beta_{a,2} L}]}_{\epsilon_2}$$

$$t_2 = e^{-\tau_2} \quad \epsilon_2 = 1 - t_2$$

2 Layer Idea for FTIR Spectra

(How to understand spectra for an atmosphere w/ temperature inversion)



$I = I(\nu) = \text{FTIR measurement of } \downarrow \text{ radiance}$

(1)
$$I = \epsilon_1 B(T_1) + \epsilon_2 B(T_2) t_1$$

Limits: ① $\epsilon_1 \approx 1 \quad t_1 \approx 0$

$$I \approx B(T_1) \quad \text{regardless of } T_2$$

② $\epsilon_1 \approx \epsilon_2 \approx t_1 \approx t_2, \quad T_2 \ll T_1$

$$I = \epsilon_1 B(T_1) + \epsilon_2 B(T_2) t_1$$

[Usual temperature profile spectra, $T_2 < T_1$]

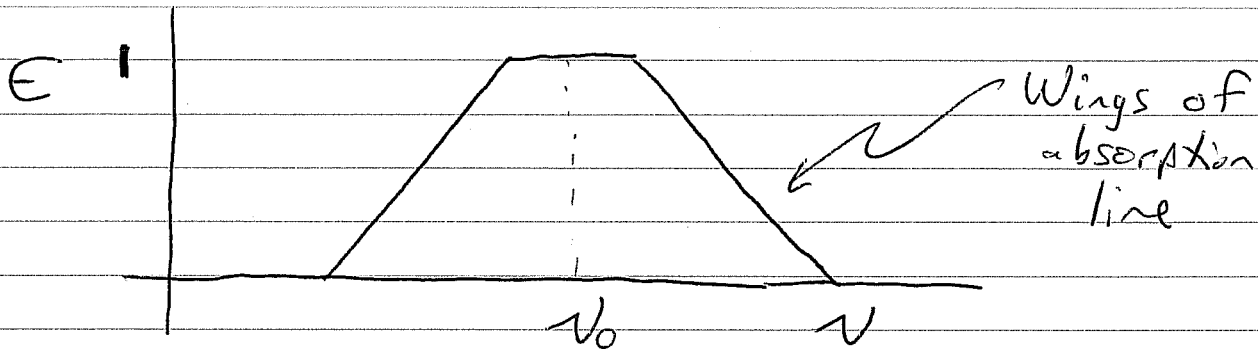
③ $\epsilon_1 \approx \epsilon_2 \approx t_1 \approx t_2 \quad T_2 \gg T_1$

$$I = \epsilon_1 B(T_1) + \epsilon_2 B(T_2) t_1$$

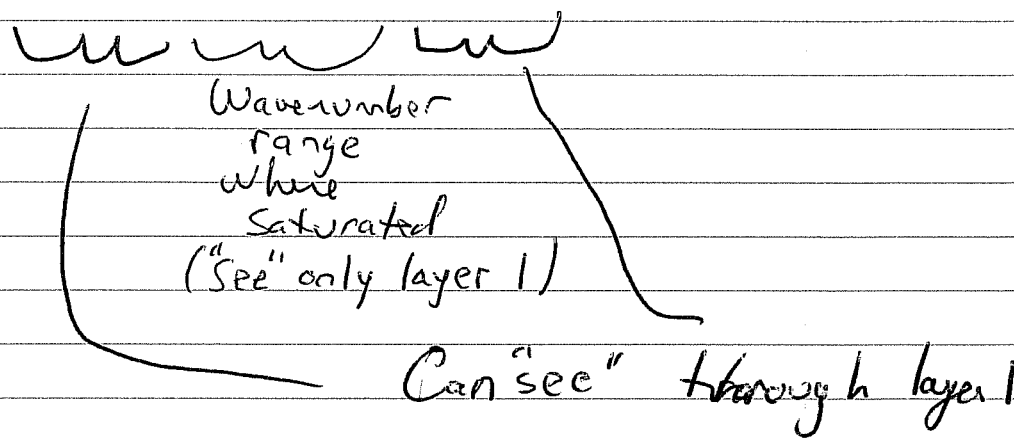
[temperature inversion]

All cases can be explored with a single strong line

Center of absorption line



Generic shape, could be region near strong absorption by CO₂ @ ~15 μm.



Does the simple model really work

Exercise: Suppose $T_1 = T_2 = T_0$, then should find $I = B(T_0) E_0$, $E_0 \equiv 1 - t_0$
 Show this → $t_0 = e^{-(\tau_1 + \tau_2)}$

Proof: $I = \epsilon_1 B(T_1) + \epsilon_2 B(T_2) t_1$

$$I = B(T_0) [\epsilon_1 + \epsilon_2 t_1]$$

$$= " [1 - t_1 + (1 - t_2) t_1]$$

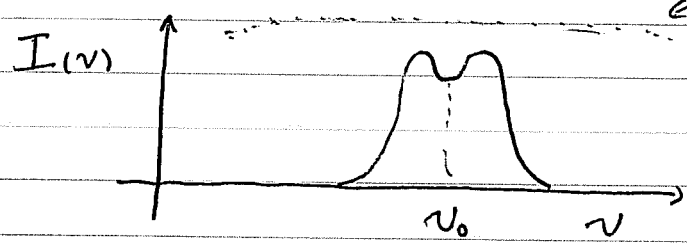
$$= " [1 - \cancel{t_1} + \cancel{t_1} - t_1 t_2]$$

Since $t_1 t_2 = e^{-\tau_1} e^{-\tau_2} = e^{-(\tau_1 + \tau_2)} = e^{-\tau_0}$

$$I = B(T_0) \underbrace{(1 - e^{-\tau_0})}_{E_0} = B(T_0) E_0$$

Condition for the Schawler Profile:

or Radiance for Blackbody - $\frac{5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}}{\text{m}^2 \text{ K}^{-4}}$



For understanding, consider a single strong spectral line at ν_0

ϵ_2, t_2, T_2
 $\epsilon_1, t_1, T_1 \downarrow I(\nu) = \epsilon_1 B(T_1) + \epsilon_2 B(T_2) t_1$
 / / /

@ ν_0 $\epsilon_1 = 1, t_1 = 0 \quad I = B(T_1) = B(T_1, \nu_0)$

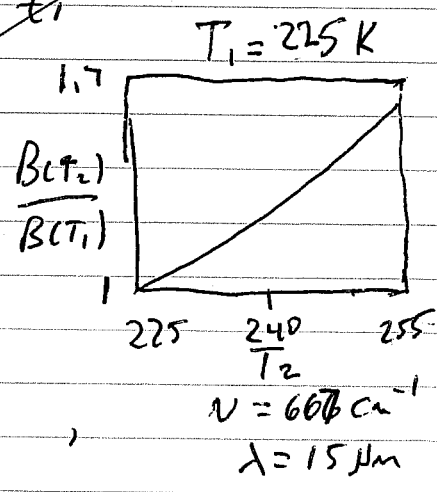
?? Away from ν_0 , what are the conditions for $I > B(T_1, \nu)$

$\epsilon_1 B(T_1, \nu) + \epsilon_2 B(T_2, \nu) t_1 > B(T_1, \nu_0)$

Considering $B(T_1, \nu) \approx B(T_1, \nu_0)$ in local area of ν ,

$\epsilon_2 B(T_2) t_1 > B(T_1) (1 - \epsilon_1)$

or $\frac{B(T_2)}{B(T_1)} > \frac{1}{\epsilon_2}$



Could solve by using

$\frac{B(T_2)}{B(T_1)} = \frac{e^{hc\nu/k_B T_1} - 1}{e^{hc\nu/k_B T_2} - 1}$