Efficient Prediction of Ground Surface Temperature and Moisture, With Inclusion of a Layer of Vegetation

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An efficient time-dependent equation for predicting ground surface temperature devised by Bhumralkar (1975) and Blackadar (1976) is tested against a 12-layer soil model and compared with five other approximate methods in current use. It is found to be generally superior if diurnal forcing is present and very much superior to the use of the insulated surface assumption. An analogous method of predicting ground surface moisture content is presented which allows the surface to become moist quickly during rainfall or to become drier than the bulk soil while evaporation occurs. These improved methods are not of much relevance unless the main influences of a vegetation layer are included. An efficient one-layer foliage parameterization is therefore developed that extends continuously from the case of no shielding of the ground by vegetation to complete shielding. It includes influences of both ground and foliage albedos and emissivities, net leaf area index, stomatal resistance, retained water on the foliage, and several other considerations. When it is tested against data for wheat measured by Penman and Long (1960), it appears quite adequate despite the many simplifying assumptions. The parameterization predicts that errors of up to a factor of 2 in evapotranspiration can be incurred by ignoring the presence of a vegetation layer.

1. INTRODUCTION

The determination of ground surface temperature within a numerical weather prediction model is usually accomplished by solution of a surface energy balance equation. A troublesome component, however, is the soil heat flux, which apparently requires the time-dependent solution for soil temperature within six or more layers of soil for reasonably good accuracy [Benoit, 1976]. Since the number of layers in the soil may then be comparable to the number of atmospheric layers comprising the model, abbreviations are usually sought for dealing with the soil heat flux. However, the abbreviations presently in use are very crude.

The prediction models of the Geophysical Fluid Dynamics Laboratory (GFDL) of Princeton have ignored the soil heat flux entirely [see Manabe et al., 1974], as has also a version of the UCLA two-level model [Gates, 1975]. The National Center for Atmospheric Research (NCAR) model attempts to improve upon this omission by assuming that the soil heat flux, \( G = -\lambda\partial T/\partial z \), is one third the sensible heat flux \( H_s \) to the atmosphere. Here \( \lambda \) is the soil thermal conductivity, \( T \) is temperature, and subscript zero refers to evaluation at the ground surface. (As is customary, \( G \) is defined as positive when directed downward; atmospheric fluxes will be defined as positive when directed upward.) A list of symbols used in this paper is found in the notation list at the end of this paper. The proportionality constant, \( k \), was chosen by Kasahara and Washington [1971] on the basis of the study of Sasamori [1970], who simulated a measurement period of the O'Neill experiment [Lettau and Davidson, 1957]. Another possibility is to assume that \( G \) is proportional to the net radiative flux \( R_{net} \): a proportionality constant of about -0.4 is suggested by the study of Idso et al. [1975a], while a value of -0.19 when the net radiative flux is downward and -0.32 when it is upward has been recommended by Nickerson and Smiley [1975] for O'Neill type conditions. However, since the negative soil heat flux equals the sum of all the atmospheric fluxes (as stated by the surface energy balance equation), any assumption that it is proportional to any particular component, or partial set of such components, seems dangerously nongeneral.

An entirely different approach to the soil heat flux has been proposed by Shaffer and Long [1975]. It is based upon the analytic solution to the diffusion equation [Carlslaw and Jaeger, 1959], which makes use of a time-weighted summation of all past values of \( G \) and theoretically requires that on each time step a fresh summation be constructed from all past values. In practice, a regrouping and truncation of terms within the summation is performed periodically. However, it is not clear if this method can be made as efficient as the use of, say, six soil layer temperatures with layer thickness increasing with depth while retaining high accuracy.

A method developed independently by Arakawa [1972] and by the British Meteorological Service [Corby et al., 1972; Rowntree, 1975] utilizes a rate equation for the ground surface temperature \( T_g \), dependent upon forcing by the sum of the atmospheric energy fluxes. It seems promising because it allows the temperature of a slab of soil to depend explicitly upon soil properties in the proper way; however, as was pointed out by Bhumralkar [1975], it omits the influence of soil heat flux on the underside of the slab.

Recently, Bhumralkar [1975] and Blackadar [1976] have independently proposed a method similar to the above \( T_g \) rate equation, which, however, contains the mechanism by which a deeper soil layer can influence the surface temperature. This method appears even more promising and is still much more efficient than the use of multiple soil layers.

With the multiplicity of methods presently in use it seems appropriate to point out their similarities and differences and to compare their accuracies for different types of soil.

The prediction of specific humidity of the air at the ground surface is in one respect usually treated slightly better than the prediction of \( T_g \) because an additional time-dependent equation for soil wetness is usually carried [Manabe, 1969; Washington and Williamson, 1977]. However, for purposes of short-range prediction this method also needs improvement. The evaporation rate at the ground, \( E_a \), is assumed to be given by

\[
E_a = \rho_a \alpha \rho H_a \left[ q_m(T_g) - q_a \right]
\]  
(1a)

\[
E_a = \rho_a \alpha \rho H_a (q_g - q_a)
\]  
(1b)

\[
\alpha = \min (1, W_s/W_a)
\]  
(1c)
where $W_s$ is the net soil moisture (depth of extracted liquid water) within a thick upper layer of soil, $W_a$ is the critical depth this layer is capable of holding before the surface is considered to act as if it were saturated, $q_{sa}$ is the saturation specific humidity at the ground surface temperature $T_g$ and pressure $p_a$, $q_s$ is the specific humidity of the air at height $z_a$ within the surface layer, and $q_g$ is the surface value of $q$. Also, $\rho_a$ is the air density, and $u_a$ is the wind speed at $z = z_a$, to which height the bare ground moisture or heat transfer coefficient $c_{wo}$ applies.

In conjunction with (1) the time-dependent equation for $W_s$ is

$$\frac{\partial W_s}{\partial t} = -\left(E_g - P\right)/\rho_w \quad 0 \leq W_s \leq W_{\text{max}} \quad (2)$$

where $P$ is the precipitation rate (mass of water exchanged per unit area and unit time), $\rho_w$ is the density of water, and $W_{\text{max}}$ is the maximum value of water depth, or field capacity, which exceeds $W_s$. (Runoff is considered to prevent $W_s$ from exceeding $W_{\text{max}}$.)

The definition of $q_g$ implied by (1) is

$$q_g = a q_{sa}(T_g) + (1 - a) q_s \quad (3a)$$

but an obvious restriction that should be recognized [Benoit, 1976; Rowntree, 1975] is

$$q_g \leq q_{sa}(T_g) \quad (3b)$$

The main shortcoming of (1) is that $E_g$ does not respond to short-period occurrences of precipitation and evaporation which only gradually change $W_s$ according to (2) and therefore only gradually change $q_g$ and $E_g$ according to (1). For example, a rainfall of 1 cm in 3 hours would increase $W_s$ from 4 to 5 cm, say, relative to a saturated value of 10 cm, while in this period $E_g$ as predicted from (1) would only increase 10% of the way toward its wet surface potential value. Instead, one would prefer a simple method which treats the actual surface and allows it to become saturated after only a short period of rainfall and to dry out substantially following evaporation. An accurate estimation of $E_g$ is very important for the prediction of $T_g$.

Finally, there is little knowledge of how accurate the prediction of ground temperature $T_g$ need be, when in actuality a complicated vegetative ground cover is usually present in the situation toward which the calculation of $T_g$ is applied. It might be thought that the GFDL method, which assumes an insulated lower boundary, effectively treats the very case of a foliage-covered ground surface and actually applies to the estimation of mean foliage temperature. However, this interpretation neglects the facts that (1) significant amounts of heat and moisture may diffuse from the ground up through the vegetative canopy, (2) the foliage transpires at a rate not closely dependent upon $W_s$, and (3) on a large scale a significant fraction of the surface, either ground or rocks, is usually exposed to solar and atmospheric radiation and is not totally shielded by vegetation.

The purpose of this paper is to overcome the shortcomings described above. Bhumralkar’s [1975] and Blackadar’s [1976] approximate method of calculating $T_g$ will be tested against the other methods which have been mentioned. An analogous approximate method for estimating soil surface moisture and $q_g$ will be presented and tested. Finally, a highly simplified parameterization of the influence of a vegetative layer will be presented, along with a comparison of how its inclusion affects $T_g$ and the net fluxes of heat and moisture to the atmosphere.

### 2. Brief Description of $T_g$ Prediction Methods

The various methods to be compared are designated and described in Table 1. In this table, $H_A = -G$ is the sum of the fluxes in the atmosphere at the ground, $\rho_s$ is the soil density, $c_s$ is the soil specific heat, $d_i = (\epsilon_i r_i)^{1/2}$ is proportional to the depth reached by the diurnal temperature wave, $\epsilon_i$ is the soil thermal diffusivity, $r_i$ is a period of 1 day, and $T_E$ is a deep soil temperature to be discussed later.

#### a. Multiple soil layer model.

In the comparative tests, $T_{\text{em}}$ from a 12-layer soil model will be considered the ‘true’ value of $T_g$ with which the value from the more approximate methods will be compared. The vertical coordinate transformation $\bar{T} = \ln \left(1 + z/\delta\right)$, $\delta = 1$ cm, is used, along with equal intervals in $\bar{T}$ of $\Delta \bar{T} = 0.385$. Soil grid points are located at depths of 0, 0.47, 1.11, 2.17, 3.66, 5.84, 9.05, 13.76, 20.69, 30.86, 45.80, 67.75, and 100.0 cm. Below the 100.0-cm depth the soil heat flux is assumed to be zero. The second-derivative soil diffusion terms are finite differenced by the Dufort-Frankel method by using a time step of $\Delta t = 15$ s. Only for the sake of accuracy is the $T_{\text{em}}$ time step made this small, and results were essentially the same as if second-order spatial differencing with a forward time step had been used. The soil flux, $G = -\lambda (\partial T/\partial z)_h$, is obtained from $-\lambda(T - T_{\text{em}})/z_i$, where $z_i = 0.47$ cm. No consideration is given to any heating effects of water phase change within the soil below the surface or heat transport by water movement.

#### b. Solution of surface energy balance equation.

This equation is

$$H_A = \epsilon_s \sigma T_E^4 + H_{Es} + L \cdot E_g$$

$$- (1 - \alpha_s) S^1 - \epsilon_s R_L^{-1} = -G \quad (4)$$

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### TABLE 1. Methods of Calculating $T_g$

<table>
<thead>
<tr>
<th>Method</th>
<th>Designation of $T_g$</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple (12) soil layers</td>
<td>$T_{\text{em}}$</td>
<td>Finite difference solution of diffusion equation for $T(z)$; $G = -\lambda (\partial T/\partial z)_h$</td>
<td>Carstlaw and Jaeger [1959]; Benoit [1976]</td>
</tr>
<tr>
<td>Insulated surface</td>
<td>$T_{st}$</td>
<td>$G = 0$</td>
<td>Gates et al. [1971]; Manabe et al. [1974]</td>
</tr>
<tr>
<td>$H_{Es}$ dependence</td>
<td>$T_E$</td>
<td>$G = 1/3H_{Es}$</td>
<td>Kasahara and Washington [1971]</td>
</tr>
<tr>
<td>$R_{net}$ dependence</td>
<td>$T_E$</td>
<td>$G = -0.19R_{net}$; $R_{net} &lt; 0$ (down); $G = -0.32R_{net}$; $R_{net} &gt; 0$ (up)</td>
<td>Nickerson and Smiley [1975]</td>
</tr>
<tr>
<td>$H_A$ forcing</td>
<td>$T_{ftr}$</td>
<td>$\delta T_f/\delta t = -2\pi^2 H_A/(\rho c_d d_i)$</td>
<td>Arakawa [1972]; Corby et al. [1972]; Rowntree [1975]</td>
</tr>
<tr>
<td>Force restore rate equation</td>
<td>$T_{ftr}$</td>
<td>$\delta T_f/\delta t = -2\pi^2 H_A/(\rho c_d d_i)$</td>
<td>Bhumralkar [1975], Blackadar [1976]</td>
</tr>
</tbody>
</table>
where $\varepsilon_A$ is the emissivity of the ground surface in the infrared, $\sigma$ is the Stefan-Boltzmann constant, $H_A$ is the sensible heat flux at the ground to the atmosphere, $L$ is the latent heat of condensation, $\alpha_A$ is the ground albedo, $S_i$ is the magnitude of the shortwave radiative flux, and $R_L$ is the downcoming longwave radiative flux. The second, third, and fourth methods listed in Table 1 replace $-G$ on the right of (4) with the indicated assumption and then solve for $T_d$, the last two methods also make use of $H_A$.

Within $H_A$, the three terms on the left depend upon $T_d$, and the two nonlinear terms, $\varepsilon_A T^4$ and $L \cdot E$, are linearized for convenience. That is, with superscript $(n)$ referring to the $n$th time step index, $[T_d^{(n+1)}]$ is approximated by

$$[T_d^{(n+1)}] = [T_d^{(n)}] + 4[T_d^{(n)}][T_d^{(n+1)} - T_d^{(n)}]$$

and

$$q_{sat}(T_d^{(n+1)}) = q_{sat}(T_d^{(n)}) + (\partial q_{sat}/\partial T)r_m^{(n)}[T_d^{(n+1)} - T_d^{(n)}]$$

where $\partial q_{sat}/\partial T$ is obtained from the Clausius-Clapeyron equation at $T = T_d^{(n)}$ and $q_{sat}(T_d^{(n)})$ is obtained from Tetens' [1930] equation. The linear solution of (4) for $T_d^{(n+1)}$, making use of the known value of $T_d^{(n)}$, is then straightforward.

In (4), $H_A$ is specified analogously to $E_r$ in (16):

$$H_A = \rho c_p c_s c_d H_d(T_s - T_d)$$

where $c_p$ is the specific heat at constant pressure and $T_s$ is the air temperature at height $z_s$ within the surface layer.

In the solar radiation term of (4), $S_i$ is assigned the time-dependent value appropriate to 45° latitude on March 21, attenuated by 15% (or more when clouds are specified to be present).

In the last term of (4), $R_L$ is prescribed by

$$R_L = [\alpha + (1 - \alpha)0.67(1670q_a^{0.6})]aTa$$

where $\alpha$ is the Stefan-Boltzman constant, $H_{sg}$ is the sensible heat flux of values at the $(n)$ and $(n + 1)$ time levels for $E_r$ being expressed by (8a) and (8b) except that he treated the temperature within a thin 1-cm slab of soil just below the surface layer. Instead, if his 1-cm slab thickness is denoted by $\delta$, say, and carried through to (15), then that equation reduces to (8) as $\delta \to 0$.

In using the $T_{gr}$ rate equation (8) the terms within $H_A$ which contain $T_{gr}$ are also linearized and treated by the Crank-Nicolson method: i.e., $T_{gr}$ values are expressed as the average of values at the $(n)$ and $(n + 1)$ time levels for $\partial T_{gr}/\partial t$ expressed by $(T_{gr}^{(n+1)} - T_{gr}^{(n)})/\Delta t$. The $H_A$ forcing rate equation for $T_{gr}$ in (7) is treated exactly the same as it is in (8), except that $c_1$ is taken as $\pi^{1/2}$ and $c_2$ as 0.

3. Results of Testing (8) and Other Approximate Methods for $T_d$

In the tests to be described in this section, (1), (2), and (3) were utilized to obtain $q_s$ and $E_r$; a modified method for obtaining $q_s$ is described in section 4. The value of $c_{ns}$ utilized in (1) and (5) was 0.0025.

Five different sets of soil parameters and other relevant values were stipulated, as indicated in Table 2. They are chosen to be roughly consistent with data presented by Sellers [1965], Lettau [1951], and Lettau and Davidson [1957].


In this set of tests the atmospheric temperature $T_a$ and specific humidity $q_s$ at $z = z_s$ were held constant at 280 K and $5 \times 10^{-4}$, respectively, except in case 5 of a simulated deep snow layer for which they were changed to 270 K and $3.0 \times$

### Table 2. Soil Parameters

<table>
<thead>
<tr>
<th>Property</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s$, cm s$^{-1}$</td>
<td>0.0040</td>
<td>0.0120</td>
<td>0.0020</td>
<td>0.0015</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\rho c_p$, cal cm$^{-2}$ K$^{-1}$</td>
<td>0.37</td>
<td>0.56</td>
<td>0.30</td>
<td>1.00</td>
<td>0.10</td>
</tr>
<tr>
<td>$\alpha_A$</td>
<td>0.90</td>
<td>0.95</td>
<td>0.80</td>
<td>0.94</td>
<td>0.90</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.25</td>
<td>0.15</td>
<td>0.40</td>
<td>0.10</td>
<td>0.65</td>
</tr>
<tr>
<td>$W_{sa}$, cm</td>
<td>2</td>
<td>8</td>
<td>(sat)</td>
<td>(sat)</td>
<td>(sat)</td>
</tr>
<tr>
<td>$W_{sa}$, cm</td>
<td>11</td>
<td>15</td>
<td>(12)</td>
<td>(sat)</td>
<td>(sat)</td>
</tr>
<tr>
<td>Description</td>
<td>O'Neil average</td>
<td>clay pasture</td>
<td>dry quartz sand</td>
<td>still muddy water</td>
<td>snow</td>
</tr>
</tbody>
</table>
Fig. 1. Ground surface temperatures calculated for the second 24 hours of a 2-day period as a function of the local time of day. The different methods listed in the upper left corner of Figure 1a are explained in Table 1, and the key for the different symbols is also in the upper left corner. Figures 1a–1e refer to cases 1–5, respectively, of Table 2 for constant atmospheric forcing except for variable solar radiation. Values are only plotted about every hour rather than every 10 min.

1892

DEARDORFF: SOIL TEMPERATURES

10−4. The wind speed $u_a$ was held constant at 4 m s−1. No clouds or precipitation were introduced. Initial soil temperatures, specified at about local sunset time, were assigned the value of $T_a$.

Predicted bare ground values of $T_{am}$, $T_{af}$, $T_{ac}$, $T_{at}$, $T_{ar}$, and $T_{air}$ are shown in Figures 1a–1e for a 1-day period with clear skies. A time step of 10 min was used (except for 15 s for $T_{am}$). Emphasis is given to the 10-min time step because that value is typical of the value used within a global circulation model.

In the figures we notice that $T_{af}$ precedes the true solution in phase by about an hour, while $T_{air}$ lags it by 1–2 hours. In cases 1–4, $T_{af}$ overshoots the peak amplitude by 2–6 K and $T_{air}$ by 1.5–2.0 K. The approximate methods have the most difficulty with case 2, for which the medium has the greatest values of $k_8$ and $\lambda$. In this case, $T_{af}$ overshoots the peak value by 3 K, $T_{air}$ overshoots it by 2 K, and $T_{air}$ undershoots it by 0.8 K. The approximate methods have difficulty in simulating the evening hours when $G$ is relatively most important and, except for $T_{air}$, predict $T_a$ values 1–2 K too low.

In case 4 (still muddy water) the occurrence of evaporation at the full potential rate somewhat moderates the diurnal amplitude and causes the various solutions to be more similar to each other than they would be otherwise. In case 5 (snow layer) all the approximate methods appear more satisfactory because of the melting temperature constraint. However, because of the insulating property of snow, $T_{sf}$ is in this one case (with $\Delta t = 10$ min) superior to the other approximate methods; $T_{sf}$ is then the least accurate.

The relative error

$$e_r = \left[\frac{(T_a - T_{am})}{\Delta T_{am}}\right]^{1/2}$$

during the second 24-hour period of the comparisons is presented in Table 3 for each of the approximate methods. The quantity $T_a$ in $e_r$ here stands for the calculated ground temperature by the approximate method tested, $T_{am}$ in $e_r$ always refers to the $T_{am}$ calculation with the 15-s time step, the angle brackets are the average over the second 24-hour period, and $\Delta T_{am}$ is the diurnal range in $T_{am}$. Overall, for a 10-min time step the error in $T_{air}$ is seen to be only about 0.62 as large as for $T_{am}$ or $T_{at}$ and only one third as large as for $T_{air}$. The results confirm the assertion of Bhumralkar [1975] that $T_{air}$ does not realistically reproduce the diurnal variation of $T_a$.

A test was made on the fidelity of $T_{am}$ with its 15-s time step by comparing it with the exact solution (11) for $G$ varying sinusoidally. The $e_r$ value for $T_{am}$ relative to $T_a$ (exact), after 9 days, averaged only 0.008 for the five different sets of soil properties, and extreme values of $T_{am}$ on the second day differed from those on the ninth day by no more than 0.68% of the diurnal range. Hence it is concluded that second-day values of $T_{am}$ were a viable standard against which to compare the more approximately calculated temperatures.

Tests were also conducted on the influence of the time step...
upon the approximate methods, and results are included in Table 3. As was pointed out by a reviewer, for constant atmospheric forcing but variable solar radiation the values of $T_{eq}$, $T_{eq}$, and $T_{eq}$ would reach maxima at the time of maximum solar radiation (noon) were it not for the linearization procedure already discussed. This procedure causes the calculated times of maxima to lag more and more as $\Delta t$ is increased. In addition, the soil heat flux assumptions involved in the $T_{eq}$ and $T_{eq}$ solutions also involve quantities evaluated on the previous time step, and this explains why the $T_{eq}$ and $T_{eq}$ maxima lag the maximum insolation more than the $T_{eq}$ maximum does. Thus it turned out that the $T_{eq}$ solution has least error for a time step between 10 and 30 min, while the $T_{eq}$ and $T_{eq}$ solutions have least error for $\Delta t \approx 10$ min. The $T_{eq}$ values are seen to retain superior accuracy even for large time steps.

Included in Table 3 are $e_r$ values for $\Delta t = 10$ min and for equatorial latitude. In this test the initial soil temperature and $T_{eq}$ were increased to 287 K. The $e_r$ values are little changed upon the approximate methods, and results are included in Table 3. As was pointed out by a reviewer, for constant atmospheric forcing the $e_r$ values were larger in all cases by an average factor of 1.37.

The steady state form of (8) was also tested in the five cases, indicated in the table, increased by up to 38%.

In other tests with a 10-min time step the assumption $G = -0.4R_{net}$ was tried, but this led to $e_r$ values about twice as large in cases 1, 3, and 4. Also, the assumption $G = -0.10R_{net}$ ($R_{net}$ incoming) and $-0.50R_{net}$ ($R_{net}$, outgoing) utilized by Gadd and Keers [1970] was tested, but $e_r$ values were larger in all cases by an average factor of 1.37.

The values listed first are for constant air properties; those in parentheses are for varying air properties.

### Table 3. Relative 24-Hour Averaged Error $e_r$ in Ground Temperature Calculated by the Five Approximate Methods for the Five Cases and Four Different Time Steps

<table>
<thead>
<tr>
<th>$\Delta t$, min</th>
<th>$T_{eq}$ Designation</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case Average Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$T_{eq}$</td>
<td>0.140(0.163)</td>
<td>0.276(0.294)</td>
<td>0.103(0.101)</td>
<td>0.138(0.161)</td>
<td>0.026(0.037)</td>
<td>0.137(0.151)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.075(0.079)</td>
<td>0.159(0.137)</td>
<td>0.057(0.054)</td>
<td>0.092(0.086)</td>
<td>0.038(0.024)</td>
<td>0.084(0.076)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.067(0.072)</td>
<td>0.128(0.112)</td>
<td>0.047(0.048)</td>
<td>0.077(0.070)</td>
<td>0.080(0.051)</td>
<td>0.080(0.071)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.143(0.133)</td>
<td>0.199(0.188)</td>
<td>0.118(0.087)</td>
<td>0.144(0.111)</td>
<td>0.060(0.070)</td>
<td>0.133(0.118)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.041(0.044)</td>
<td>0.062(0.061)</td>
<td>0.033(0.036)</td>
<td>0.040(0.041)</td>
<td>0.022(0.037)</td>
<td>0.040(0.044)</td>
</tr>
<tr>
<td>10</td>
<td>$T_{eq}$</td>
<td>0.134(0.149)</td>
<td>0.270(0.340)</td>
<td>0.097(0.121)</td>
<td>0.133(0.141)</td>
<td>0.026(0.049)</td>
<td>0.132(0.160)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.053(0.047)</td>
<td>0.121(0.138)</td>
<td>0.044(0.052)</td>
<td>0.076(0.074)</td>
<td>0.040(0.029)</td>
<td>0.077(0.068)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.043(0.041)</td>
<td>0.097(0.117)</td>
<td>0.029(0.045)</td>
<td>0.059(0.063)</td>
<td>0.081(0.054)</td>
<td>0.062(0.064)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.147(0.123)</td>
<td>0.203(0.194)</td>
<td>0.122(0.104)</td>
<td>0.149(0.121)</td>
<td>0.067(0.069)</td>
<td>0.138(0.122)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.043(0.039)</td>
<td>0.064(0.066)</td>
<td>0.035(0.039)</td>
<td>0.043(0.049)</td>
<td>0.029(0.030)</td>
<td>0.043(0.045)</td>
</tr>
<tr>
<td>30</td>
<td>$T_{eq}$</td>
<td>0.119(0.122)</td>
<td>0.251(0.347)</td>
<td>0.087(0.108)</td>
<td>0.119(0.129)</td>
<td>0.077(0.063)</td>
<td>0.131(0.178)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.101(0.097)</td>
<td>0.213(0.264)</td>
<td>0.083(0.079)</td>
<td>0.095(0.077)</td>
<td>0.112(0.118)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.116(0.117)</td>
<td>0.226(0.264)</td>
<td>0.074(0.078)</td>
<td>0.076(0.073)</td>
<td>0.113(0.074)</td>
<td>0.121(0.121)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.163(0.162)</td>
<td>0.218(0.282)</td>
<td>0.139(0.142)</td>
<td>0.166(0.144)</td>
<td>0.097(0.090)</td>
<td>0.157(0.164)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.056(0.059)</td>
<td>0.072(0.094)</td>
<td>0.050(0.056)</td>
<td>0.057(0.048)</td>
<td>0.063(0.059)</td>
<td>0.060(0.063)</td>
</tr>
<tr>
<td>60</td>
<td>$T_{eq}$</td>
<td>0.129(0.155)</td>
<td>0.236(0.386)</td>
<td>0.120(0.118)</td>
<td>0.125(0.163)</td>
<td>0.172(0.133)</td>
<td>0.156(0.191)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.215(0.211)</td>
<td>0.288(0.250)</td>
<td>0.156(0.137)</td>
<td>0.178(0.196)</td>
<td>0.188(0.137)</td>
<td>0.205(0.186)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.229(0.220)</td>
<td>0.287(0.256)</td>
<td>0.173(0.150)</td>
<td>0.174(0.190)</td>
<td>0.184(0.139)</td>
<td>0.209(0.191)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.192(0.176)</td>
<td>0.243(0.198)</td>
<td>0.169(0.150)</td>
<td>0.195(0.207)</td>
<td>0.160(0.118)</td>
<td>0.192(0.170)</td>
</tr>
<tr>
<td></td>
<td>$T_{eq}$</td>
<td>0.082(0.084)</td>
<td>0.093(0.089)</td>
<td>0.078(0.075)</td>
<td>0.086(0.100)</td>
<td>0.135(0.110)</td>
<td>0.094(0.091)</td>
</tr>
<tr>
<td></td>
<td>$\Delta T_{sm}$, K</td>
<td>22.5(26.1)</td>
<td>15.8(14.0)</td>
<td>24.4(25.2)</td>
<td>16.3(16.1)</td>
<td>5.9(6.8)</td>
<td></td>
</tr>
</tbody>
</table>

Values listed first are for constant air properties; those in parentheses are for varying air properties.
of the approximate methods is very similar to that portrayed in Figures 1a–1e so that cases 3–5 are not presented. $T_{rfr}$ values have degraded only very slightly and, on the average, remain superior to the other $T_{r}$ approximations tested for all values of the time step, as shown in Table 3 by the $e_r$ values given in parentheses. A new feature which these $e_r$ values disclose is that the relative error in $T_{st}$ in the presence of some random forcing but in the absence of diurnal forcing, increases with increasing $w$ in (13b), which corresponds to stage 2 of a drying soil [Idso et al., 1974] and to the soil moisture diffusivity's being an increasing function of soil moisture content. Equation (13a) corresponds to stage 1 (potential evaporation); (13b) corresponds to stage 3 (evaporation rate governed by vapor transfer and adsorption), except that (18b) is not really applicable [Philip, 1957]. It is nevertheless used even in

The same data show how strong the influence of the diurnal moisture cycle is upon the soil moisture fraction in the uppermost 5 mm of bare soil and provide justification for using the diurnal period in the denominator of the last term in (12). Earlier data of Schiff and Dreibelbis [1949] lack the very high resolution necessary for evaluation of the two constants but also suggest that $C_1$ is usually larger than $C_2$.

The time-dependent equation for $w_2$ is

$$\frac{\partial w_2}{\partial t} = - \frac{(E_g - P)}{\rho_w d'_s}$$

(12)

where $\rho_w$ is the density of liquid water, $C_1$ and $C_2$ are constants analogous to $c_1$ and $c_2$, $d'_s$ is a depth to which the diurnal soil moisture cycle extends, $w_2$ is the vertically averaged value of $w$ over a thicker layer $d'_s$ below which the moisture flux is negligible, and $w_{max}$ is the maximum value of $w_2$ which exceeds $w_9$. Runoff of precipitation reaching the ground is considered to occur when $w_2$ exceeds $w_{max}$.

Although the coefficients $C_1$ and $C_2$ are somewhat uncertain and depend upon soil type, the data of Jackson [1973] for Adelanto loam suggest that

$$C_1 = 0.5 \quad w_2/w_{max} \geq 0.75$$

$$C_1 = 14 - 22.5 \left[ (w_2/w_{max}) - 0.15 \right]$$

$$0.15 < w_2/w_{max} < 0.75$$

$$C_1 = 14 \quad w_2/w_{max} \leq 0.15$$

and

$$C_2 = 0.9$$

with

$$d'_s = 10 \text{ cm} \quad d'_s = 50 \text{ cm}$$

The form similar to (1a) which $E_g$ takes upon using (18a) is

$$E_g = \rho_w C_h \left[ q_{u,2}(T_g) - q_e \right]$$

(18b)

where

$$\alpha' = \min \left(1, \frac{w_2}{w_9} \right)$$

(18c)

For convenience, $d'_s$ in (15) has been taken to be constant and its dependence upon $w_9$ absorbed into $C_1$. Thus $C_1$ decreases with increasing $w_9$ in (13b), which corresponds to stage 2 of a drying soil [Idso et al., 1974] and to the soil moisture diffusivity's being an increasing function of soil moisture content. Equation (13a) corresponds to stage 1 (potential evaporation); (13b) corresponds to stage 3 (evaporation rate governed by vapor transfer and adsorption), except that (18b) is not really applicable [Philip, 1957]. It is nevertheless used even in
stage 3, since the evaporation rate is then very small and its absolute error of estimate is also small.

Equation (18a) does not give the unrealistic behavior that (3a) does of $q_a$ reaching its saturation value only when the mean soil moisture value $W_s$ reaches its saturated value. Instead, in (18a), local surface saturation can occur during moderate precipitation while $w_s$ is still far below its critical or saturated value. Following cessation of precipitation, $w_s$ in (12) starts dropping below $w_{max}$ owing to both evaporation and redistribution to the deeper soil and may even drop far below $w_s$ during a sunny afternoon because of evaporation.

Another advantage of using (12) is that the soil algae may be formulated to be dependent upon $w_s$; the dependency is much more accurate and unique if $w_s$ is used rather than $w_s$ [see Idso et al., 1975b].

It should be recognized that (18), although an improvement over (3a), still suffers the inconsistency that the height of evaluation of $q_a$ within the surface layer does not appear explicitly.

The behavior of (12)-(16) and (18) will be examined by utilizing this set of equations in place of (1)-(3). However, a demonstration of the behavior will be postponed until the latter part of the next section.

5. INCLUSION OF A LAYER OF VEGETATION

a. The assumptions and parameterization. A single layer of vegetation which has negligible heat capacity is assumed to be present. Its density will be characterized by the single quantity $\sigma_r$, which is an area average shielding factor associated with the degree to which the foliage prevents shortwave radiation from reaching the ground. The limits of $\sigma_r$ are $0 \leq \sigma_r \leq 1$, $\sigma_r = 0$ signifying no foliage and at = 1 signifying complete radiative clover 30 cm high, 0.83 for winter rye 80 cm high, 0.30 for the degree to which the foliage prevents shortwave radiation at, which is an area average shielding factor associated with young elms with dense undergrowth in summer, 0.50 for the present. Its density will be characterized by the single quantity vegetation which has negligible heat capacity is assumed to be

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represents the fraction of potential evaporation, \( \delta \) is a step function which is zero if condensation is occurring onto the leaf (if \( q_{fl} > q_{sat}(T) \)) and is unity otherwise, \( r_s \) is a generalized stomatal resistance and \( r_a \) is the 'atmospheric resistance.' \( W_{dew} \) is the mass of any liquid water retained on the foliage per unit ground area, and \( W_{dew,max} \) is the maximum value of \( W_{dew} \) beyond which runoff to the ground occurs.

The net foliage evaporation rate per unit horizontal ground area, \( E_f \), is then

\[
E_f = N E_{water} = N \rho_c v_{sat}[q_{sat}(T) - q_{fl}]r^* = r^*(E_f)_{pot}
\]

where \( (E_f)_{pot} \) is the potential (maximum possible) foliage evaporation rate. By definition of the leaf area index \( N \) the evaporative resistance of the canopy is only \( 1/N \) as large as that of the representative leaf.

With the inclusion of \( r^* \) in (25c), \( E_f \) may seem to be the transpiration rate when \( \delta = 1 \) and no dew or retained water is present, to be the rate of condensation onto the foliage when \( \delta = 0 \), and to be the rate of transpiration plus evaporation of dew when \( \delta = 1 \) and \( W_{dew} > 0 \). When evaporating dew is present, the fraction of the foliage surfaces coated with moisture is assumed to be given by \( (W_{dew}/W_{dew,max})^{1/2} \); the reason for the fractional exponent is explained later. For \( \delta = 1 \) and \( W_{dew} = 0 \) (the usual daytime case), (25c) and (25b) together represent the transpiration rate as formulated by Monteith and Szeicz.

The transpiration rate itself, per unit ground area, is seen to be given by

\[
E_tr = \delta_c (E_f)_{pot}[(r_s + r_a)]^{-1} - (W_{dew}/W_{dew,max})^{1/2}
\]

since the quantity in the second set of brackets represents the fractional foliage surface not covered by dew.

The facts that many types of leaves transpire from only the underside and that older leaves transpire less than newer ones are supposed to be accounted for by a representative choice of generalized stomatal resistance \( r_s \):

\[
r_s = 2.0 \text{ (s cm}^{-1})[S_{max}^{-1}/(S_1' + 0.03 S_{max}^{-1})] + S \text{ (s cm}^{-1})[w_{dew}/w_s]^{1/2}
\]

where \( S_{max}^{-1} \) is the maximum noon incoming solar radiation which can be achieved, \( S \) is a seasonal dependence, \( w_{dew} \), is a wilting point value of soil moisture relative to its saturated value, and \( w_s = 0.9 w_w + 0.1 w_d \) is a soil moisture value in the root zone assumed to lie closer to the bulk value \( w_b \) than to the surface \( w_s \). The specified dependence of \( r_s \) upon solar radiation is suggested from studies of Cline and Campbell [1976] for forests, Waggoner and Reifsnyder [1968] for certain crops, and Monteith et al. [1965] for barley. The minimum value of 2 s cm\(^{-1}\) is near the lower side of the range of values suggested by these investigators, by Fetcher [1976] for lodgepole pine, and by Monteith and Szeicz [1962] for various types of foliage when the difference between leaf resistance and canopy resistance is considered. The last term within the brackets of (27) is designed to give a strong enhancement to the transpirative resistance if \( w_d \) drops close to or below \( w_{dew} \) in magnitude. Besides the seasonal dependence, the two dependencies, daylight and soil moisture, appear to be the most important [Cline and Campbell, 1976] of many factors involved in the transpirative resistance. In temperate latitudes, \( S \) is set to zero during the growing season and to a value much larger than unity during the rest of the year.

With this approach of Monteith and Szeicz the atmospheric resistance is simply

\[
r_a = (c_f r_d)^{-1}
\]

since the transfer coefficient \( c_f \) is defined with respect to the mean flow inside the foliage layer rather than to the mean flow above the foliage.

The conservation equation for \( W_{dew} \) is taken to be

\[
\frac{\partial W_{dew}}{\partial t} = \sigma P - (E_f - E_{tr}) \quad 0 \leq W_{dew} \leq W_{max}
\]

since the difference, \( E_f - E_{tr} \), just represents evaporation or condensation of liquid water from or onto the foliage surfaces. When dew is evaporating, \( E_f - E_{tr} \) becomes \( (W_{dew}/W_{dew,max})^{1/2} \) \( (E_f)_{pot} \). The reason for specifying a fractional power dependence upon \( W_{dew}/W_{dew,max} \) is to allow dew to evaporate more rapidly than it does at an exponentially decreasing rate; for zero exponent it would evaporate much too fast with the implication that the dew is present in a continuous thin film over the entire leaf as evaporation proceeds. Formulations (25b) and (29) are a compromise between those two extreme positions and simulate the dew's occupying only a fraction of the leaf area during its evaporation and the entire area during its formation.

Condensation which may occur on the soil surface is not treated as dew but is simply added to the bulk soil moisture budget.

A gross energy budget for the foliage layer must be established in order to estimate \( T_f \). The values at the top of the canopy being denoted by subscript \( h \), those at the ground by subscript \( g \), and the direction of radiative fluxes by arrows, the assumption of no canopy heat storage leads to

\[
S_n + R_{hn} - S_h - R_{hn} = (S_t + R_{lt}) - (S_t + R_{lt}) \quad (30)
\]

where \( S \) is the shortwave and \( R \) the longwave flux, \( H \) the sensible heat, and \( E \) the evaportranspiration as seen at level \( z = h \). On the left side of (30), \( S_n \) and \( R_{hn} \) are assumed known, while by definition of \( \sigma_g \), \( S_h \) is given by

\[
S_h = (1 - \sigma_g) S_n \quad (31a)
\]

By definition of \( \sigma_g \) the reflected flux \( S_t' \) is given by

\[
S_t' = \sigma_g (1 - \sigma_g) S_n \quad (31b)
\]

The upward longwave flux just above the ground, \( R_{t_i} \), is obtained by interpolating with \( \sigma_i \) between the expression applicable above bare soil and that applicable just above soil overlain with a dense canopy:

\[
\begin{align*}
R_{t_i} &= (1 - \sigma_i)[\epsilon_t \sigma T_i^4 + (1 - \epsilon_t) R_{lh}^4] + \sigma_i \epsilon_t \sigma T_i^4 \quad (31c)
\end{align*}
\]

For \( \sigma_i = 1 \) this expression for \( R_{t_i} \) reverts to that for the radiative flux between two parallel surfaces [Fleagle and Businger, 1963] of emissivities \( \epsilon_t \) and \( \epsilon_i \). For \( \sigma_i = 0 \) the expression similarly applies for the upward reflection of \( R_{lh} \) from the ground when \( \epsilon_t < 1 \). The three remaining radiative fluxes are similarly obtained:

\[
\begin{align*}
S_h' &= (1 - \sigma_i) \alpha_g S_h + \sigma_i \alpha_t S_h' \quad (31d)
R_{lh}' &= (1 - \sigma_i)[\epsilon_t \sigma T_i^4 + (1 - \epsilon_t) R_{lh}^4] + \sigma_i [\epsilon_t \sigma T_i^4 + (1 - \epsilon_t) R_{lh}^4] \quad (31e)
R_{t_i}' &= (1 - \sigma_i) R_{t_i} + \sigma_i \epsilon_t \sigma T_i^4 + (1 - \epsilon_t) \epsilon_t \sigma T_i^4 \quad (31f)
\end{align*}
\]
With these substitutions of (31) into (30) and with the definitions

\[ H_{st} = H_{s} - H_{sg} \]

and

\[ E_{t} = E_{s} - E_{g} \]

(30) becomes

\[
\sigma_{t} \left[ (1 - \alpha_{g})S_{g}^{-1} + \frac{\epsilon_{r} \epsilon_{s}}{(\epsilon_{t} - \epsilon_{g})} \right] \sigma T_{g}^{4} - \frac{(\epsilon_{t} + \epsilon_{g} - \epsilon_{s})}{(\epsilon_{t} - \epsilon_{g})} \epsilon_{t} \sigma T_{t}^{4} = H_{st} + LE_{t} \]

(32)

which is to be solved for \( T_{g} \). It may be noted that \( \sigma_{g} \) in (32) can be cancelled from all terms (since the right-hand terms contain \( N \), which contains \( \sigma_{g} \)), allowing \( T_{g} \) to be obtained even in the limit of just no foliage. It may also be noticed from (31d) that \( \alpha_{g} \) in (32) is the foliage albedo as would be measured with a radiometer above a dense canopy, so that variable leaf angle relative to the sun and to the zenith is incorporated within its definition. For \( \epsilon_{t} \) and \( \epsilon_{g} \) close to unity the coefficient of \( \epsilon_{t} \sigma T_{t}^{4} \) term in (32) is about 2, which accords with the expectation that the foliage layer emits longwave radiation both upward and downward.

Within a sunlit canopy there will be a wide range of individual leaf temperatures differing by as much as 12 K [Miller, 1971], and the intent is for the leaf energy budget equation (32) to yield a representative value for \( T_{g} \), which taken along with the values for \( T_{r}, q_{r}, u_{r}, c_{r}, N, \) and \( r \) will yield the correct values for sensible heat flux and transpiration rate from foliage to air.

The foliage surface specific humidity \( q_{r} \) needed in (21b) is obtained from

\[ q_{r} = r' q_{sat}(T_{r}) + (1 - r') q_{at} \]

(33a)

with the usual restriction

\[ q_{r} \leq q_{sat}(T_{r}) \]

(33b)

Expression (33a) is derived by equating \( E_{r} \) from (25c) with the alternate expression involving \( q_{r} \):

\[ E_{r} = N \rho_{c} c_{v} u_{a}(q_{r} - q_{at}) \]

(34)

Because of the ground cover the soil energy balance of (4) must be modified in order to obtain \( T_{g} \) needed in (32). With \( H_{sg} \) denoting the sensible heat flux at the ground surface and \( E_{g} \) the evaporation rate there, we make the simplest possible generalization of (5) and (16):

\[ H_{sg} = \rho_{c} c_{v} u_{a}(T_{g} - T_{sg}) \]

(35a)

\[ E_{g} = \rho_{c} c_{v} u_{a}(q_{g} - q_{at}) \]

(35b)

Equation (35) makes use of the property that \( (c_{v} u_{a}, T_{sg}, q_{at}) \rightarrow (c_{v}, u_{a}, T_{g}, q_{at}) \) as \( \sigma_{g} \rightarrow 0 \); (35a) and (5) further neglect the small distinction between temperature and potential temperature differences within the foliage and atmospheric surface layers.

The ground surface energy balance then becomes

\[ -G = H_{sg} + LE_{g} - (1 - \alpha_{g})S_{g}^{-1} + R_{sg}^{-1} - R_{sg}^{-1} \]

(36)

with \( S_{g}^{-1} \) given by (31a), \( R_{sg}^{-1} \) by (31c), and \( R_{sg}^{-1} \) by (31f).

When the force restore method is used to get \( T_{g} \), the soil surface heat flux \( -G \) in (36) replaces \( H_{g} \) in (8) and in (9).

The soil properties used in (8) and (9) appear to depend more upon soil moisture than soil type, and a parameterized dependence for \( \lambda \) and \( \rho_{c} c_{v} \) upon \( w_{g} \) and \( w_{s} \) is therefore attempted, following Benoit [1976] and Sellers [1965]. At the ground surface and in the bulk mean, respectively, we parameterize \( \rho_{c} c_{v} \) by

\[ (\rho_{c})_{g} = 0.27 + w_{g} \quad \text{cal cm}^{-2} \text{K}^{-1} \]

(37)

\[ (\rho_{c})_{b} = 0.27 + w_{b} \quad \text{cal cm}^{-2} \text{K}^{-1} \]

and parameterize \( \lambda = \rho_{c} c_{v} \alpha_{g} \) by

\[ \lambda_{g} = 0.001 + 0.004(w_{g})^{1/2} \quad \text{cal cm}^{-1} \text{s}^{-1} \text{K}^{-1} \]

(38a)

\[ \lambda_{b} = 0.001 + 0.004(w_{b})^{1/2} \quad \text{cal cm}^{-1} \text{s}^{-1} \text{K}^{-1} \]

(38b)

Now, in (8) it is not clear how best to define an optimal value of \( \rho_{c} c_{v} \alpha_{g} \) when the ground surface soil properties differ significantly from the bulk soil properties. However, tests were made using the 12-layer soil model with \( w_{g}/w_{b} \) being either a factor of 5 greater than or smaller than unity and with a logarithmic distribution of \( w \) with depth (linear distribution in the coordinate \( \zeta \)). Improved results were obtained by defining

\[ r' = 0.30 + 0.05 w_{g}/w_{b} \quad 0.3 < r' \leq 1 \]

(39a)

and utilizing

\[ \rho_{c} c_{v} \alpha_{g} = r'(\rho_{c})_{b}(1 - r')\alpha_{g} \]

(39b)

Without this subparameterization and with only the use of the \( w_{g} \) moisture value, the root-mean-square error in \( T_{g} \) averaged over a day was 2.6 K relative to a diurnal range of 22.7 K when the surface was moist and 1.3 K relative to a diurnal range of 21.4 K when the surface was dry. With the parameterization these errors were reduced to 0.6 and 0.9 K, respectively. Much more work is needed on this subject, however.

The ground albedo \( \alpha_{g} \) is also made a function of ground surface moisture as follows:

\[ \alpha_{g} = 0.31 - 0.17 w_{g}/w_{b} \quad w_{g} \leq w_{b} \]

(40a)

\[ \alpha_{g} = 0.14 \quad w_{g} > w_{b} \]

(40b)

as suggested by the study of Idso et al. [1975b].

The quantity \( q_{g} \) needed in (35b) and in (21b) is obtained not from (18a) but from the slightly generalized equation

\[ q_{g} = c' q_{sat}(T_{g}) + (1 - c') q_{at} \]

(41a)

again provided that

\[ q_{g} \leq q_{sat}(T_{g}) \]

(41b)

The soil moisture budget equations are modified slightly from those in sections 1 and 4 to allow for the effect of transpiration:

\[ \frac{\partial w_{g}}{\partial t} = -C_{a}(E_{g} + 0.1 E_{tr} - P_{g})/(\rho_{g} \alpha_{g} \alpha_{d} \alpha_{t}) \]

(42)

where \( P_{g} \) is the precipitation rate felt at the ground surface:

\[ P_{g} = P(1 - \sigma_{g}) \quad W_{dew} < W_{dmax} \]

(43)

\[ P_{g} = P \quad W_{dew} \geq W_{dmax} \]

(44)
shown, along with extrapolated observed values denoted by crosses. The main purpose for the development of the equations in this section is to obtain modified expressions for the vertical fluxes from the ground foliage system to the atmosphere. These are the sensible heat flux $H_{sh} = H_{se} + H_{sr}$, given from (35a) and (22b) as

$$H_{sh} = \rho_a C_p c_h u_0 (T_a - T_d) + 1.1 N p_a C_p c_h u_0 (T_r - T_a) \tag{45}$$

the evapotranspiration rate $E_a = E_g + E_f$, given from (35b) and (25c) as

$$E_a = \rho_a C_p c_h u_0 (q_f - q_a) + N p_a C_p c_h u_0 (\text{qref}(T_f) - q_a) u' \tag{46}$$

and the upward directed shortwave radiative flux $S_u$ from (31d) and upward directed longwave flux $R_{lw}$ from (31e).

An outline of the method of solution which will be used for obtaining these vertical fluxes is as follows:

1. Specify $\sigma_f$, along with soil and foliage roughness, $\alpha_s$, initial conditions, above-canopy air properties, and incoming radiative fluxes or precipitation; calculate $N$ from (24) and $C_{wa}$ from (19).
2. Calculate $\alpha_s$ from (40), $u_{so}$ from (20), $c_f$ from (23), $r_a$ from (28), $r_f$ from (27), and $r'$ from (25b).
3. Using $q_{so}(T_f)$ and $q_{so}$ from the previous time step, diagnose $q_f$ from (33). Here a more economical method of calculating $q_{so}$ may be employed than that in section 2, where testing of a large time step precluded stepping along the Clausius-Clapeyron equation.
4. Diagnose $q_{so}$ from (21b) and $q_f$ from (41), the two being solved simultaneously, and $T_{so}$ from (21a). Previous values of $T_f$, $T_{so}$, $q_f$, and $q_{so}$ are utilized.
5. Diagnose $T_f$ from the foliage energy budget equation (32) by using a time step linearization of the outgoing longwave radiation term and foliage saturation humidity term as described in section 2.
6. Obtain $\rho_a C_d V_a$ from (37)-(39) and the components of $G_f$ from (31a), (31f), and (35), and then obtain $G_f$ from (36).
7. Predict updated values for $T_f$ by the force restore method (8), and $T_z$ from (9), by using $G_f$ from step 6.

8. Obtain $E_{so}$ from (26) and $L_f$ from (44), and then predict $w_e$ from (42), $w_z$ from (43), and $W_{dew}$ from (29).
9. Obtain the vertical fluxes to the atmosphere from (45), (46), (31d), and (31e).
10. Repeat steps 2-9 on subsequent time steps.

b. Test of the vegetation layer parameterization. A partial test is made by simulating the conditions reported by Penman and Long [1960] within a dense wheat crop in England on June 4-5, 1955. It is incomplete because measurements of $\sigma_f$, soil moisture, soil temperatures, mean foliage radiometric temperature, evapotranspiration, heat flux, and radiation balance during this period were not made or reported and because sampling errors of temperature and humidity within the crop can be appreciable.

There was apparently little or no cloudiness during this period, and solar radiation appropriate for latitude 51.8°N, attenuated by 15% to allow for absorption by ozone, water vapor, and dust, is prescribed. Parameters $w_e$ and $w_z$ are assumed to be 0.20 and 0.25 initially, $w_o$ to be 0.30, $w_{max}$ to be 1.33$w_o$, and $w_{min}$ to be 0.10. Soil properties are taken from (37) and (38) and, for $w_e = 0.25$, agree well with those reported by the authors for their site in 1956. Initially (hour 18), $T_{so}$ was taken as 286.6 K and $T_f$ as 282 K. $W_{dew}$ was set to 0.1$\sigma_f$ g cm$^{-2}$.

The wheat crop was extra thick and about 31 cm high. Although Penman and Long mention that it was typical of a canopy which completely shades the ground, it is assumed here, on the basis of earlier discussion of $\sigma_f$, that $\sigma_f$ was 0.75. The values of $c_{na}$ and $c_{wa}$, referred to a height of 2 m, are taken to be 0.0057 and 0.0096; the corresponding roughness lengths are 1 and 3 cm, respectively, with a zero displacement height.$z_o$ is the uppermost layer of relative moisture content $w_e$, only 0.1 moisture supply for transpiration is considered to lie beneath the uppermost layer of relative moisture content $w_e$, only 0.1 of $E_{tr}$, is allowed to influence $w_e$ in (42). In (43), moisture transpiring from the foliage is considered to be piped directly from the bulk layer of mean content $w_o$, while that condensing onto the foliage at night is assumed to remain in situ (unless $W_{dew} > W_{dew_{max}}$) until eventual evaporation.

Air temperatures, specific humidity, and wind speed at the $z_o$ (2 m) height for the 24-hour period beginning at 1800, June 4, are specified from Penman and Long's Figure 2. During this period, $u_o$ varied between 0.25 m s$^{-1}$ at night and 4.8 m s$^{-1}$ in the afternoon. To account for winds too weak to measure, $w_e$ is
constrained not to fall below 0.3 m s\(^{-1}\) and \(u_\infty\) not below 0.15 m s\(^{-1}\). A second 24-hour period is also treated with conditions identical to those of the first, except for simulation of 2 cm of rainfall between hours 18 and 22, accompanied then by overcast skies and a relative humidity of 95%. The numerical integration utilized a 10-min time step.

The diurnal variation of \(T_s\) calculated for the 2-day period for \(\sigma_t = 0.75\) is shown in Figure 3; included are calculated \(T_s\) values obtained for \(\sigma_t = 0, 0.5,\) and 1.0 for comparison, all else being held the same, and observational estimates from Penman and Long [1960]. The observed values (crosses) are in-canopy downward extrapolations from nickel resistance thermometer measurements at 5.0 and 30 cm except in three instances (circled crosses) which make use also of measurements at 2.5 and 7.5 cm. Although neither the model nor the measurements are so good that the crosses fall along any particular \(\sigma_t\) curve, the value of 0.75 selected for \(\sigma_t\) yields satisfactory \(T_s\) values overall. The observed maximum at hour 10 seems peculiar but could be associated with the microdensity of the wheat canopy in the immediate vicinity of the instruments in relation to the solar azimuth angle. The parameterization predicts a 2.5-fold reduction in the diurnal range of \(T_s\) from \(\sigma_t = 0\) to \(\sigma_t = 1\) and a corresponding lag in the time of the occurrence of peak ground temperature of about 1 hour. The maximum bare ground temperature is 5 K less on the second day than on the first because of the effect of simulated rainfall during the intervening evening.

The diurnal variations calculated for \(T_r, T_{ar}, T_s,\) and the prescribed variation of \(T_a\) are shown in Figure 4, and those for \(q_r, q_{ar}, q_s,\) and \(q_a\) are shown in Figure 5, all for \(\sigma_t = 0.75\).

The representative foliage temperature is seen to exceed substantially the maximum value which \(T_a\) reaches during midday but to be up to 5° cooler than \(T_a\) for bare soil. \(T_r\) exceeds the surrounding air temperature \(T_{ar}\) by up to 2 K. This result is consistent with data compiled by Geiger [1965, Figures 136 and 137] and observations [Waterhouse, 1955] which show the air near the top of a crop during the day to be warmer than either the air above or the ground below. The air must therefore have been heated by foliage considerably warmer than itself. However, this conclusion has been disputed by Lomas et al. [1971], and the difficulty in obtaining representative measurements is probably responsible for the uncertainty. If the model is rerun with the stomatal resistance increased by a factor of 100, but with all else the same, the maximum foliage temperature reaches a value 5.5 K warmer (12.0 K warmer for \(\sigma_t = 1\)), which is an example of ‘foliage fever.’

A downward jump in \(T_r\) is discernible in Figure 4 just following the simulated rainfall; this is associated with the decrease in downcoming longwave radiation after the skies clear.

Figure 5 shows that \(q_r\) becomes less than \(q_s\) and \(q_{ar}\) at night, thus promoting condensation onto the foliage, while \(q_r\) exceeds \(q_s\) during the early morning hours. The peak between hours 18 and 21 of June 5 is caused by the simulated rainfall which wets both the foliage and ground. The peak in \(q_t\) between 0700 and 0930 on June 6 is associated with warming of the extra-wet foliage before the retained water evaporates. Other wiggles are associated with variations in the observed values of \(q_r, T_a,\) and \(u_\infty\) utilized.

A sensitive measure of the performance of the parameterization is the temperature difference \(T_s - T_{ar}\), since \(T_s\) and \(T_{ar}\) each comprises rather independent elements of the model. This difference is shown in Figure 6 for \(\sigma_t = 0, 0.5,\) and 1.0 as well as for 0.75. Estimated values from the observations (crosses

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**Fig. 6.** Variation of \(T_s - T_{ar}\) during the 24-hour period from June 4 to 5, 1955, as calculated from the model for three values of \(\sigma_t\), curves, and observational estimates from Penman and Long [1960], crosses. Circled crosses are obtained from observations with greater vertical resolution within the canopy.

**Fig. 7.** Variation of the vapor pressure difference \(e - e_{mr}\) during the 24-hour period from June 4 to 5, 1955, as calculated from the model for \(\sigma_t = 0.75\), dash-dot curve, and observational estimates from Penman and Long [1960], crosses.

**Fig. 8.** Predicted variation of surface \((w_s)\) and bulk \((W_s)\) soil moisture, and of \(W_{aw}\) (right-hand scale) over the 2-day period which includes a 4-hour period of simulated rain, for \(\sigma_t = 0.75\).
and circled crosses) are also presented, with in-canopy measurements at a height of 30 cm utilized as a measure of $T_{sT}$. Although predicted differences are seen to be critically dependent upon $\sigma_f$, the constant value of $\sigma_f = 0.75$ captures the observed behavior fairly well in both phase and magnitude. We see that the sensible heat transfer from ground to canopy is positive at night and negative in the daytime, just opposite from the behavior experienced over bare ground.

A similar plot of the vapor pressure difference, $e - e_{st}$, is shown in Figure 7. Here the $\sigma_f = 0.75$ curve agrees even closer with the observations than could be expected. The positive values of the difference at night mean that part of the dew formation then comes from moisture supplied from the ground, a process called ‘distillation’ by Monteith [1957].

Predicted values of $w_d$, $w_s$, and $W_{dew}$ during the 48-hour period for $\sigma_f = 0.75$ are presented in Figure 8. Predicted dewfall commences at 2240 (sunset being calculated for 2016), and $W_{dew}$ reaches a maximum value of 0.08 mm per unit ground area by sunrise (predicted for 0408 h). Although the dewfall was not measured, Penman and Long stated that the night of June 4–5 at their site ‘was one of exceptionally intense dew formation,’ heavy dew being characterized by 0.15 mm or more [Long, 1958]. (For $\sigma_f = 1$ the predicted dewfall reached 0.16 mm.) Evaporation of the dew is predicted to have occurred by 0740 for $\sigma_f = 0.75$.

Although the ground is emitting moisture at night while the foliage is receiving it, the rate of dew accumulation is predicted to exceed $E_h$ more and more as the night proceeds. Shortly after onset of dew, $E_h$ exceeded the dew accumulation rate, but by 0400 the latter exceeded $E_h$ by a factor of 16. Distillation was therefore unimportant after the first hour of dewfall. If the model is rerun with all wind speeds doubled but all else the same, the maximum dew accumulation is 7% less. This result can be traced mainly to a potential for increased dewfall through increased turbulent mixing to be more than compensated by warmer foliage at night and a smaller difference in $\text{q}_{st} - \text{q}_{st}(T_f)$.

The ground surface moisture fraction, $w_s$ in Figure 8, is predicted to decrease considerably between 0800 and 1800 on June 5 owing to evapotranspiration, but the amount of decrease for bare soil under similar conditions is very marked. During the simulated rainy period, $w_s$ quickly increases beyond the saturated value. Following cessation of the rain the restoring term in (42) causes the soil surface moisture content to decrease slowly, as does evaporation the following day. The retained rainwater on the foliage starts disappearing after sunrise, but because of the increased amount in comparison with dewfall it is not predicted to vanish until 0940 on June 6.

The evapotranspiration rate $E_h$ of (46) is shown in Figure 9 for this case of $\sigma_f = 0.75$ and also for $\sigma_f = 0.0, 0.5,$ and 1.0. Unfortunately, there is no observational check from Penman and Long, except that the net value predicted over the first 24-hour interval, which is 0.43 cm, lies within the range of values 0.33–0.61 cm that they found using a large field balance during June 12–19, 1957, with generally clear skies for a wheat crop at the same site. It may be mentioned that the net 24-hour evapotranspiration is predicted to be 1.9 times larger for $\sigma_f = 1$ than for $\sigma_f = 0$, all else being held constant. This result follows from the enhanced dryness of the soil surface which develops during the day in the absence of foliage. To capture this effect, a model needs to include both a foliage parameterization and a prediction of soil surface moisture content as in (12). The skewness of the daytime June 5 $E_h$ curve for $\sigma_f = 0$ is also caused by the progressive drying of the soil surface and is typical of stage 2 evaporation [Idso et al., 1974].

If $w_s$ is doubled in this test of the model, while all else is being held the same, the net 24-hour evapotranspiration for $\sigma_f = 0.75$ is found to be increased from 0.43 to only 0.49 cm. This somewhat surprising result stems from the foliage surfaces being about 2 K cooler, owing to increased ventilation (and transpiration), and this leads to a correspondingly greater reduction in leaf surface saturation humidity due to its nonlinear temperature dependence.

If the stomatal resistance is halved and all else is held constant, the net 24-hour evapotranspiration for $\sigma_f = 0.75$ is found to increase from 0.43 to 0.51 cm. If $r_s$ is increased by 50%, the net 24-hour evapotranspiration is found to decrease from 0.43 to 0.38 cm.

For $\sigma_f = 0$ a pronounced peak in $E_h$ occurs during the simulated rainy period (Figure 9) when the ground becomes wet while it is still relatively warm. The bare soil evaporation rate is seen to be much increased on the day following the simulated rain, whereas the transpiration rate is little affected.
The sensible heat flux to the atmosphere, $H_{sn}$ in (45), is shown in Figure 10 for the same four values of $\sigma_t$. An upward spike occurs briefly at the beginning of the rainfall period, associated with increased downward longwave radiation to which the foliage quickly responds. If this rainfall is not simulated, the maximum sensible heat flux over the bare soil increases by 12% on the second day in comparison with the first and then exceeds $H_{sn}$ for $\sigma_t = 0.75$ by 78%.

The investigation of Penman and Long was not designed for the purpose of obtaining the sensible heat flux, and as a gross check of the $H_{sn}$ values obtained here the Bowen ratio ($H_{sn}/\Delta E_n$) is compared with Bowen ratios measured by Black and McNaughton [1971] above a young Douglas fir forest in British Columbia in summer. This comparison can be made because there is little in this parameterization to distinguish a field of grass from a forest. The present model is seen to predict Bowen ratios in rough agreement with those measured above the forest, as indicated by Figure 11. Daytime values of Bowen ratio near 3.4 over meadowland on clear days in spring and summer are reported by Geiger. One estimate of Bowen ratio from the data of Penman and Long can be made for 1000-1100 h, June 5, and is 0.48 ± 0.15; at that time the model gives a value of 0.62 for $\sigma_t = 0.75$. Hence the model prediction of $H_{sn}$ appears reasonable.

c. Discussion of vegetation parameterization. Although there are some existing foliage models [Paltridge, 1970; Waggoner and Reifsnyder, 1968], these are multilayer (e.g., eight layers within the foliage) and do not treat the fluxes from the soil surface underneath the canopy nor allow the canopy density to approach small values.

The present parameterization has purposely been kept extremely simple, from the viewpoint of an agricultural meteorologist, so that it might be useful to atmospheric modelers who cannot afford to become too entwined in a host of vegetative canopy details. Another reason for the simplicity is that the parameterization should be capable of working reasonably well for a given large-scale shielding factor, whether the foliage cover is homogeneous in space with $\sigma_t = 0.5$ or very dense over half the area and nonexistent over the other half. Because of the linear interpolations upon $\sigma_t$ built into this parameterization, it is subject to either interpretation of the foliage distribution.

The additional computer running time required of this parameterization is not great. The numerical program which involves prediction of the vegetation layer properties as well as of $T_{sir}$, $T_s$, $W_{bl}$, and $W_s$ was found to require only 1.5 times the computer time of a similar program stripped of all vegetative canopy and related statements. (Both programs were stripped of all superfluous or output statements.) The parameterization could therefore be made considerably more comprehensive without acquiring excessive computer requirements. Additional foliage variables requiring computer storage are $T_f$, $W_{brown}$, and $q_{at}$. Presumably $\sigma_t$ will also be a variable which could be made a function of season and location (or latitude, elevation, seasonal soil moisture, and land use).

It is perhaps worth mentioning some of the considerations omitted from this parameterization. These include effects of stratification or free convection upon both $C_{bl}$ and $C_{sh}$; influence of different types of canopy, for given $\sigma_t$, upon mean wind, temperature, humidity, thermal radiation, and leaf area index within the canopy; different rates at which air within the canopy may be modified in temperature and humidity; influence of solar elevation angle upon $\sigma_t$, $\alpha_f$, and $\alpha_s$; dependence of liquid water retention upon canopy height and type; influence of soil water potential and of atmospheric temperature and humidity upon stomatal resistance; influence of soil properties upon the prediction of $w_s$; interaction of snowfall with a foliage layer, and effects of a thin layer of organic litter atop the soil.

As the parameterization now stands, its chief components subject to tuning are two of the three factors which sum to unity in (21a) and (21b), the stomatal factor 2 s cm$^{-1}$ appearing in (27), the factor 0.83 in (20), the factor 7 for the maximum leaf area index in (24), and the values assigned to $\sigma_t$, $\alpha_f$, $W_{brown}$, and $w_{at}$. To a certain extent, different types or combinations of canopies could be represented by somewhat different values of these parameters.

6. SUMMARY AND CONCLUSIONS

Bhumalkar's [1975] and Blackadar's [1976] 'force restore' method of predicting ground surface temperature has been tested against five other approximate methods that are about equally efficient numerically and do not require calculation of multiple soil layer temperatures. In cases with substantial diurnal solar forcing this force restore method is found in general to be superior to the other methods. When the atmospheric forcing has a substantial random component to it, the insulated surface assumption is found to be especially poor. If no substantial diurnal forcing is present, the force restore method is still found to be superior to the insulated surface method. The procedure of relating the soil heat flux to either the sensible heat flux or the net radiative flux yields surface temperatures of intermediate but surprisingly acceptable accuracy if the time step does not exceed 10 or 20 min.

A method is presented for predicting ground surface moisture which is analogous to the force restore method of predicting surface temperature. The specific humidity at the surface is then related to the ground surface moisture content rather than to the bulk soil moisture content as is present practice. This permits evaporation to dry out the ground surface and so reduce the evaporation rate from bare soil in comparison with evapotranspiration. Also the method permits the new variable, ground surface moisture, to be treated with comparable care and time scale resolution as is afforded ground surface temperature.

A simple parameterization for a vegetation layer is developed and tested against observations of Penman and Long [1960] and others. It involves solution of an abbreviated energy budget equation to obtain the temperature of a representative foliage element and diagnosis of mean air temperature and humidity within the vegetation layer. The force restore method is utilized to calculate ground surface temperature and moisture. With no shielding of the ground from solar radiation ($\sigma_t = 0$), one recovers the bare soil values of ground temperature and surface heat flux. With complete shielding ($\sigma_t = 1$) the diurnal ground surface temperature wave is strongly damped but is still present because of heat and moisture exchange between the ground and the vegetation. With intermediate shielding of $\sigma_t = 0.75$ in simulation of the data of Penman and Long [1960], the difference between in-canopy and ground surface temperature and the corresponding difference in vapor pressure are predicted surprisingly well during the course of a day. The evapotranspiration rate above a dense foliage layer is predicted to exceed the bare soil evaporation rate by a factor of about 2, while the sensible heat flux in the afternoon over bare soil can typically exceed that over the foliage by a similar factor. There is therefore strong need to make use of a simple parameterization of the vegetation layer...
if one wishes to improve upon the method of calculating ground surface temperature and at the same time predict more accurate fluxes to the atmosphere.

The introduction of this vegetation parameterization does add some 50% to the computer time otherwise necessary for calculating the bulk plus ground surface values of temperature and moisture. However, this modest expansion of a minor part of an atmospheric prediction model seems fully warranted in view of the gross errors which can occur when the foliage layer is ignored. Introduction of a foliage layer which may have variable density and an albedo different from that of the ground seems mandatory for testing a climate theory like that of Charney et al. [1975], for example, which involves interactions between surface albedo, soil moisture, large-scale weather and precipitation, and crop type and amount. In their paper, Charney et al. do point out the need for including a model of the biosphere within the atmospheric model.

### Notation

**Arabic**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>specific heat in general.</td>
</tr>
<tr>
<td>c_f</td>
<td>dimensionless heat or moisture transfer coefficient for the foliage element.</td>
</tr>
<tr>
<td>c_H</td>
<td>dimensionless heat or moisture transfer coefficient applicable to bare soil, c_{H0} to soil under a canopy, c_{Hn} to the top of a dense canopy.</td>
</tr>
<tr>
<td>c_p</td>
<td>specific heat of air at constant pressure.</td>
</tr>
<tr>
<td>c_s</td>
<td>specific heat of soil.</td>
</tr>
<tr>
<td>c_1, c_2</td>
<td>constants in force restore rate equation (8) for ground surface temperature.</td>
</tr>
<tr>
<td>C_1, C_2</td>
<td>coefficients in the rate equation (12) for ground surface moisture.</td>
</tr>
<tr>
<td>d_i</td>
<td>a soil depth influenced by the diurnal temperature cycle, equal to (1 + z_i T_1)/2.</td>
</tr>
<tr>
<td>d_i'</td>
<td>a soil depth (10 cm) influenced by the diurnal soil moisture cycle.</td>
</tr>
<tr>
<td>d_a</td>
<td>a soil depth influenced by the annual temperature cycle, equal to 19.1 d_i.</td>
</tr>
<tr>
<td>d_s</td>
<td>a soil depth (50 cm) influenced by seasonal moisture variations.</td>
</tr>
<tr>
<td>e</td>
<td>vapor pressure.</td>
</tr>
<tr>
<td>E</td>
<td>evaporation rate in general.</td>
</tr>
<tr>
<td>(E_v)_opt</td>
<td>potential evaporation rate from foliage.</td>
</tr>
<tr>
<td>e_v</td>
<td>root-mean-square error of approximately calculated ground temperature, relative to the diurnal range.</td>
</tr>
<tr>
<td>E_v</td>
<td>foliage transpiration rate.</td>
</tr>
<tr>
<td>G</td>
<td>soil heat flux at the surface (positive when directed into the soil).</td>
</tr>
<tr>
<td>H_A</td>
<td>sum of fluxes to atmosphere (positive when directed upward).</td>
</tr>
<tr>
<td>H_r</td>
<td>sensible heat flux (positive when directed upward).</td>
</tr>
<tr>
<td>K</td>
<td>degrees Kelvin.</td>
</tr>
<tr>
<td>L</td>
<td>latent heat of vaporization.</td>
</tr>
<tr>
<td>n</td>
<td>time step index.</td>
</tr>
<tr>
<td>N</td>
<td>net leaf area index.</td>
</tr>
<tr>
<td>P</td>
<td>precipitation rate (mass per unit time and area).</td>
</tr>
<tr>
<td>q</td>
<td>specific humidity in general.</td>
</tr>
<tr>
<td>q_a</td>
<td>a mean specific humidity of air within a canopy.</td>
</tr>
<tr>
<td>q_{sat}(T)</td>
<td>saturation specific humidity at temperature T.</td>
</tr>
<tr>
<td>r_a</td>
<td>atmospheric resistance, equal to (c_{Hn})^{-1}.</td>
</tr>
<tr>
<td>r_s</td>
<td>generalized stomatal resistance (dimensions of inverse velocity).</td>
</tr>
<tr>
<td>r'</td>
<td>soil moisture interpolation factor.</td>
</tr>
</tbody>
</table>

**Greek**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>degree of bulk soil water saturation.</td>
</tr>
<tr>
<td>α'</td>
<td>degree of soil surface water saturation.</td>
</tr>
<tr>
<td>α_f</td>
<td>foliage albedo.</td>
</tr>
<tr>
<td>α_s</td>
<td>ground surface albedo.</td>
</tr>
<tr>
<td>δ</td>
<td>soil depth increment, equal to 1 cm.</td>
</tr>
<tr>
<td>δ_0</td>
<td>step function, equal to 1, except 0 during condensation.</td>
</tr>
<tr>
<td>Δ</td>
<td>a difference operator.</td>
</tr>
<tr>
<td>ε_f</td>
<td>foliage emissivity.</td>
</tr>
<tr>
<td>ε_s</td>
<td>ground surface emissivity.</td>
</tr>
<tr>
<td>ξ</td>
<td>transformed depth coordinate in soil, equal to ln (1 + z/δ).</td>
</tr>
<tr>
<td>κ_s</td>
<td>soil thermal diffusivity.</td>
</tr>
<tr>
<td>λ</td>
<td>thermal conductivity of soil.</td>
</tr>
<tr>
<td>ω</td>
<td>Stefan-Boltzmann constant.</td>
</tr>
<tr>
<td>σ</td>
<td>cloud fraction.</td>
</tr>
</tbody>
</table>
\( \sigma_t \) foliage shielding factor of ground from shortwave radiation, area average.

\( \tau_1 \) diurnal period.

\( \rho_a \) density of air.

\( \rho_s \) density of soil.

\( \rho_w \) density of water.

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References


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